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# Thermal Design and Optimization of Heat Sinks

J. Richard Culham



# Outline

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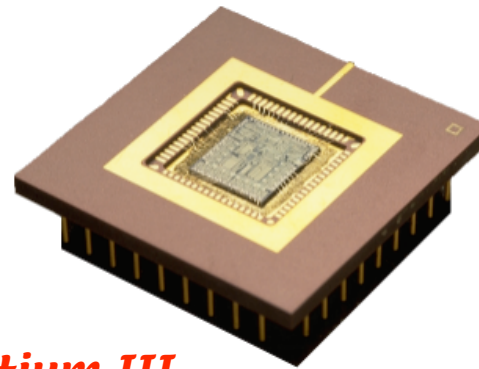
- Background
  - Modelling Approach
  - Validation
  - Optimization
  - Future Work
  - Summary
-

# 40 Watts! What's the big deal?



## *Light Bulb*

- Power: 40 W
- Area: 120 cm<sup>2</sup>
- Flux: 0.33 W/cm<sup>2</sup>



## *Pentium III*

- \* 0.25 micron CMOS technology
- \* 9.5 million transistors
- \* 450 - 550 MHz

## *Silicon*

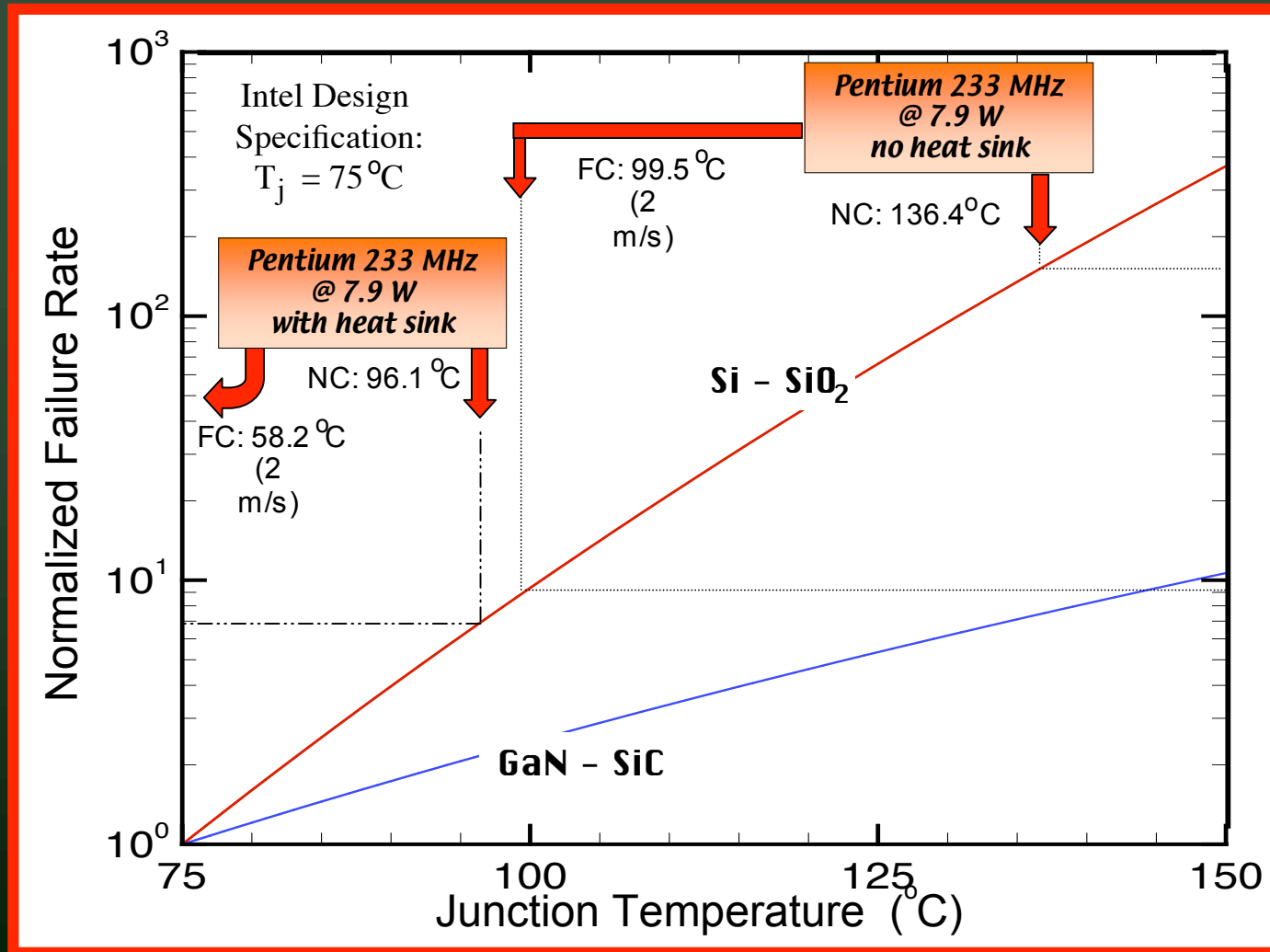
- Power: 40 W
- Area: 1.5 cm<sup>2</sup>
- Flux: 26.7 W/cm<sup>2</sup>

## *Package*

- Rj-c: 0.94 C/W
- Rj-a: 6.8 C/W (no heat sink)
- Rj-a: 2.5 C/W (heat sink)

↑ 80 x ↑

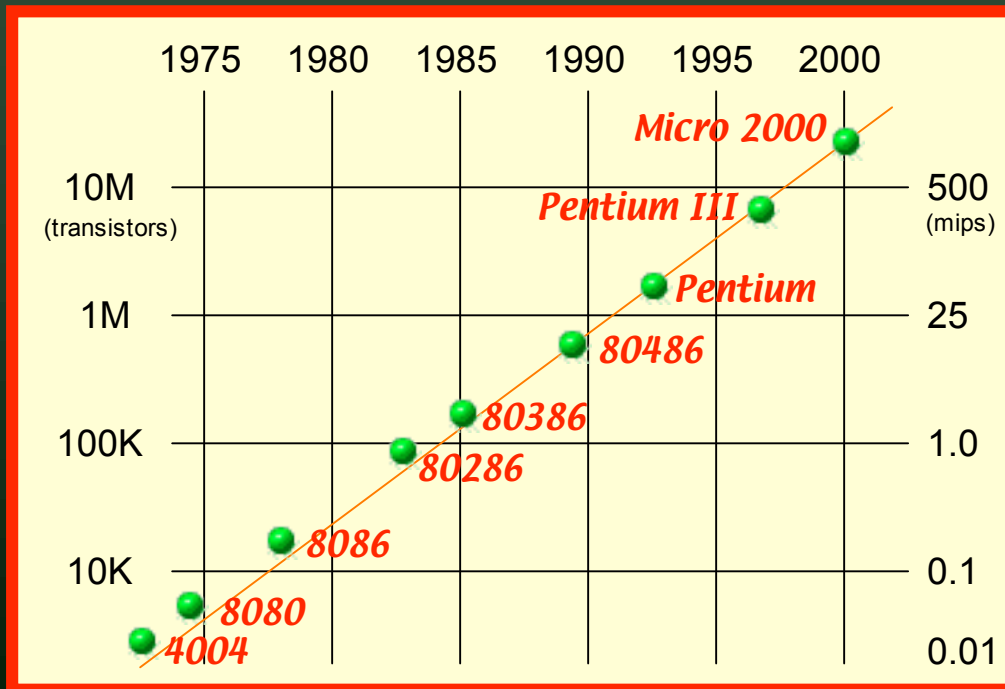
# Component Failure Rate





# Moore's Law (1965)

- each new chip contains roughly twice as much capacity as its predecessor
- a new generation of chips is released every 18 - 24 months



*From: [www.intel.com](http://www.intel.com)*

↳ *in 26 years, the population of transistors per chip has increased by 3,200 times*

# IC Trends: Past, Present & Future

	<b>1980</b>	<b>1999</b>	<b>2003</b>	<b>2006</b>	<b>2012</b>
<b>Comp. Per Chip</b>	0.2 M	6.2 M	18 M	39 M	100 M
<b>Frequency (MHz)</b>	5	1250	1500	3500	10000
<b>Chip Area (sq. cm)</b>	0.4	4.45	5.60	7.90	15.80
<b>Max. Power (W)</b>	5	90	130	160	175
<b>Junction Temp. (C)</b>	125	125	125	125	125

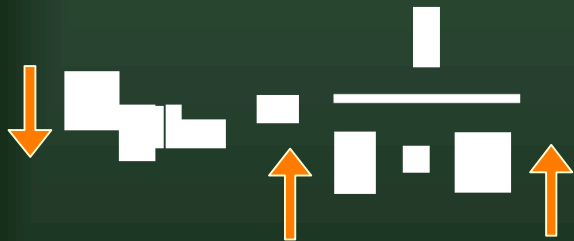
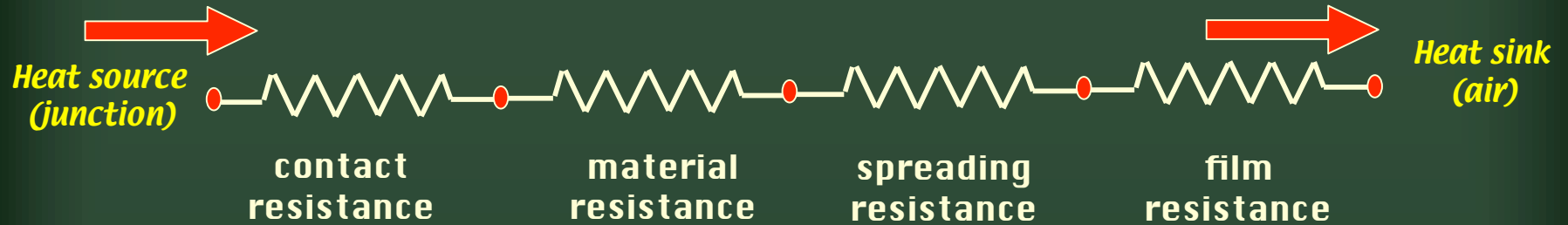
From: David L. Blackburn, NIST

# Why Use Natural Convection?

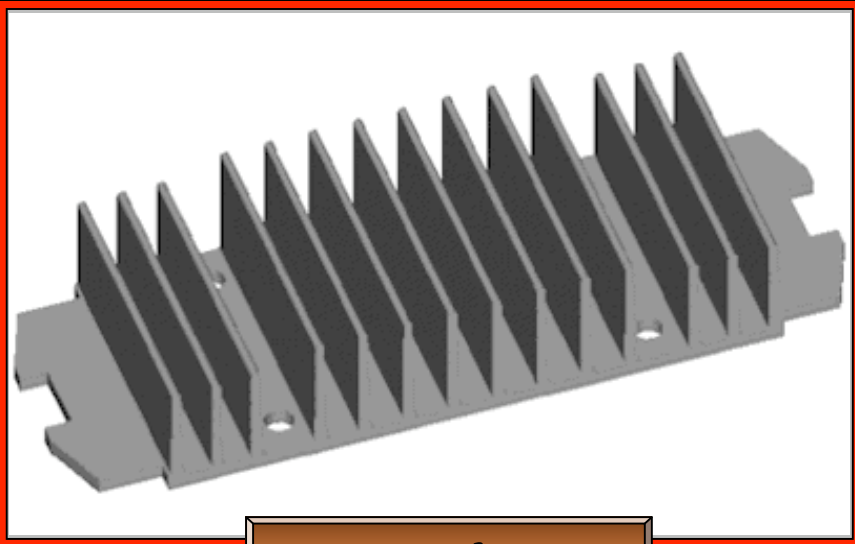
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- simplicity:
    - ↳ low maintenance
    - ↳ lower power consumption
    - ↳ less space (notebook computers)
  - less noise
  - fail safe heat transfer condition
-

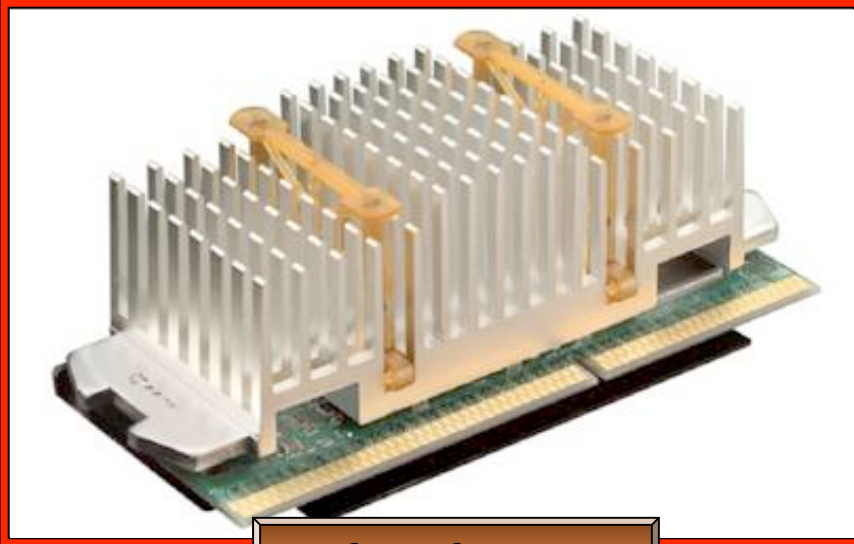
# Thermal Resistance



- increased heat transfer coefficient
  - ↳ immersion cooling (boiling)
  - ↳ impingement cooling
  - ↳ forced air
  - ↳ natural convection
- increased surface area
  - ↳ spreaders
  - ↳ heat sinks

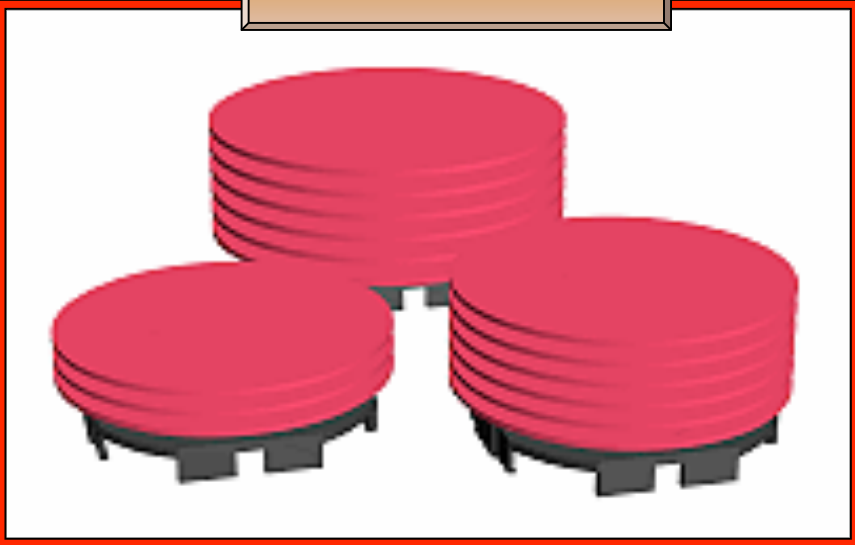


*Plate Fin H.S.*

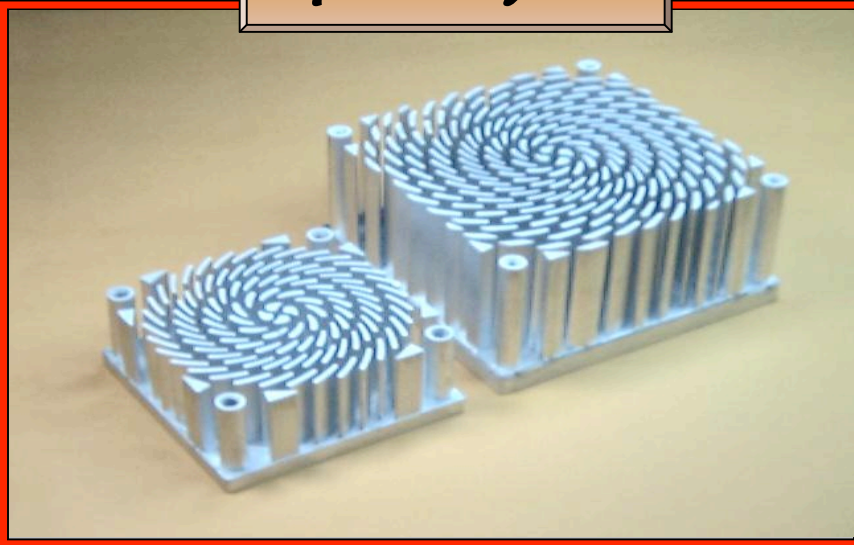


*Pin Fin H.S.*

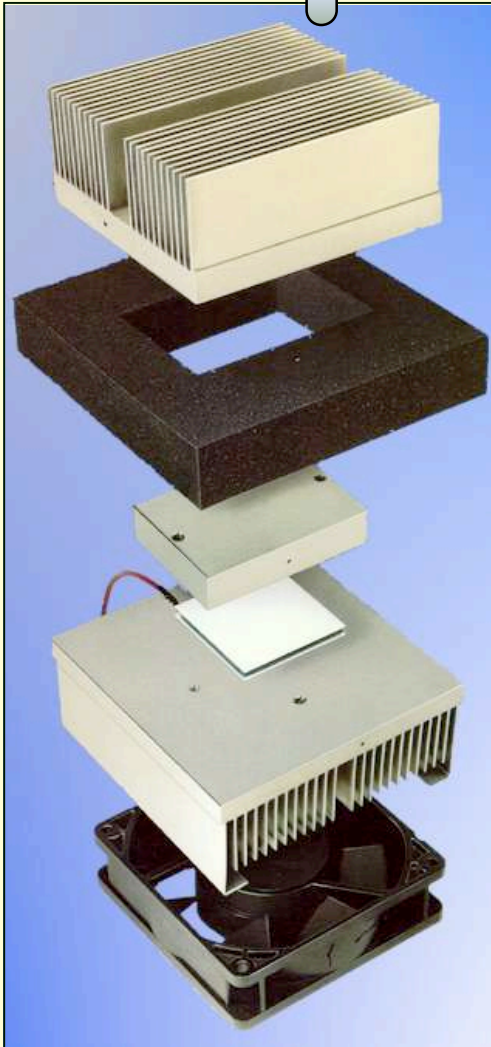
*Radial Fin H.S.*



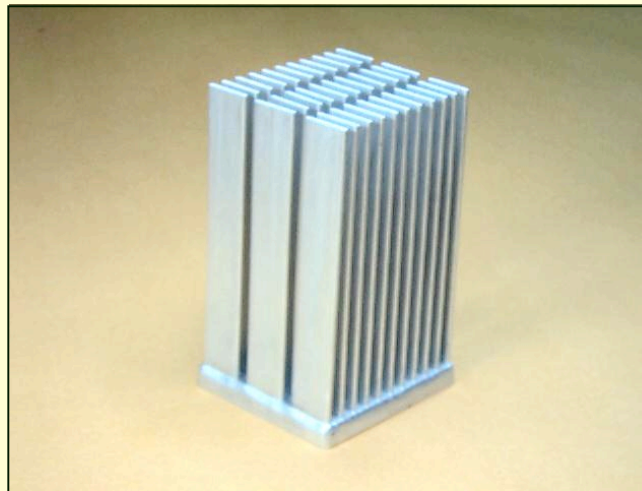
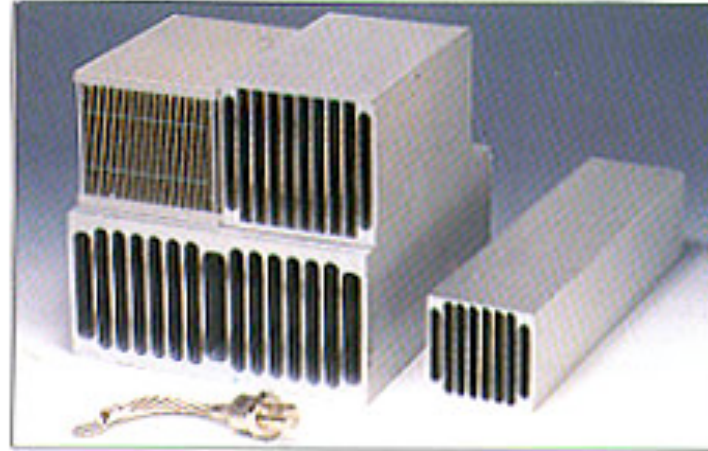
*Specialty H.S.*



# Plate Fin Heat Sinks

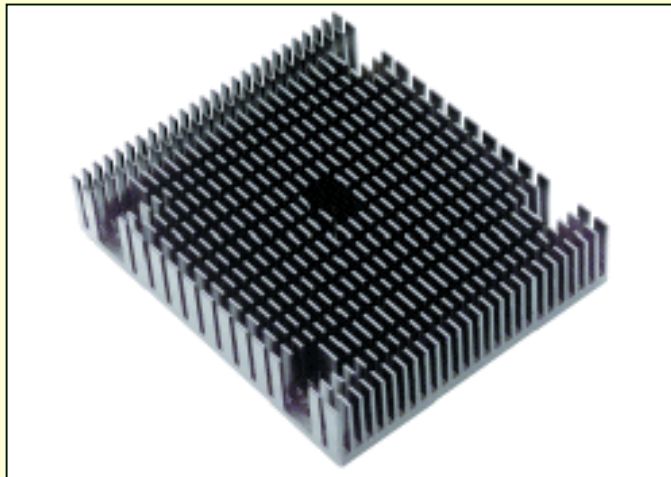
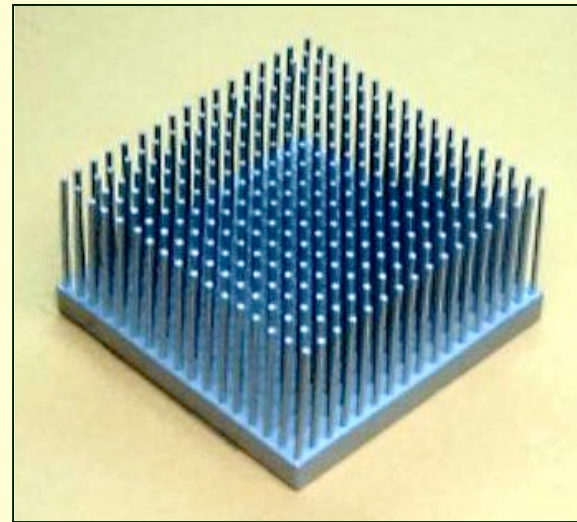
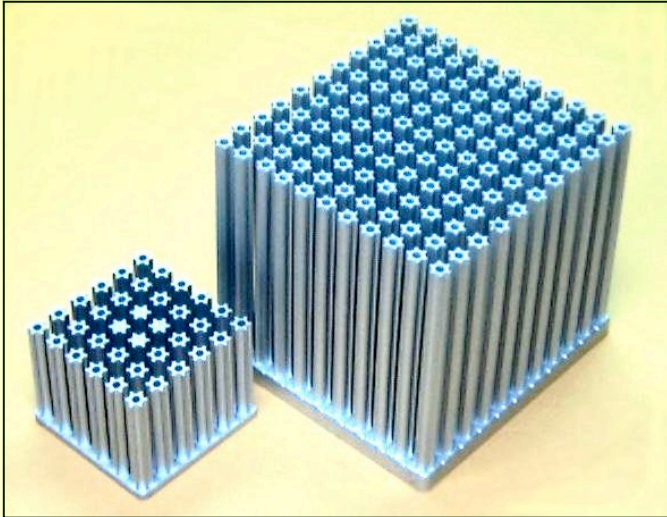


UPS, AVR, SUBWAY HEAT SINK





# Pin Fin Heat Sinks

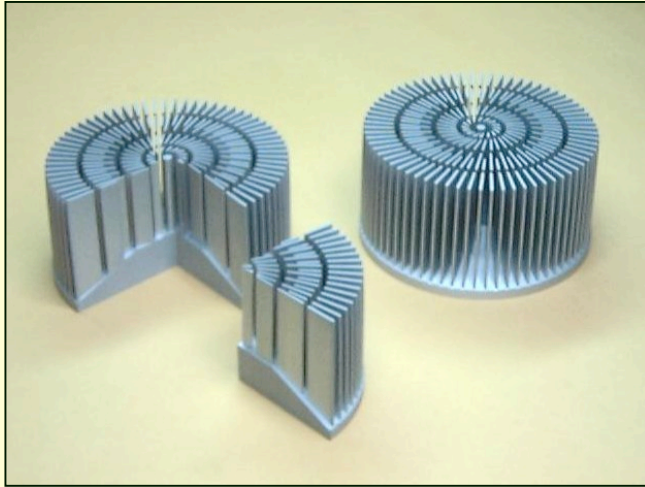


# Radial Fin Heat Sinks

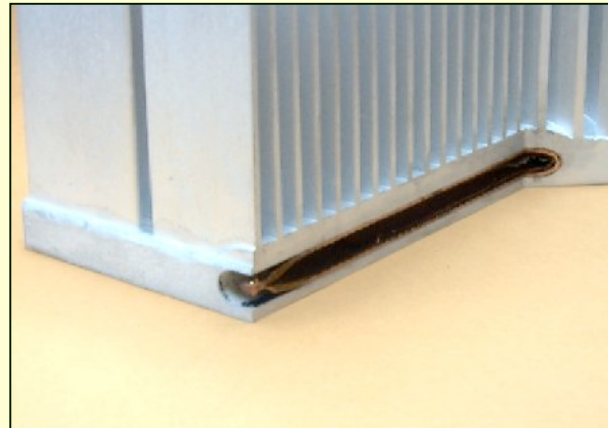
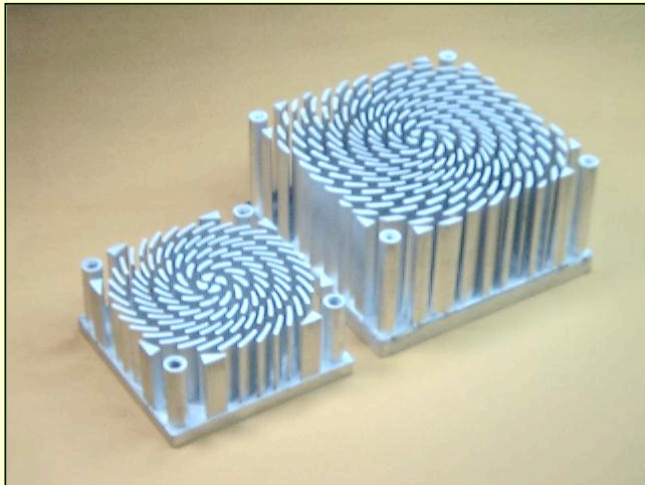
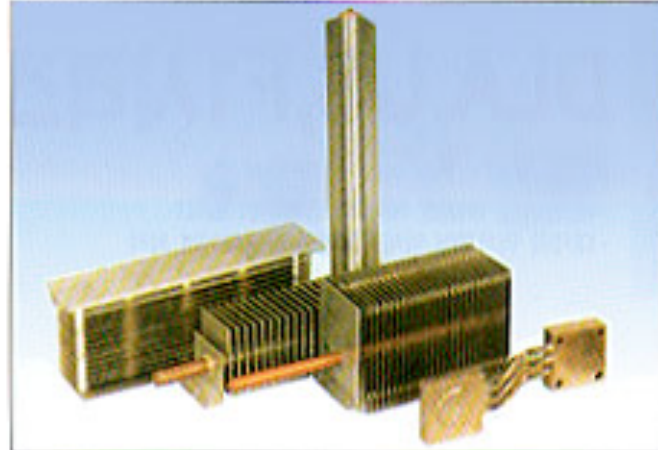




# Specialty Heat Sinks



**HEAT PIPE HEAT SINK**

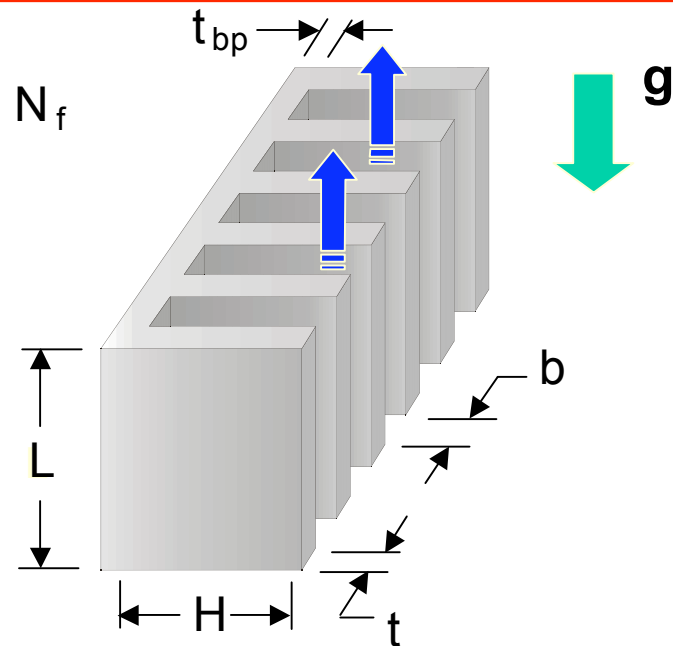


# Heat Sink Model

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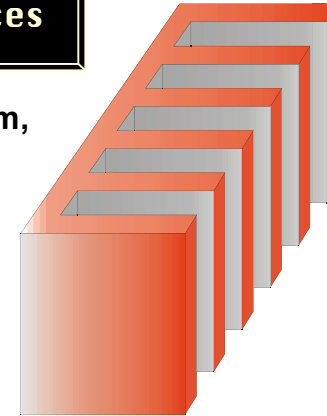
- Plate fin heat sink
  - Natural convection
  - Isothermal
  - Steady state
  - Working fluid is air i.e.  $Pr = 0.71$
-

# Modelling Procedure



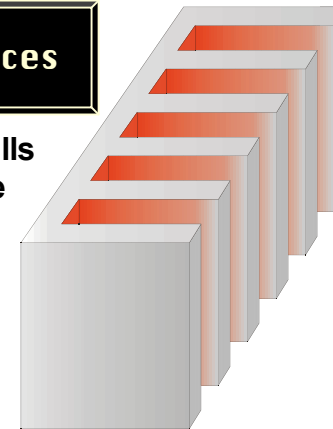
## Exterior surfaces

- fins : top, bottom, ends & tip
- base plate: top, bottom, ends and back



## Interior surfaces

- fins : side walls
- channel base

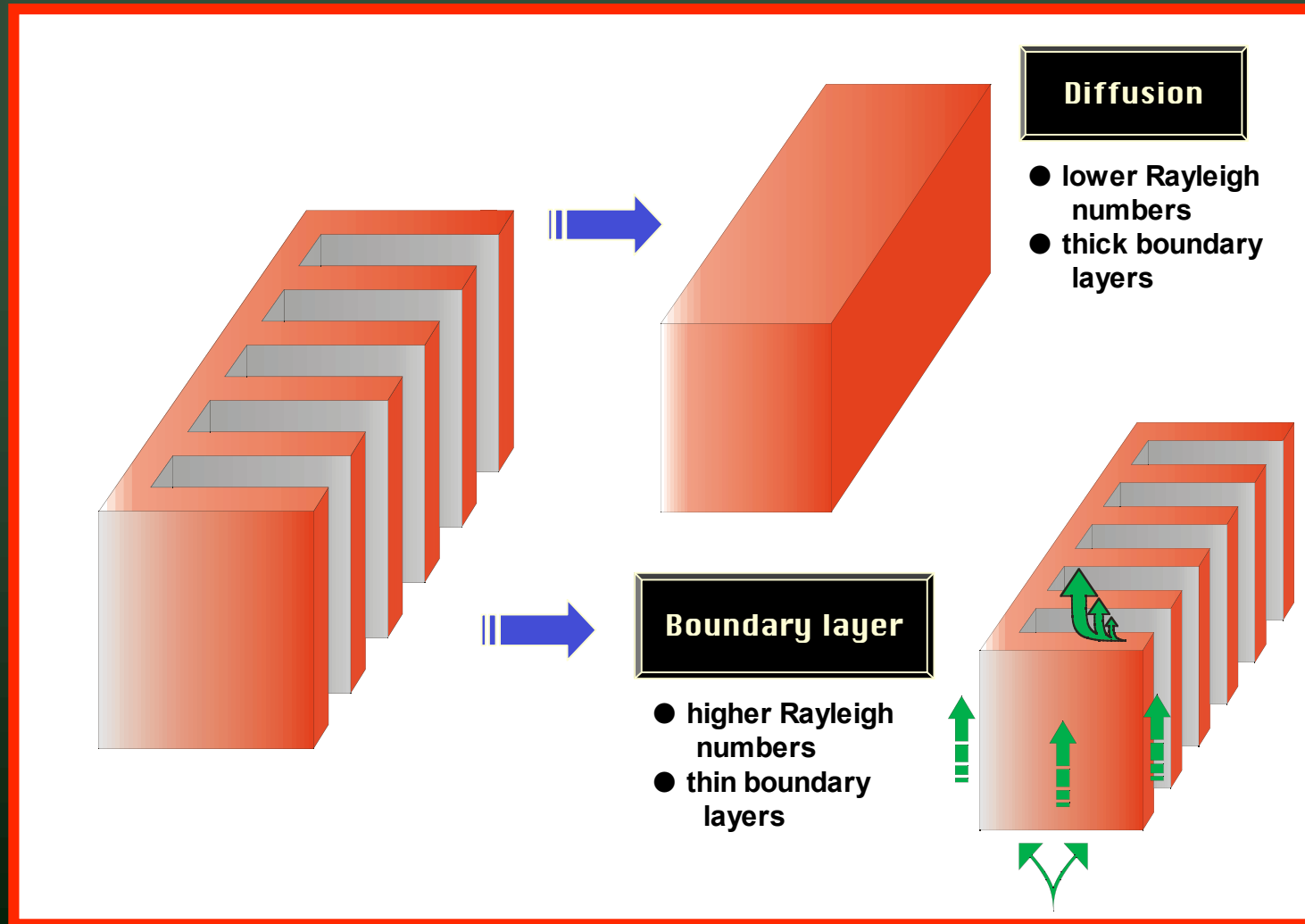


**Given:** dimensions & temperature

**Find**  $Nu_b$  vs.  $Ra_b$

$$: \quad \frac{hb}{k_f} = \frac{g\beta\Delta T b^3}{\alpha\nu} \cdot \frac{b}{L}$$

# Exterior Surfaces



# Diffusion Model

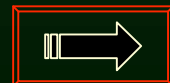
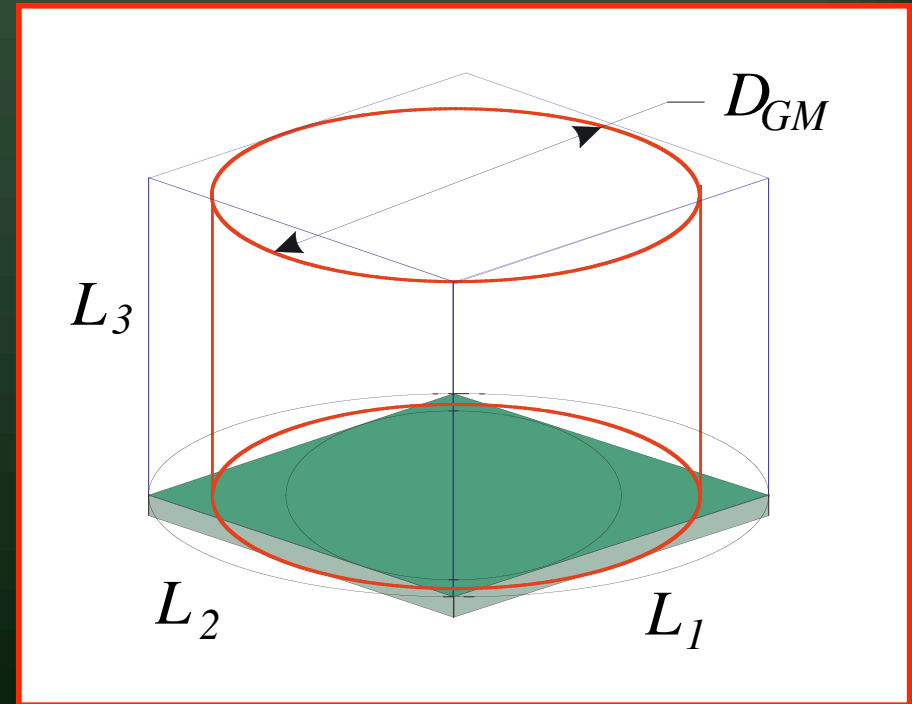
$$\frac{1}{\sqrt{4\pi D t}} \exp\left(-\frac{x^2}{4Dt}\right)$$

$$\frac{1}{\sqrt{4\pi D t}}$$

$$\frac{\sqrt{4\pi D t}}{\sqrt{4\pi D t}}$$

$$\frac{1}{\sqrt{4\pi D t}}$$

$$\frac{\sqrt{4\pi D t}}{\sqrt{4\pi D t}}$$



# Exterior Boundary Layer Model

$$\frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) - \frac{d}{dx} \left( \int_0^{\delta} u v dy \right) = \nu \frac{d^2 u}{dx^2} \Big|_{y=0}$$

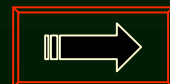
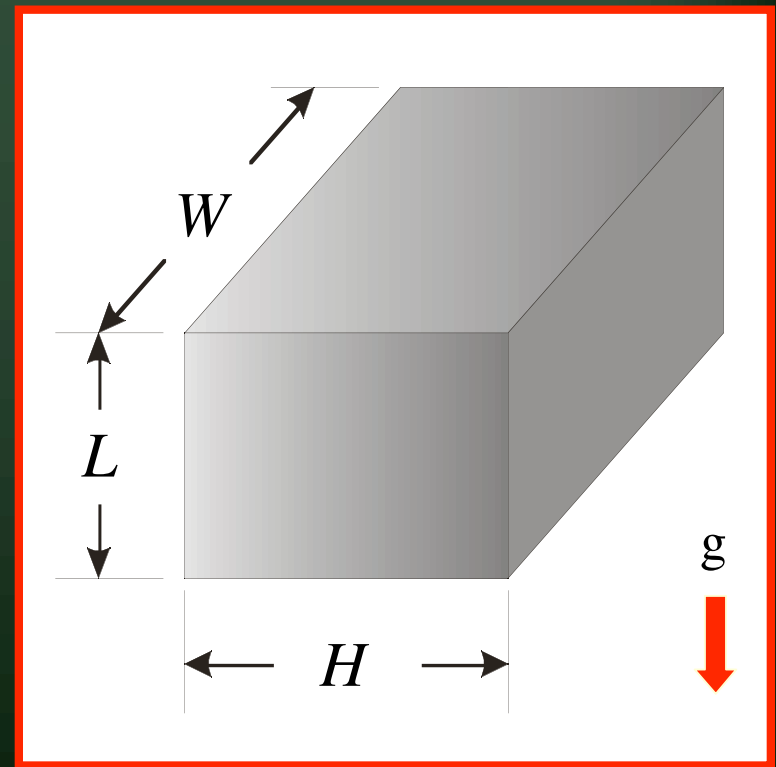
(in terms of the surface area)

Where:

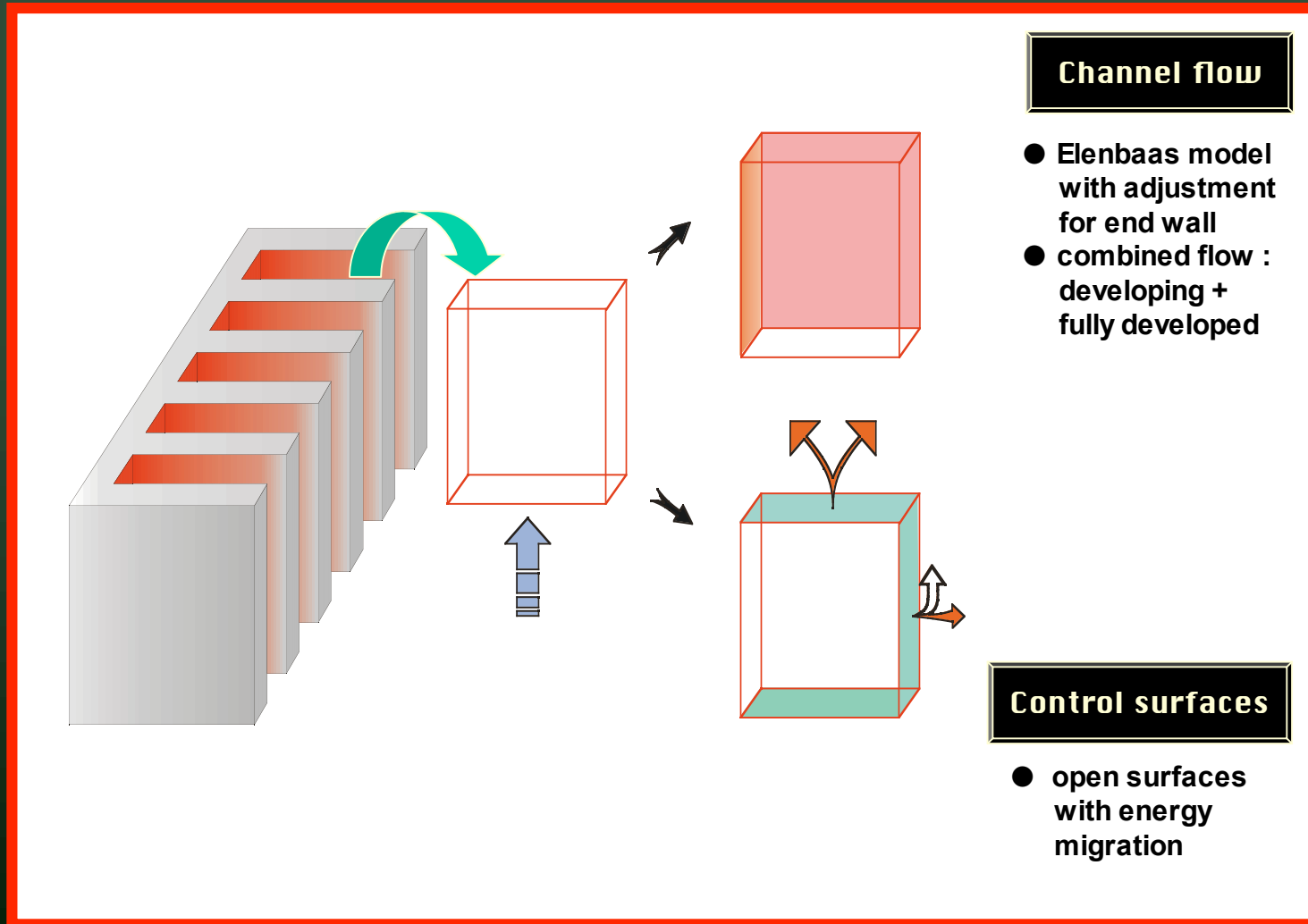
$$\frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) = \frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) + \frac{d}{dx} \left( \int_0^{\delta} u v dy \right)$$

$$\frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) = \frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) + \frac{d}{dx} \left( \int_0^{\delta} u v dy \right)$$

$$\frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) = \frac{d}{dx} \left( \int_0^{\delta} u^2 dy \right) + \frac{d}{dx} \left( \int_0^{\delta} u v dy \right)$$



# Interior Surfaces

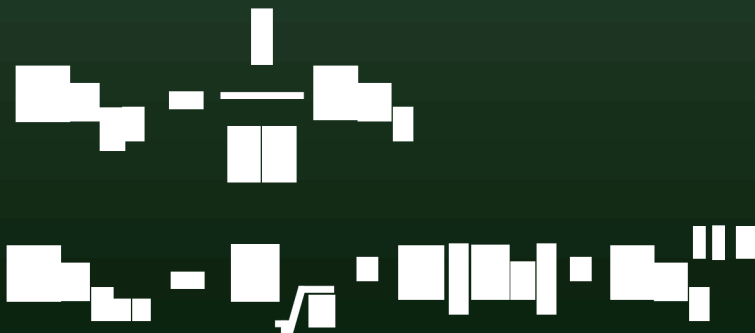


# Parallel Plates Model

Elenbaas, 1941



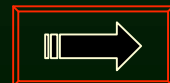
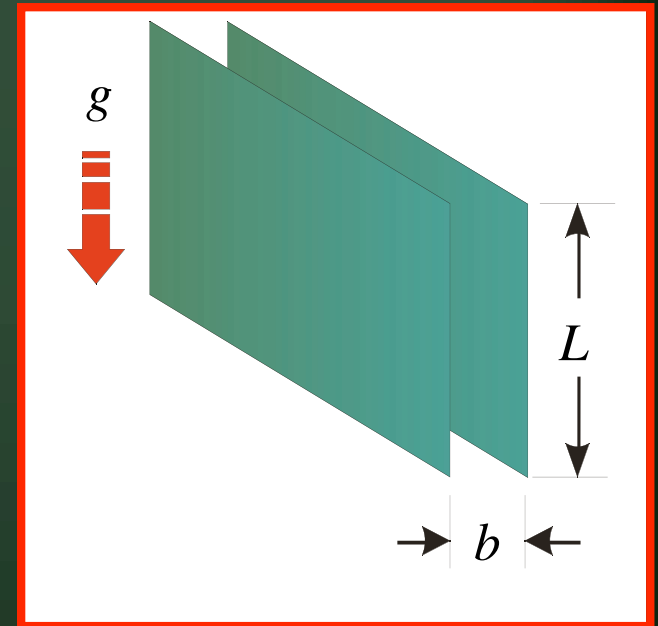
Churchill, 1977



*fd* - fully developed flow

$\sqrt{\cdot}$  - body gravity function

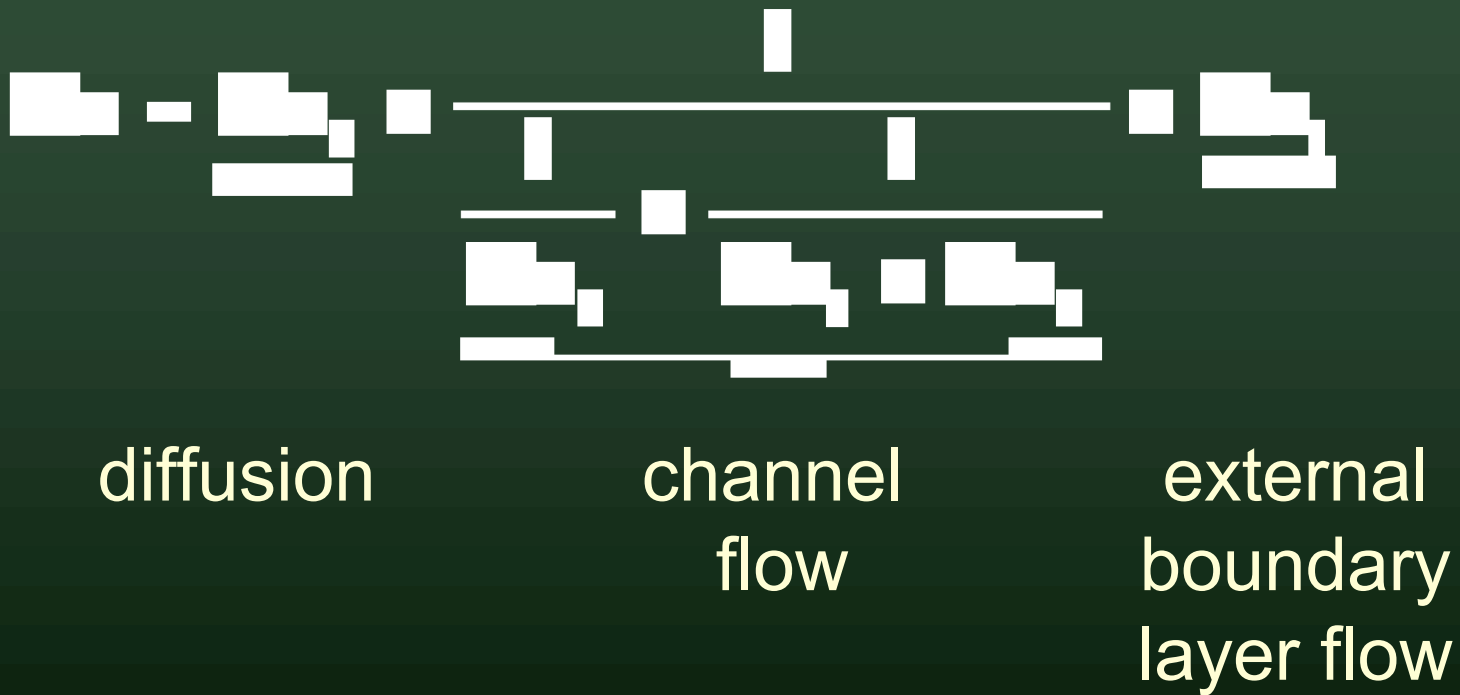
$\frac{1}{\sqrt{\cdot}}$  - Prandtl number function





# Comprehensive Model

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# Model Validation

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## *Limiting Cases*

### ➤ cuboids

- ➊ plate - Karagiozis (1991), Saunders (1936)
- ➋ cube - Chamberlain (1983), Stretton (1988)
- ➌ rectangular prism - Clemes (1990)

### ➤ parallel plates

- ➊ Elenbaas (1942), Aihara (1973), Kennard (1941)

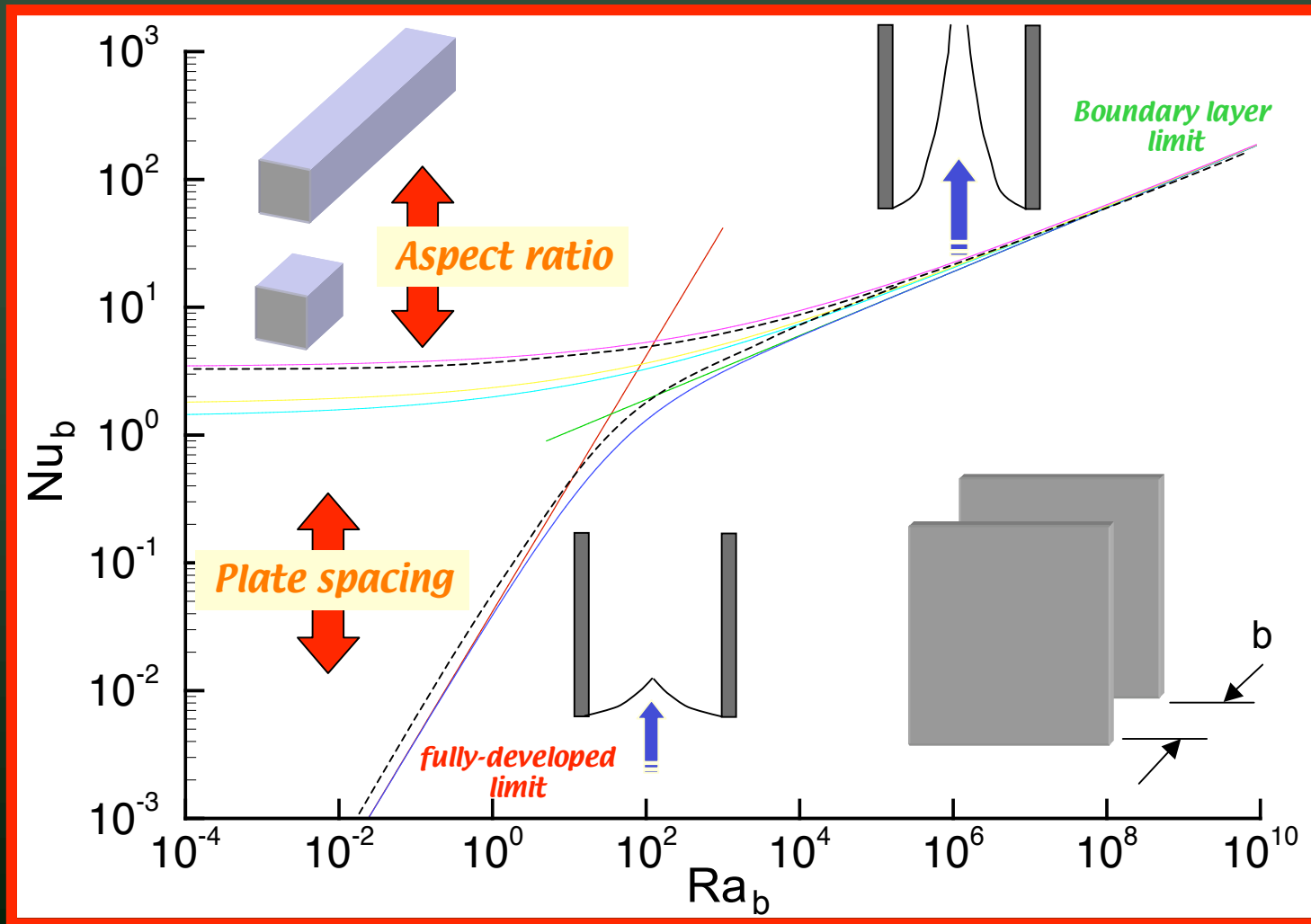
## *Heat Sinks*

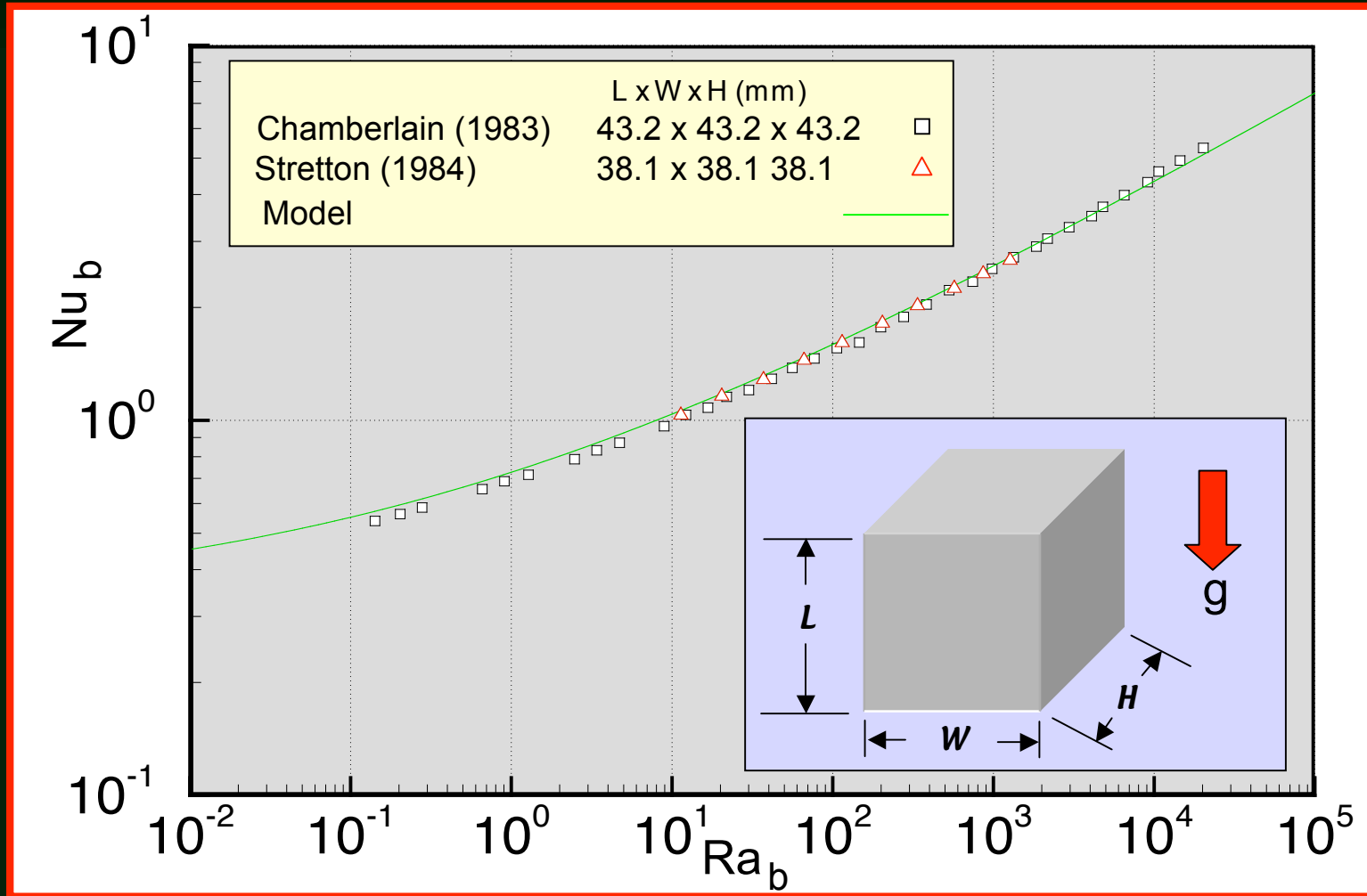
### ➤ Karagiozis (1991)

### ➤ Van de Pol & Tierny (1978)

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# Modelling Domain





$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[ Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

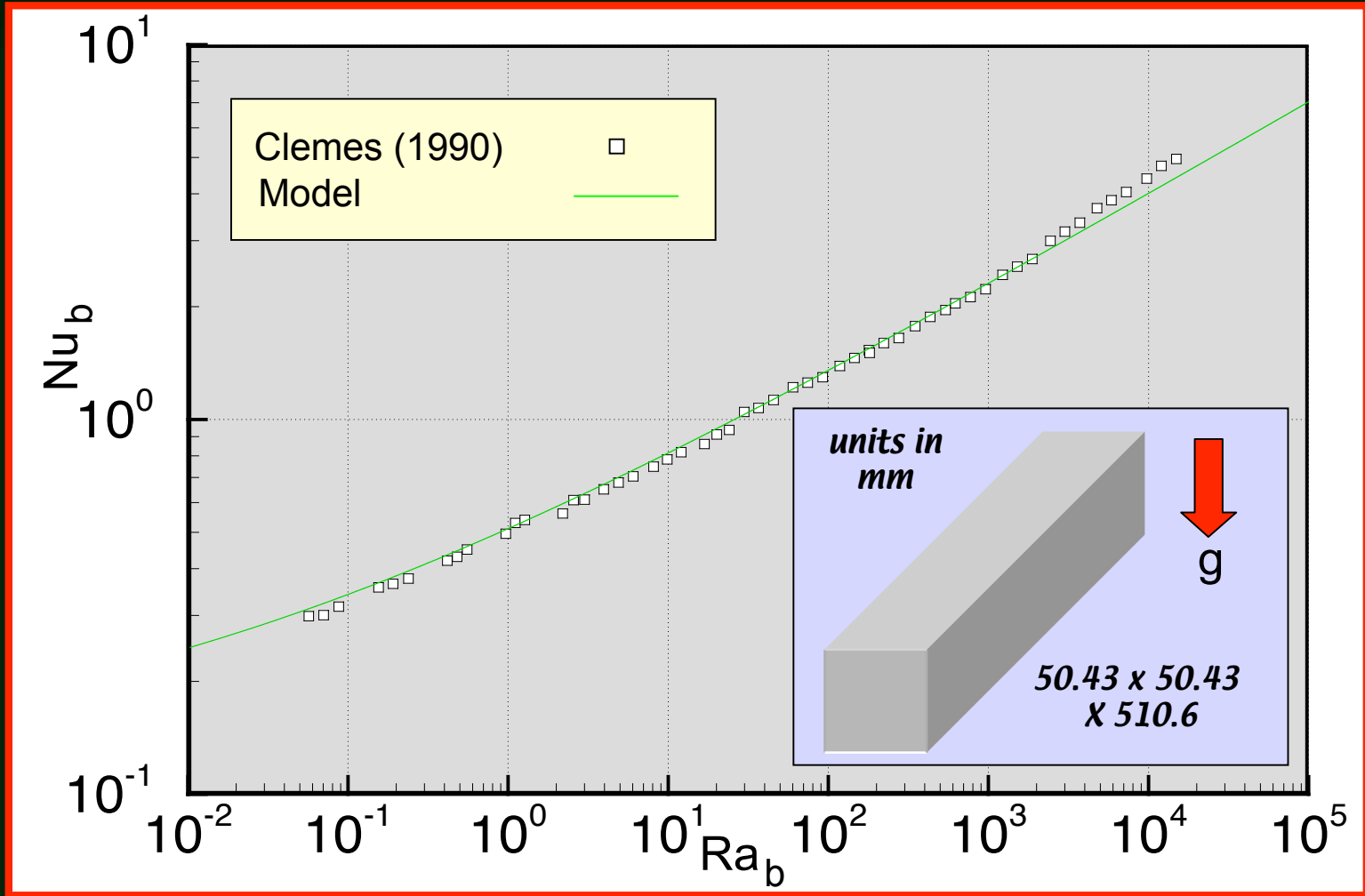
CUBE

PRISM

FLAT PLATE

|| PLATES

HEAT SINK



$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[ Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

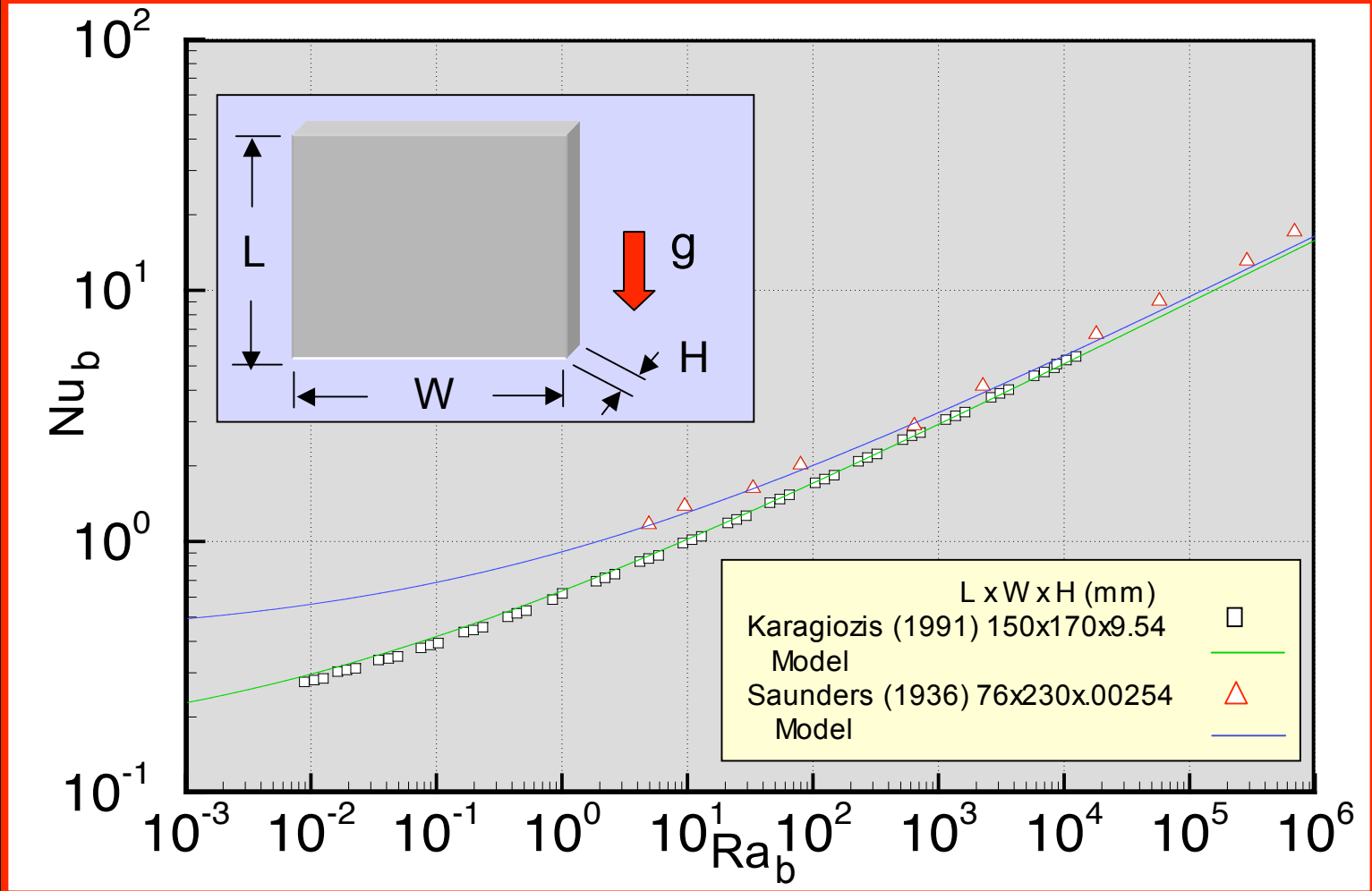
CUBE

PRISM

FLAT PLATE

|| PLATES

HEAT SINK



$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[ Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

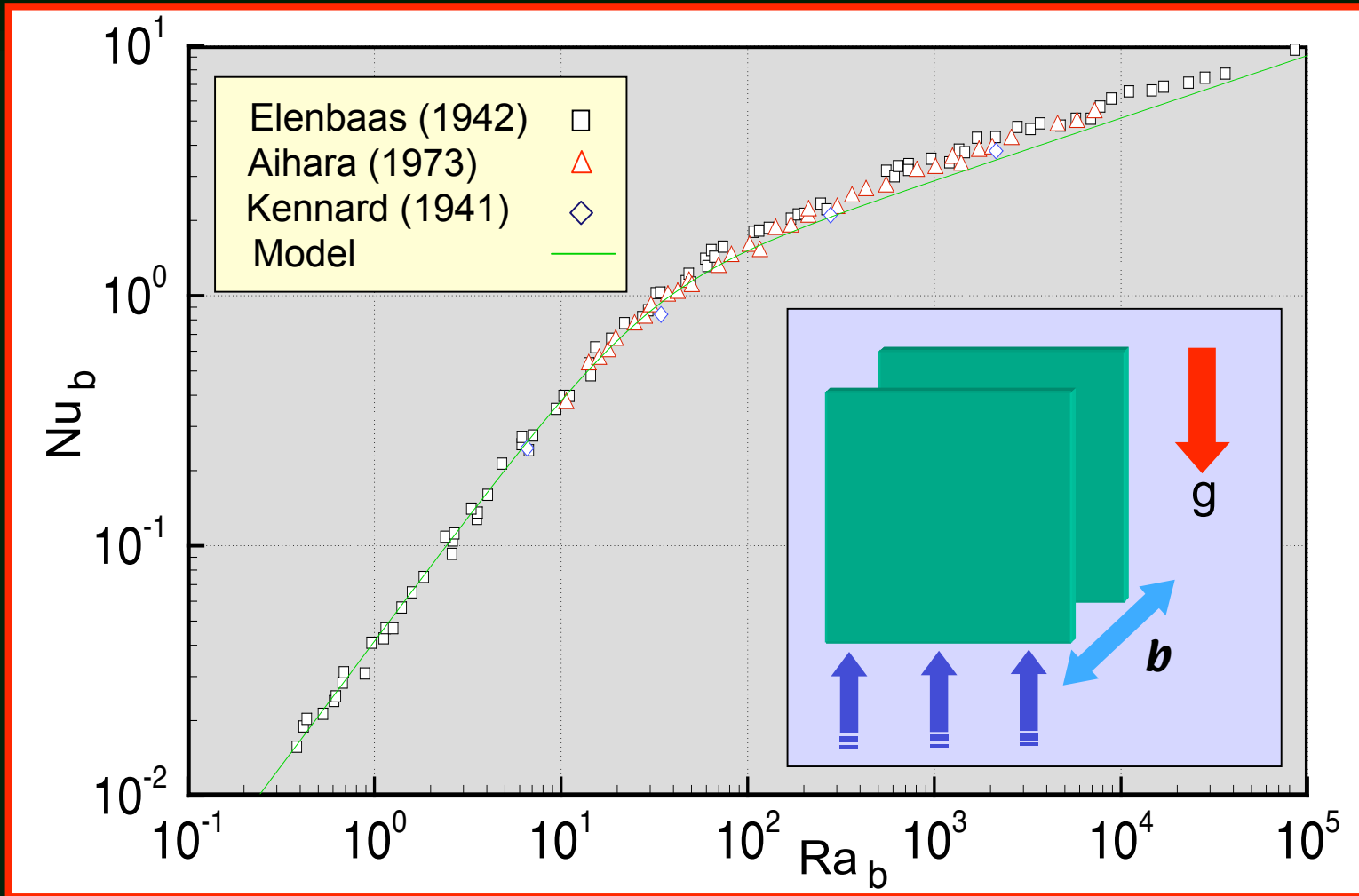
CUBE

PRISM

FLAT PLATE

|| PLATES

HEAT SINK



$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[ Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

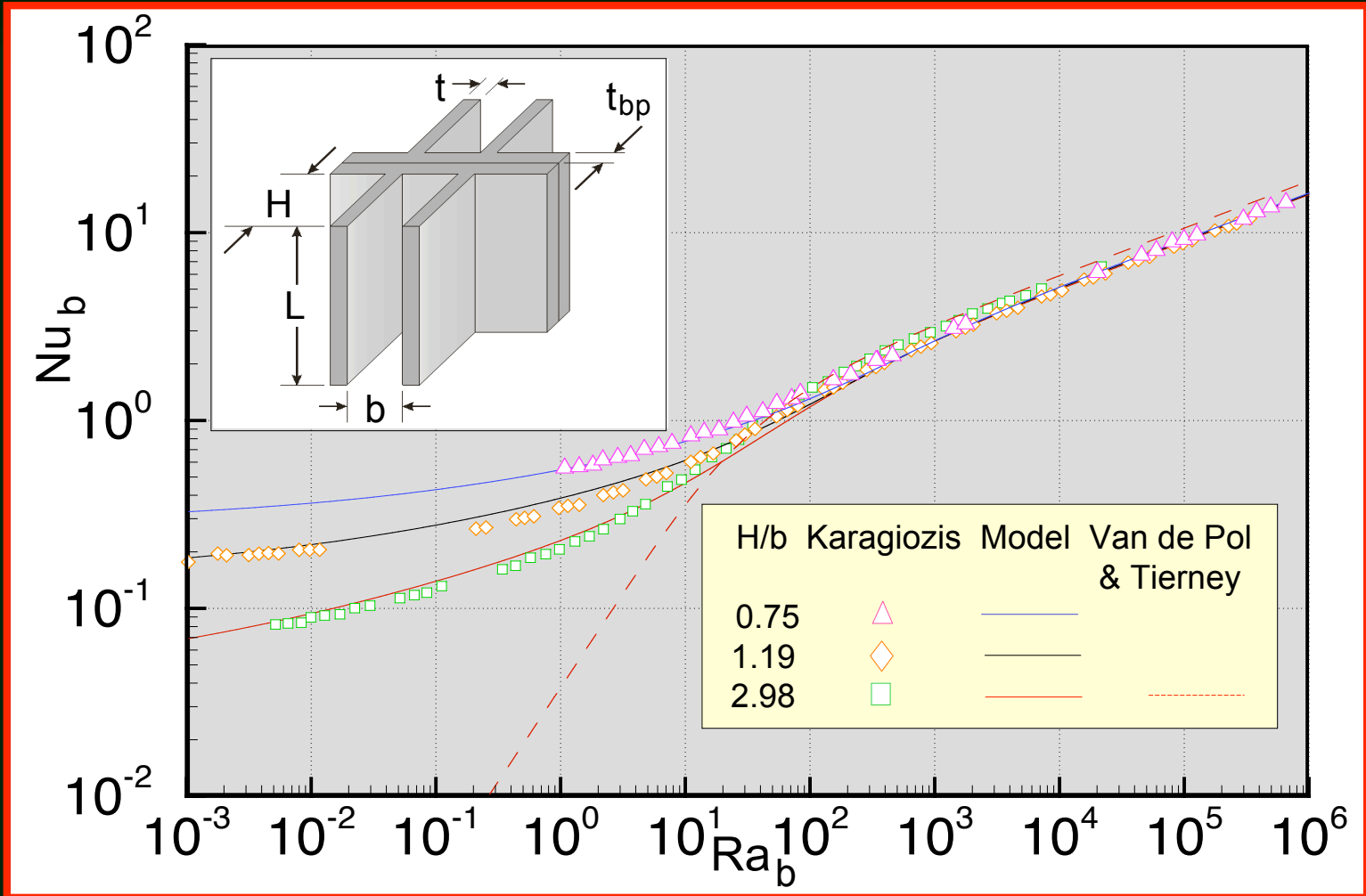
CUBE

PRISM

FLAT PLATE

|| PLATES

HEAT SINK



$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[ Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

CUBE

PRISM

FLAT PLATE

|| PLATES

HEAT SINK



# Which is the Right Tool?

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## *Analysis Tool*

vs.

## *Design Tool*

- ↳ design is known a priori
- ↳ used to calculate the performance of a given design, i.e.  $Nu$  vs.  $Ra$
- ↳ cannot guarantee an optimized design

- ↳ used to obtain an optimized design for a set of known constraints  
i.e. **given:**
  - heat input
  - max. temp.
  - max. outside dimensions**find:** the most efficient design

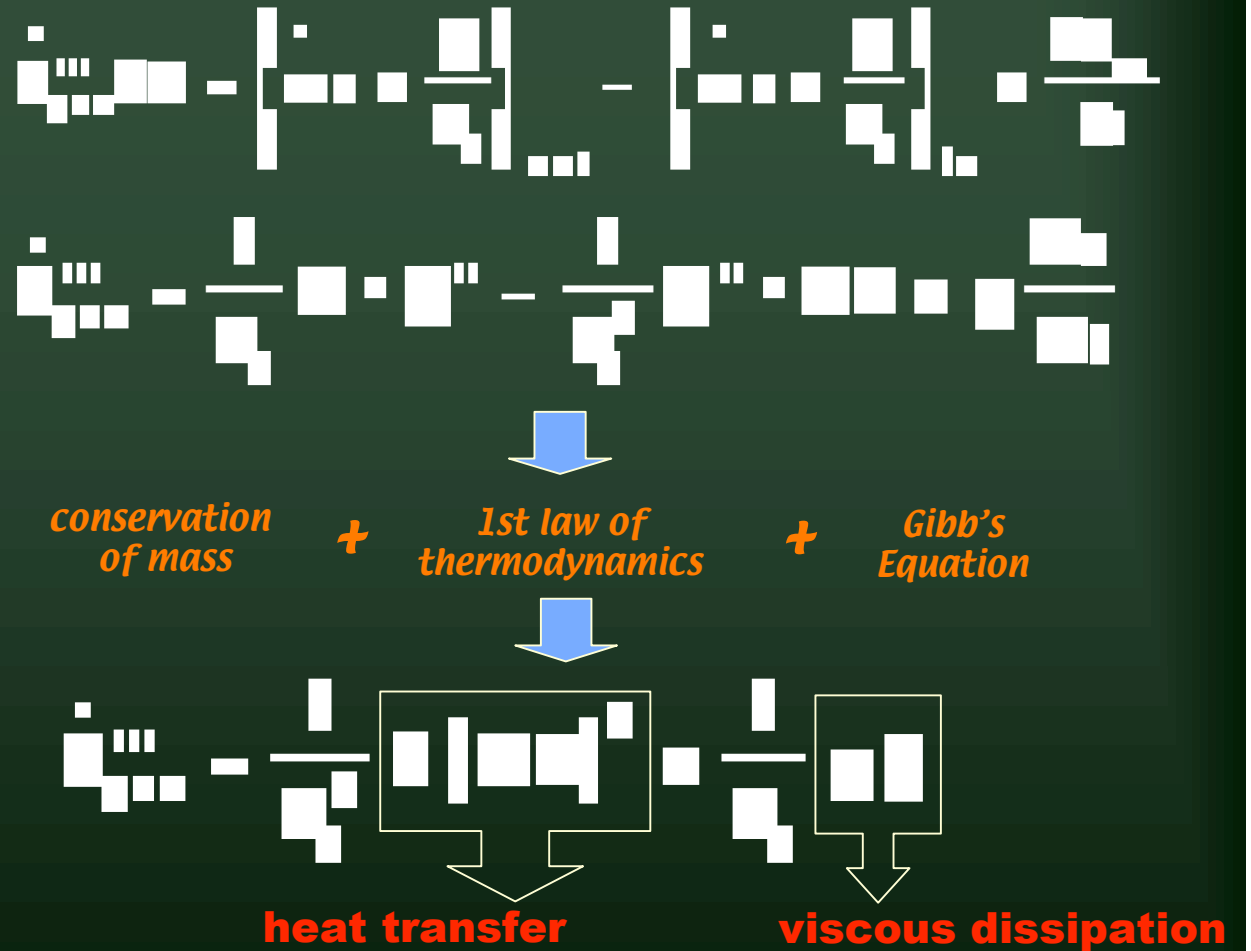
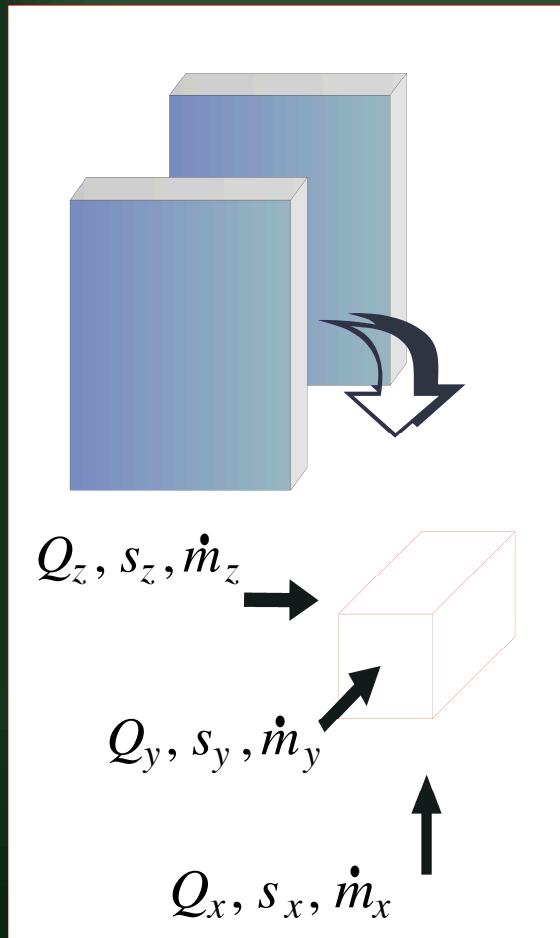
# Optimization Using EGM

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## Why use Entropy Generation Minimization?

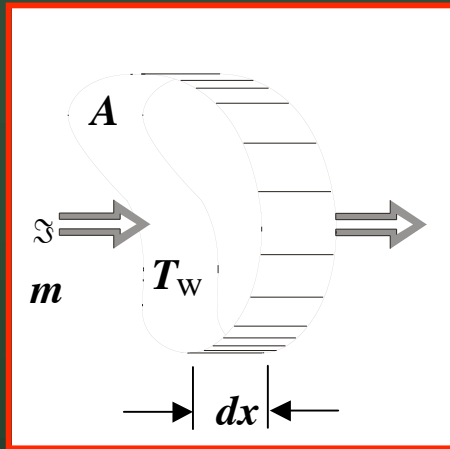
- ↳ entropy production  $\propto$  amount of energy degraded to a form unavailable for work
  - ↳ lost work is an additional amount of heat that could have been extracted
  - ↳ degradation process is a function of thermodynamic irreversibilities e.g. friction, heat transfer etc.
  - ↳ minimizing the production of entropy, provides a concurrent optimization of all design variables
-

# Entropy Balance (local)

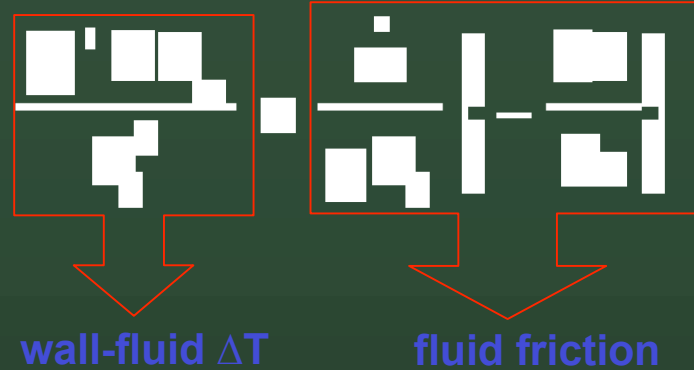


# Entropy Balance (external & internal)

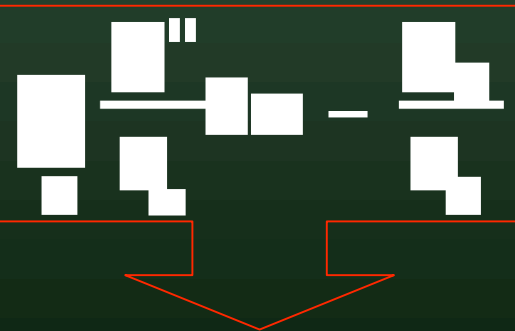
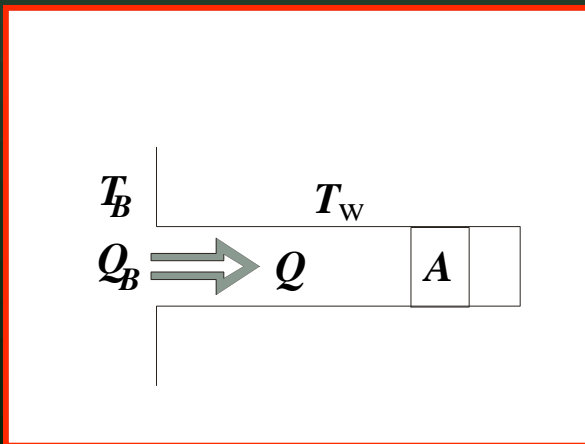
## Passage geometry



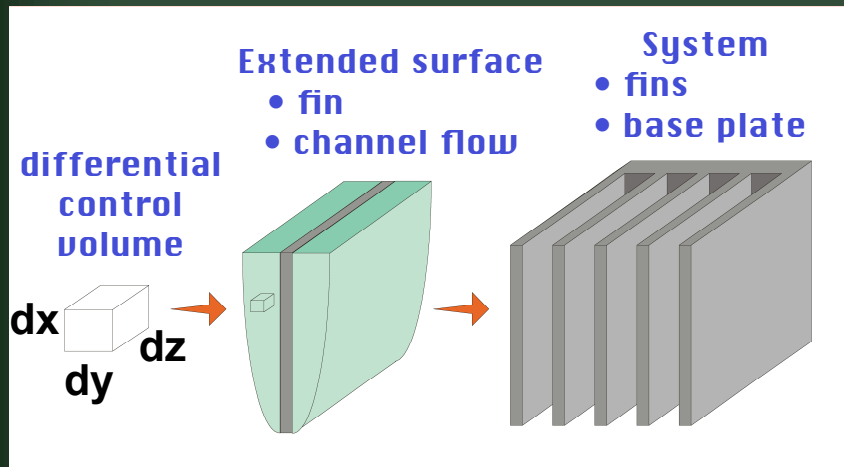
irreversibilities  
due to:








## Extended surface

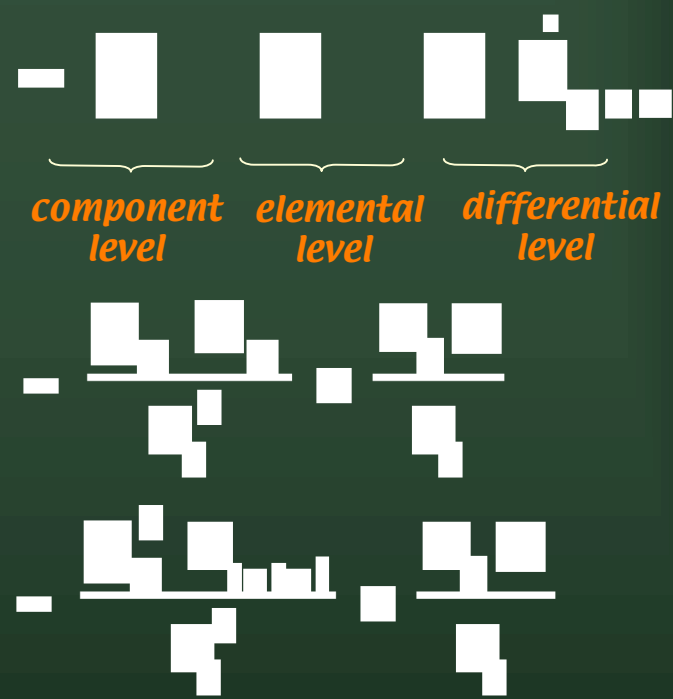





# Total Entropy Generation



where:

-  - base heat flow rate
-  - base - stream temp. difference
-  - ambient temperature
-  - drag force
-  - total fin resistance

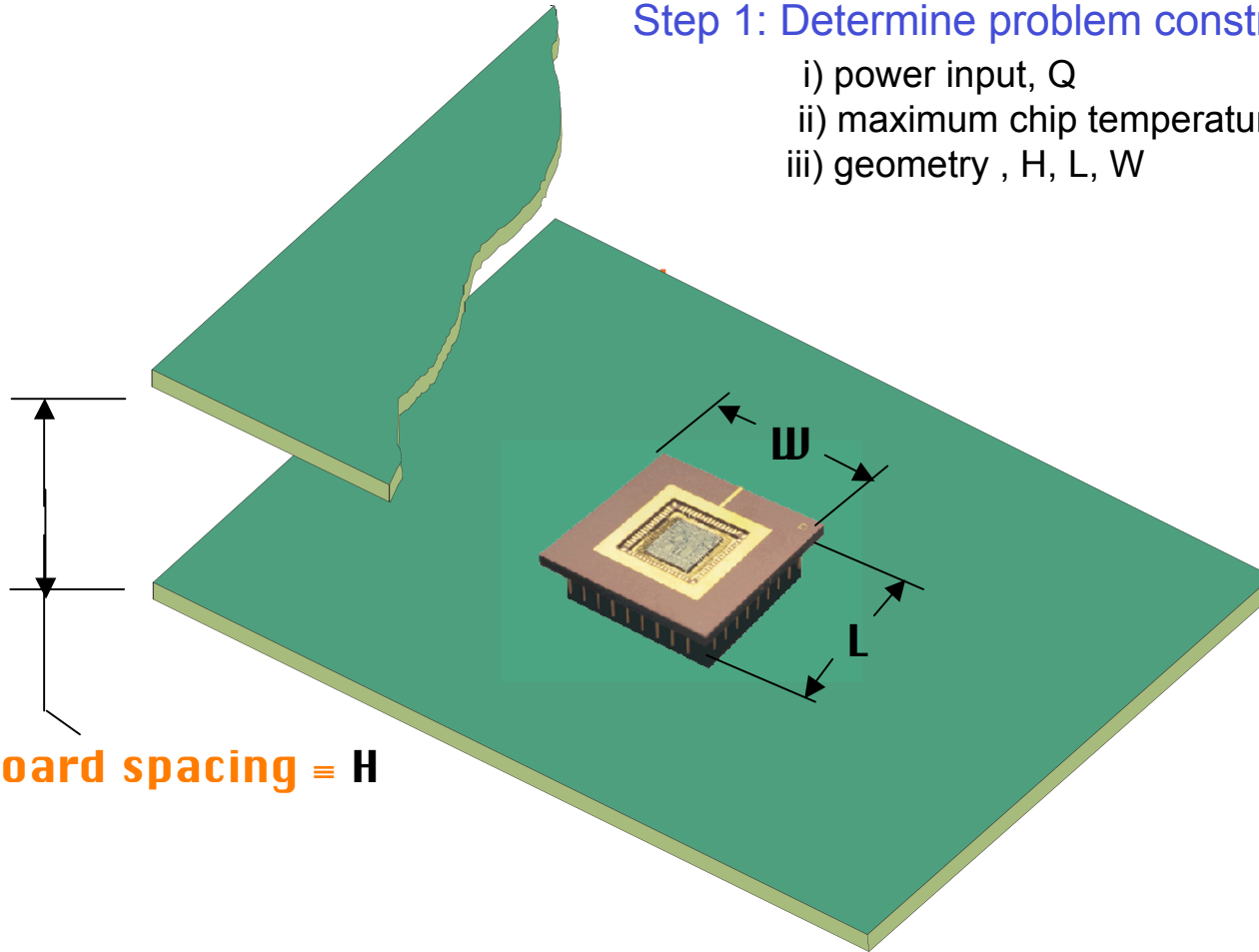


-  - specified
-  - fan curve
-  - buoyancy induced

# Example: Heat Sink Optimization

## Step 1: Determine problem constraints

- i) power input,  $Q$
- ii) maximum chip temperature,  $T_{max}$
- iii) geometry,  $H, L, W$

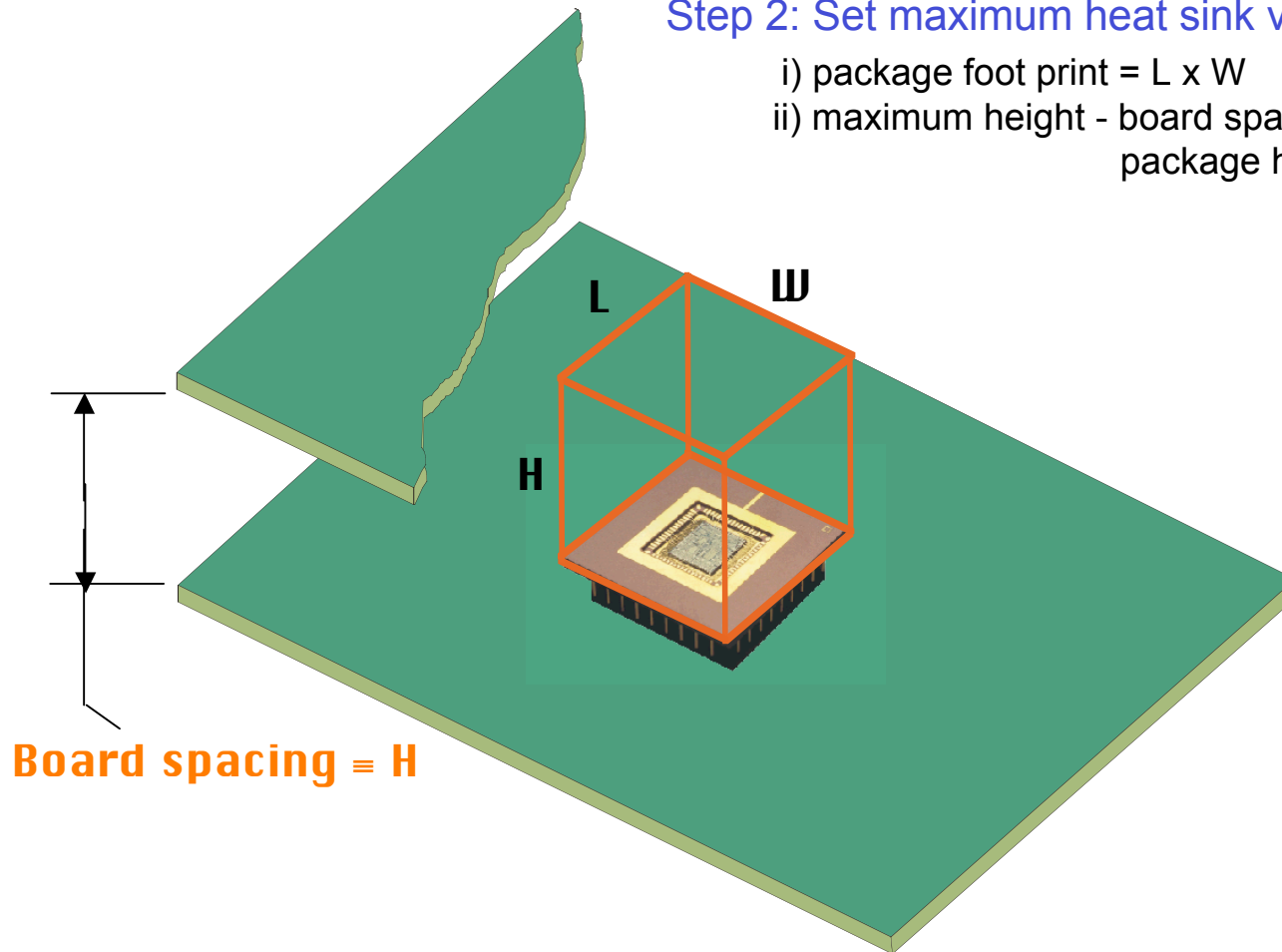


Board spacing  $\equiv H$

# Example: Heat Sink Optimization

## Step 2: Set maximum heat sink volume

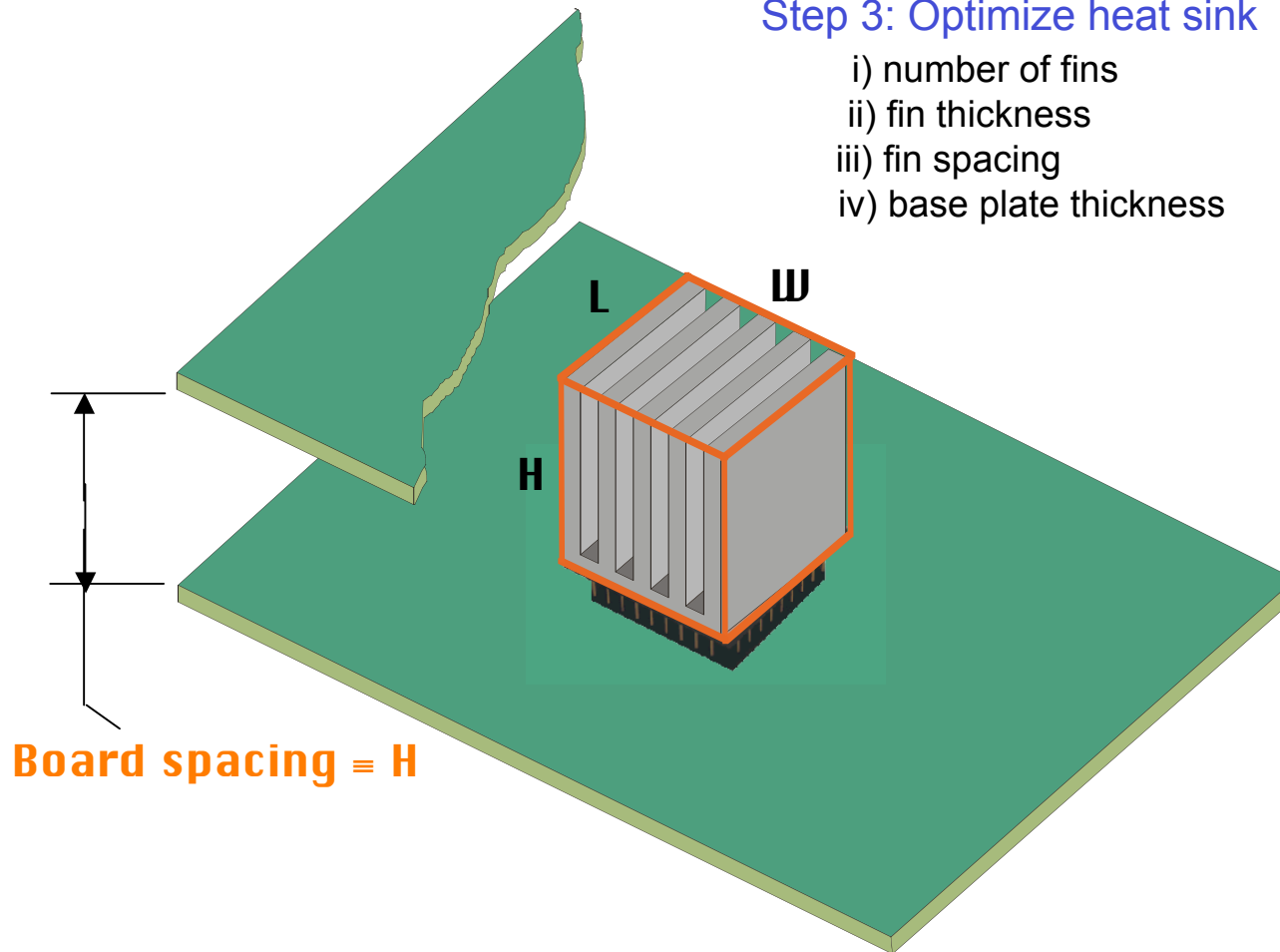
- i) package foot print =  $L \times W$
- ii) maximum height = board spacing minus package height



# Example: Heat Sink Optimization

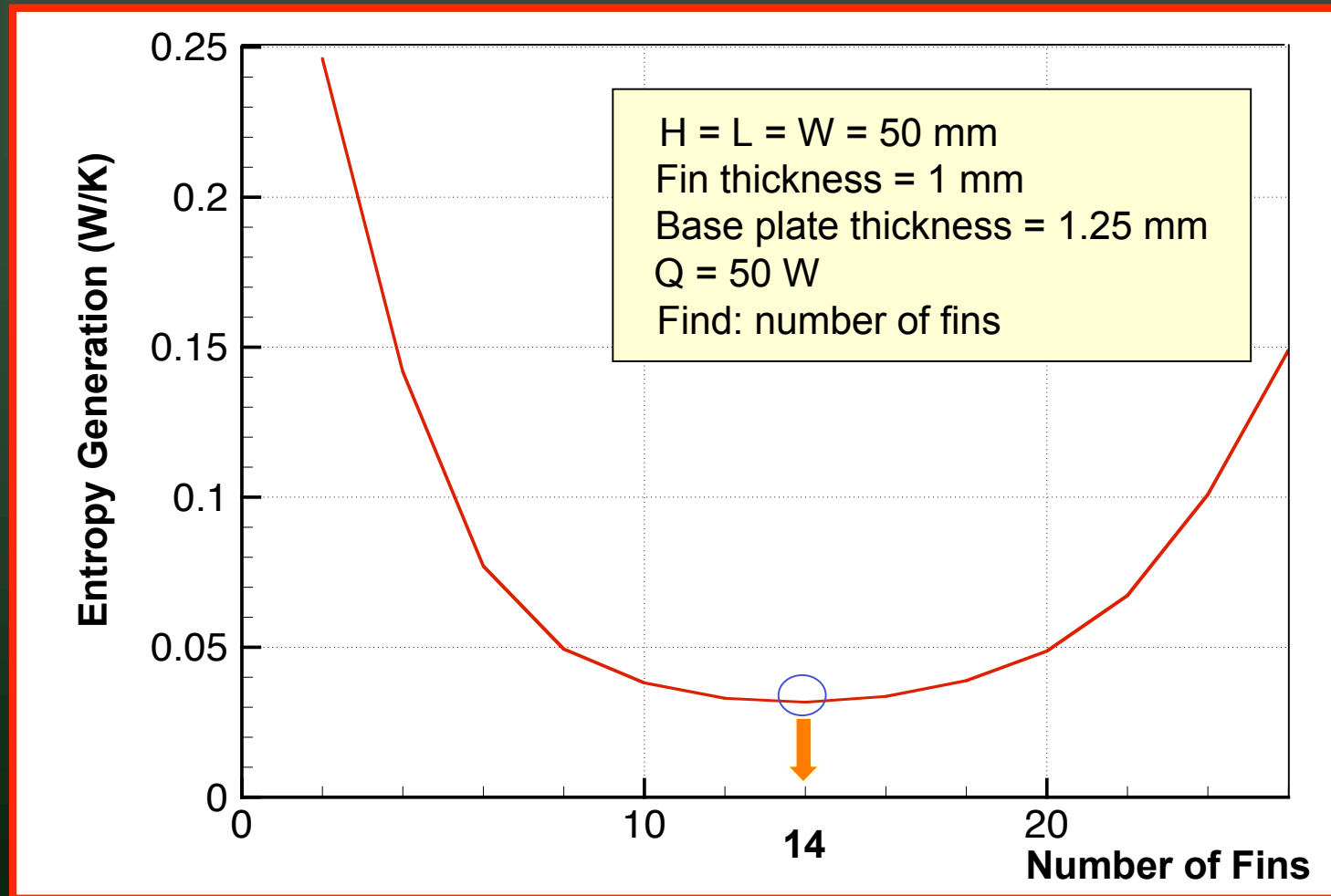
## Step 3: Optimize heat sink

- i) number of fins
- ii) fin thickness
- iii) fin spacing
- iv) base plate thickness





# Single Parameter EGM



# Multi-Parameter Minimization Procedure

$$f(\mathbf{p}) = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

$$\frac{\partial f}{\partial p_j} = -2 \sum_{i=1}^n (y_i - \hat{y}_i) \frac{\partial \hat{y}_i}{\partial p_j}$$

## Newton-Raphson Method with Multiple Equations and Unknowns

$$\begin{bmatrix} \frac{\partial^2 f}{\partial p_1^2} & \frac{\partial^2 f}{\partial p_1 \partial p_2} & \frac{\partial^2 f}{\partial p_1 \partial p_3} \\ \frac{\partial^2 f}{\partial p_1 \partial p_2} & \frac{\partial^2 f}{\partial p_2^2} & \frac{\partial^2 f}{\partial p_2 \partial p_3} \\ \frac{\partial^2 f}{\partial p_1 \partial p_3} & \frac{\partial^2 f}{\partial p_2 \partial p_3} & \frac{\partial^2 f}{\partial p_3^2} \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \end{bmatrix} = \begin{bmatrix} -\frac{\partial f}{\partial p_1} \\ -\frac{\partial f}{\partial p_2} \\ -\frac{\partial f}{\partial p_3} \end{bmatrix}$$

where:  $\mathbf{H} = \begin{bmatrix} \frac{\partial^2 f}{\partial p_1^2} & \frac{\partial^2 f}{\partial p_1 \partial p_2} & \frac{\partial^2 f}{\partial p_1 \partial p_3} \\ \frac{\partial^2 f}{\partial p_1 \partial p_2} & \frac{\partial^2 f}{\partial p_2^2} & \frac{\partial^2 f}{\partial p_2 \partial p_3} \\ \frac{\partial^2 f}{\partial p_1 \partial p_3} & \frac{\partial^2 f}{\partial p_2 \partial p_3} & \frac{\partial^2 f}{\partial p_3^2} \end{bmatrix}$   $\mathbf{g} = \begin{bmatrix} -\frac{\partial f}{\partial p_1} \\ -\frac{\partial f}{\partial p_2} \\ -\frac{\partial f}{\partial p_3} \end{bmatrix}$   $\mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}$   $\Delta \mathbf{p} = \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \Delta p_3 \end{bmatrix}$  *iterate until*  $\|\Delta \mathbf{p}\| < \epsilon$

# Future Work

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**Goal:** Develop a comprehensive model to find the best heat sink design given a limited set of design constraints

## Physical Design

- heat sink type
- material
- weight
- dimensions
- surface finish

## Thermal

- maximum volume
- boundary conditions
- max. allowable temp.
- orientation
- flow mechanism

## Cost

- labour
- manufacturing
- material

## Standards

- noise
  - exposure to touch
-

# Summary

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- Heat sink design requires both a **selection** tool & an **analysis** tool
  - Selection is based on:
    - ↳ physical constraints - geometry, material, etc.
    - ↳ thermal-fluid conditions - bc's, properties, etc.
    - ↳ miscellaneous conditions - cost, standards etc.
  - Analysis is based on simulating a prescribed design
-

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The End

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# Karagiozis Heat Sink Model

$$\bar{Nu}_D = \frac{1}{\frac{1}{\bar{Nu}_{D, \text{flat}}} + \frac{1}{\bar{Nu}_{D, \text{corr}}}}$$

where:

$$\bar{Nu}_{D, \text{flat}} = \frac{0.66 + 0.4 Re_D^{1/4} + 0.75 Pr^{1/4}}{1 + 1.13 \times 10^{-6} Re_D^{2.45} + 8.71 \times 10^{-5} Pr^{0.75}} \left[ \frac{Pr}{Pr_s} \right]^{1/4}$$

$$\bar{Nu}_{D, \text{corr}} = \frac{0.14 + 0.42 Re_D^{1/4} + 0.62 Pr^{1/4}}{1 + 1.13 \times 10^{-6} Re_D^{2.45} + 8.71 \times 10^{-5} Pr^{0.75}} \left[ \frac{Pr}{Pr_s} \right]^{1/4}$$

$$Pr = \frac{\mu_f c_p}{k_f} \quad \text{Pr}_s = \frac{\mu_s c_{p,s}}{k_s}$$

$$Re_D = \frac{\rho_f V D}{\mu_f}$$

$$Re_{D, \text{crit}} = 2300$$

$$Pr_s = \frac{\mu_s c_{p,s}}{k_s}$$

$$Pr_s = \frac{\mu_s c_{p,s}}{k_s}$$

Modified flat plate model → correction term at low Ra