APPROXIMATE MODELLING PROCEDURES FOR RAPID ANALYSIS AND DESIGN

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Overview

- What is modelling?
- How can approximate modelling methods be used to understand thermal behaviour in electronics applications
- Modelling procedures: as applied to heat sinks
- Other potential applications

Modelling Alternatives

- Experimental Methods
  - prototype testing
  - empirically-based correlations
- Numerical Methods
  - approximate the governing equations over a finite, discretized domain
- Analytical Methods
  - closed form solutions
  - approximate methods

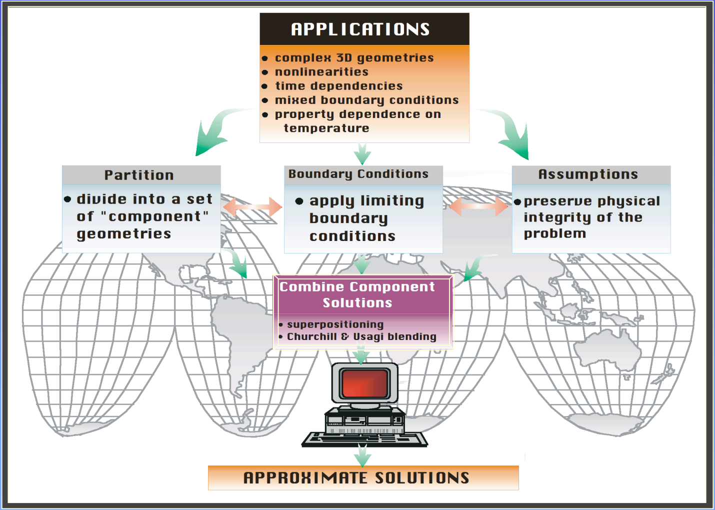
Why Use Approximate Methods?

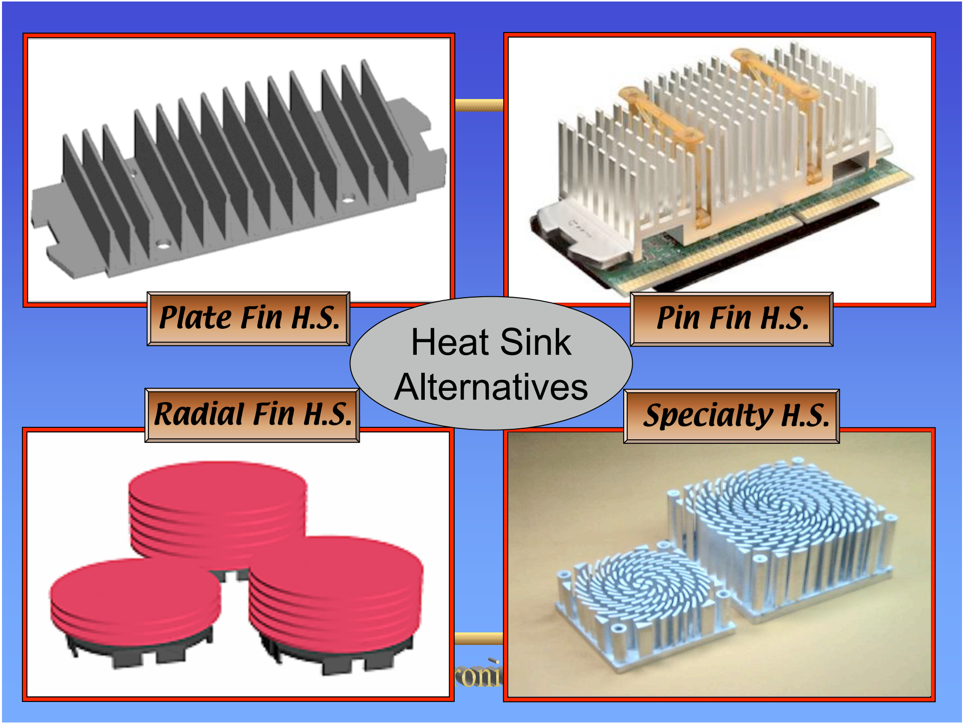
- Fast, accurate and easy to use
- Minimal hardware requirements
- Ideal for preliminary design studies
  - material selection
  - component selection and placement
  - trade-off studies
- Optimization studies
- Concurrent design

Perceived Limitations

- Limited range of applications
- Cannot be used for complex geometries
- Cannot be used with mixed or non-uniform boundary conditions
- Simplifying assumptions provide inaccurate solutions

Modelling Approach

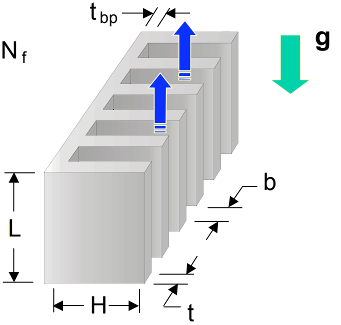



Heat Sink Alternatives
Plate Fin H.S.
Pin Fin H.S.
Radial Fin H.S.
Specialty H.S.

Heat Sink Model

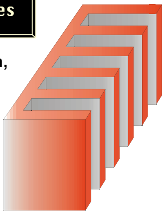
- Plate fin heat sink
- Natural convection
- Vertical orientation
- Isothermal
- Steady state
- Working fluid is air

Modelling Procedure



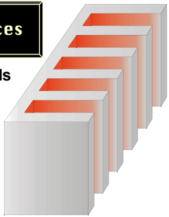
Exterior surfaces

- fins : top, bottom, ends & tip
- base plate: top, bottom, ends and back



Interior surfaces

- fins : side walls
- channel base



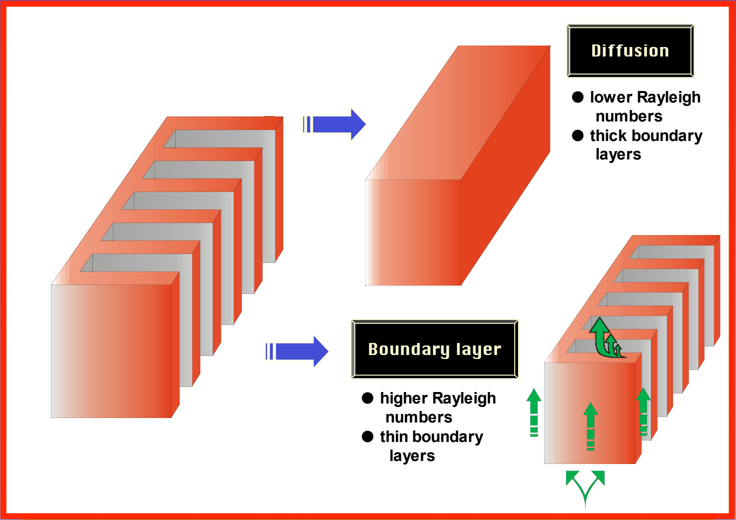
Given: dimensions & temperature

Find: Nu_b *vs.* Ra_b

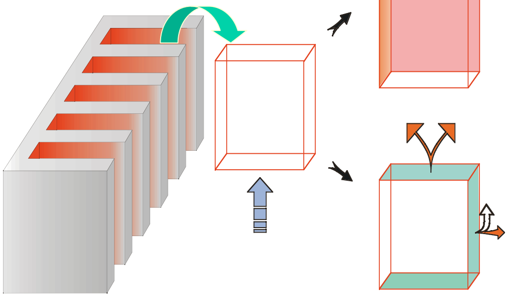
$$Nu_b = \frac{hb}{k_f}$$

$$Ra_b = \frac{g\beta\Delta T b^3}{\alpha\nu} \bullet \frac{b}{L}$$

Exterior Surfaces



Interior Surfaces



Channel flow

- Elenbaas model with adjustment for end wall
- combined flow : developing + fully developed

Control surfaces

- open surfaces with energy migration

Comprehensive Model

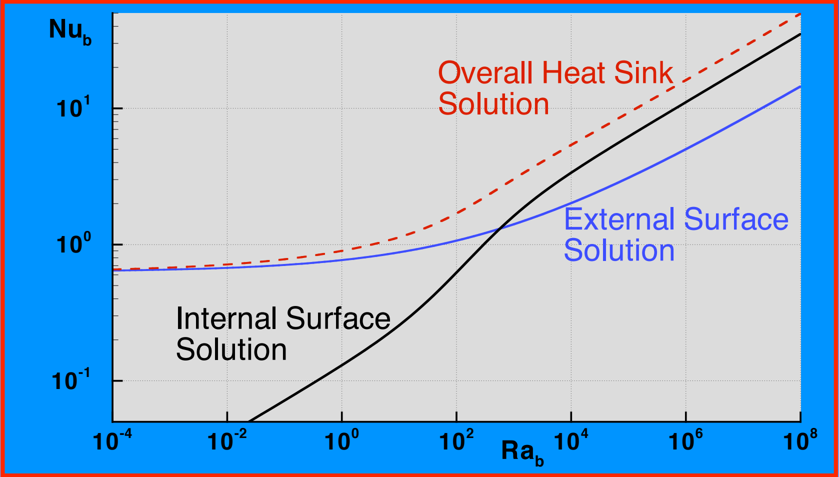
$$Nu = \underbrace{Nu_0}_{} + \underbrace{\frac{1}{\frac{1}{Nu_2} + \frac{1}{Nu_3 + Nu_4}}}_{} + \underbrace{Nu_1}_{}$$

diffusion

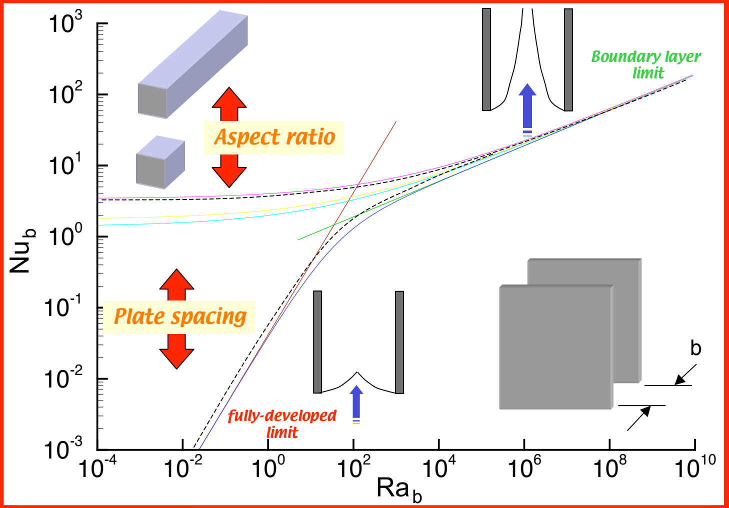
channel flow

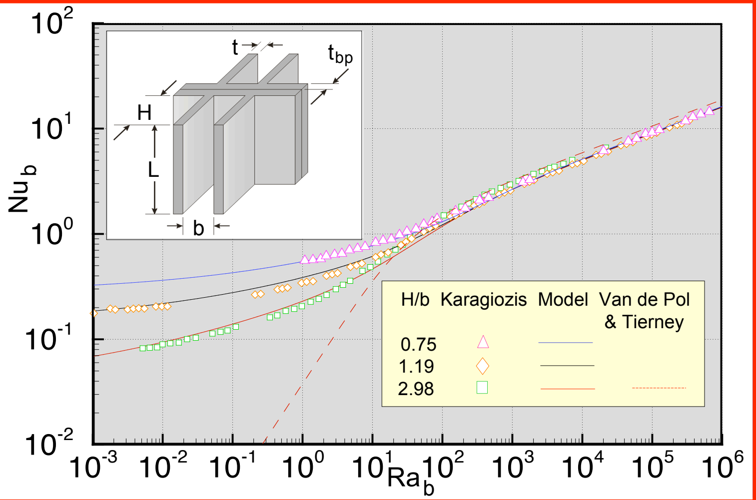
external boundary layer flow

Total Composite Solution



Modelling Domain





$$Nu = Nu_0 + \left\{ Nu_2^{-2} + \left[Nu_3 + Nu_4 \right]^{-2} \right\}^{-1/2} + Nu_1$$

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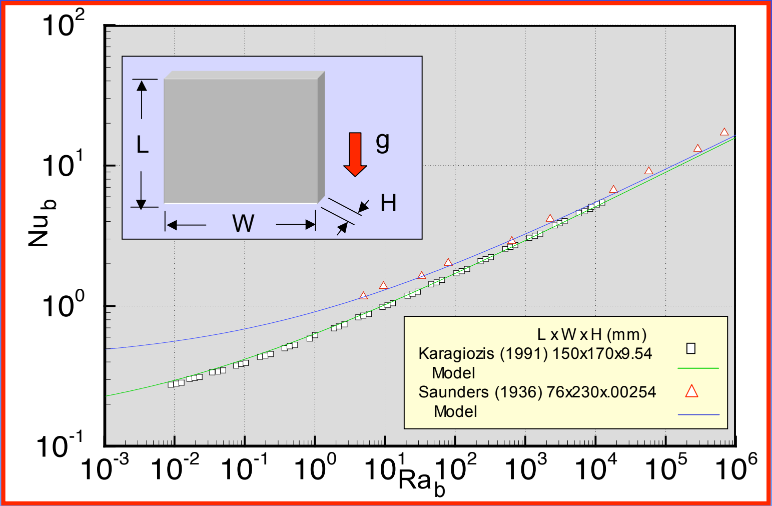

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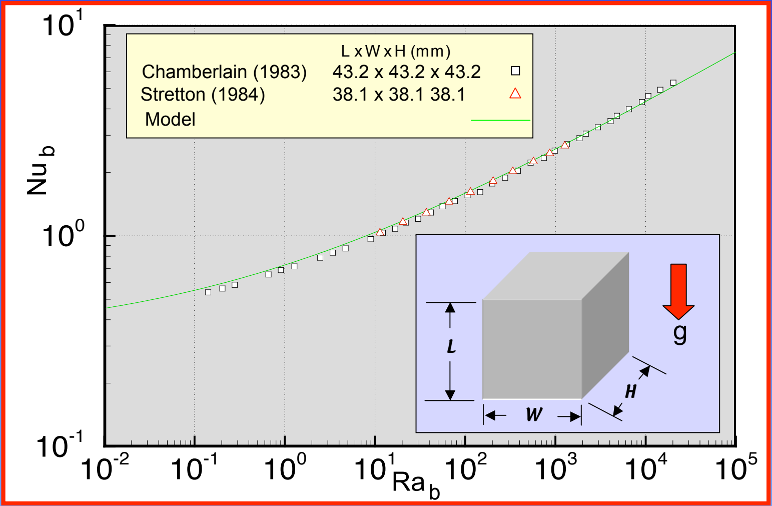

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How Do We Use These Results?

dimensionless heat transfer coefficient

dimensionless flow parameter

[illegible]

- *If max. [illegible] changes ➪ what is the max. [illegible] dissipated*
- *For a given [illegible] ➪ what is the max. [illegible] of the heat sink or the package junction*
- *How do changes in geometry affect ➪ [illegible] and [illegible]*

Future Work

Goal: **Develop a comprehensive model to find the best heat sink design given a limited set of design constraints**

Physical Design

- heat sink type
- material
- weight
- dimensions
- surface finish

Thermal

- maximum volume
- boundary conditions
- max. allowable temp.
- orientation
- flow mechanism

Cost

- labour
- manufacturing
- material

Standards

- noise
- exposure to touch

Other Examples of Approximate Models

Applications	*Asymptotic Limits*	
Heat & Mass Transfer		
• ***Boundary layer flow***	laminar	turbulent
• ***Channel flow***	fully developed flow	boundary layer flow
• ***External flow***	diffusion	boundary layer flow
• ***Internal flow***	fully developed flow	developing flow
• ***Enclosures***	diffusion	boundary layer flow
• ***Transient conduction***	short time	steady state
• ***Radiation***	opaque	transparent
• ***Steady conduction at nano-scales***	rarefied	continuum
Moving Sources	stationary	fast moving
Elasto-plastic contacts	elastic	plastic

Summary

- Approximate models offer superior speed of execution and ease of use over most conventional modelling methods
- Analytical modelling can be used for a wide range of applications previously considered to be too complex

The End