Modeling of Natural Convection in Electronic Enclosures

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Outline

- Introduction and problem description
- Model development
- Numerical simulations
- Validation
- Summary

Introduction

- Current design practice for sealed electronic enclosures
  - Numerical CFD simulations
  - Experimental prototype testing
  - Time consuming, expensive
- Analytically-based modeling
  - Quick, easy to implement
  - Ideal for preliminary design, parametric studies
- Objective: *to develop and validate a natural convection model for simple, sealed enclosures*
  - Vertical rectangular flat plate at center of a cuboid shaped enclosure
  - Full range of Rayleigh number from laminar natural convection to conduction

Problem Description

- Enclosure dimensions

$$\frac{L_o}{L_i}, \quad \frac{L_i}{W_i}, \quad \frac{L_o}{W_o}, \quad \frac{b}{L_o}$$

- Isothermal boundary conditions

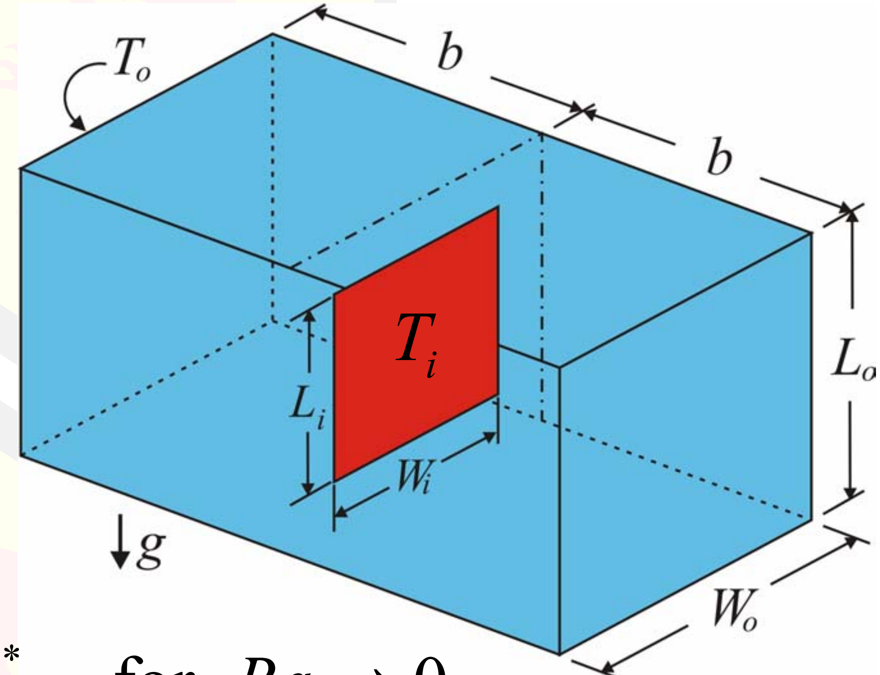
$$T_i > T_o$$

- Total heat transfer rate

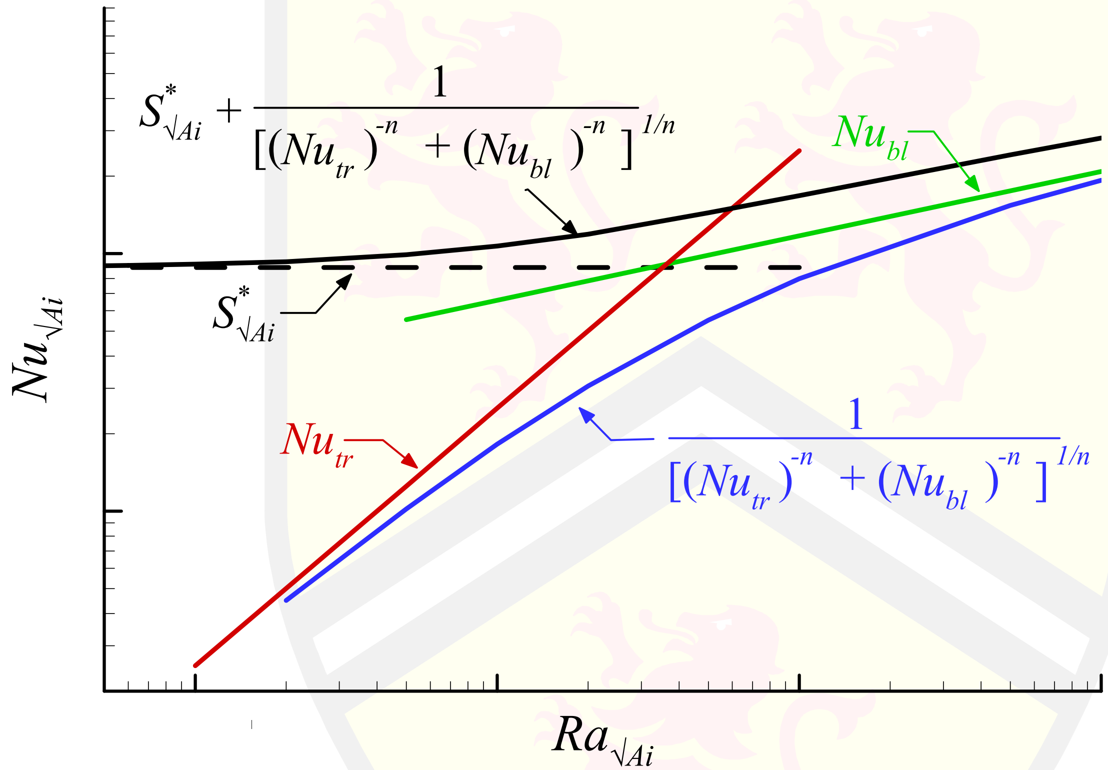
$$Nu_{\sqrt{A_i}} = \frac{Q}{k\sqrt{A_i}\left(T_i - T_o\right)} \quad = S^*_{\sqrt{A_i}} \quad \text{for} \;\; Ra \rightarrow 0$$

- Rayleigh number

$$Ra_{\sqrt{A_i}} = \frac{g\,\beta\left(T_i - T_o\right)\left(\sqrt{A_i}\right)^3}{\nu\,\alpha}$$



General Model Formulation



- Combination of three asymptotic solutions (Teertstra, 2003)

$$Nu_{\sqrt{A_i}} = S^*_{\sqrt{A_i}} + \left[\left(\frac{1}{Nu_{tr}}\right)^2 + \left(\frac{1}{Nu_{bl}}\right)^2\right]^{-1/2}$$

$S^*_{\sqrt{A_i}}$ = conduction shape factor

Nu_{tr} = transition flow convection

Nu_{bl} = laminar boundary layer flow convection

Conduction Shape Factor

- Composite model (Churchill and Usagi, 1972)

$$S^*_{\sqrt{A_i}} = \left[\left(S^*_{b/L\to 0}\right)^{3/2} + \left(S^*_{b/L\to\infty}\right)^{3/2}\right]^{2/3}$$

- $b/L \to 0$ one dimensional conduction in gap

$$S = \frac{A_i}{b} \qquad S^*_{b/L\to 0} = \frac{S}{\sqrt{A_i}} = \frac{\sqrt{A_i}}{b}$$

- $b/L \to \infty$ shape factor independent of b/L

$$S^*_{b/L\to\infty} = \frac{1}{k\sqrt{A_i}R} \qquad R = R_{plate} - R_{sphere}$$

R_{plate} = isothermal flat plate in full space region

R_{sphere} = equivalent sphere in full space region

$$d_{eff} = \left(L_o + W_o\right)/2$$

Laminar Boundary Layer

- Assumptions
  - T_b uniform
  - Non-intersecting boundary layers
- Series combination of resistances

$$R_{conv} = R_i + R_o \qquad Nu_{bl} = \frac{1}{k\sqrt{A_i}}\frac{1}{R_{conv}} = \frac{1}{k\sqrt{A_i}}\frac{(1/R_i)}{(1+R_o/R_i)}$$

$$R_i = \frac{T_i - T_b}{Q} = \frac{1}{k\sqrt{A_i}}\frac{1}{Nu_i} \qquad R_o = \frac{T_b - T_o}{Q} = \frac{1}{k\sqrt{A_o}}\frac{1}{Nu_o}$$

- Convection at boundaries modeled using Lee, Yovanovich and Jafarpur (1991)

$$Nu_{\sqrt{A}} = F(\Pr)G_{\sqrt{A}}\,Ra_{\sqrt{A}}^{1/4}$$

Laminar Boundary Layer

$$Nu_{bl} = \frac{F(\Pr) G_{\sqrt{A_i}} Ra_{\sqrt{A_i}}^{1/4}}{\left[1 + \left(\frac{A_i}{A_o}\right)^{7/10} \left(\frac{G_{\sqrt{A_i}}}{G_{\sqrt{A_o}}}\right)^{4/5}\right]^{5/4}}$$

- Prandtl number function $F(\Pr) = 0.513$ for air at STP
- Body gravity functions

$$G_{\sqrt{A_i}} = 2^{1/8} \left(W_i / L_i\right)^{1/8}$$ (Lee et al., 1991)

$$G_{\sqrt{A_o}} = 2^{1/8} \left[\frac{0.625(2b)^{4/3} W_o + L_o (2b + W_o)^{4/3}}{\left(L_o W_o + 2b(L_o + W_o)\right)^{7/6}}\right]^{3/4}$$

(Jafarpur and Yovanovich, 1993)

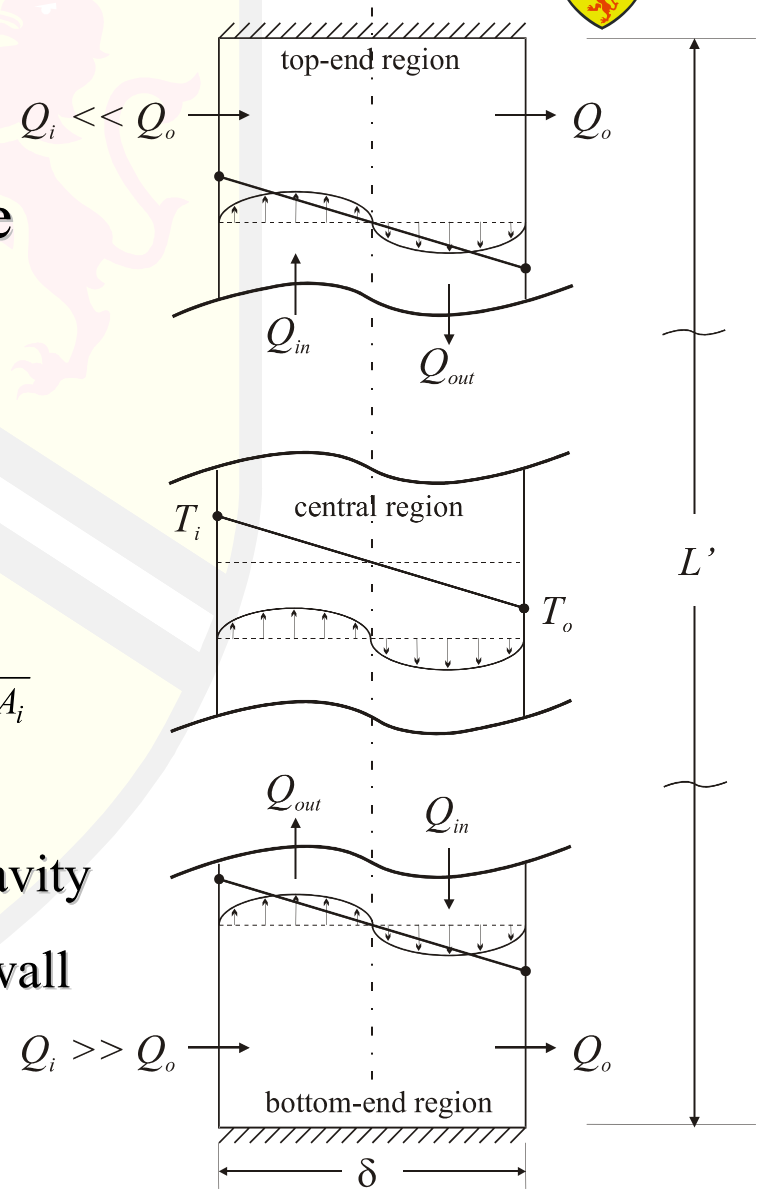
Transition Flow

- Boundary layers merge at low Rayleigh numbers
- Linear temperature distribution in core
- Convective heat transfer in top and bottom recirculation regions
- Enthalpy balance in end regions

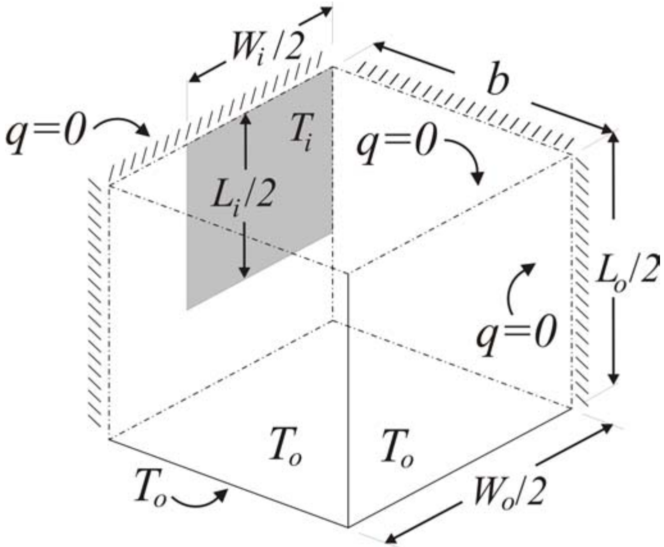
$$Nu_{tr} = \frac{\sqrt{2}}{360} \frac{\sqrt{W_i / L_i}}{(1 + L_o / L_i)} \left(\frac{\delta_{\text{eff}}}{\sqrt{A_i}} \right)^3 Ra_{\sqrt{A_i}}$$

δ_{eff} = gap spacing of equivalent spherical cavity

L_o , L_i = effective flow length on outer, inner wall



Numerical Simulations



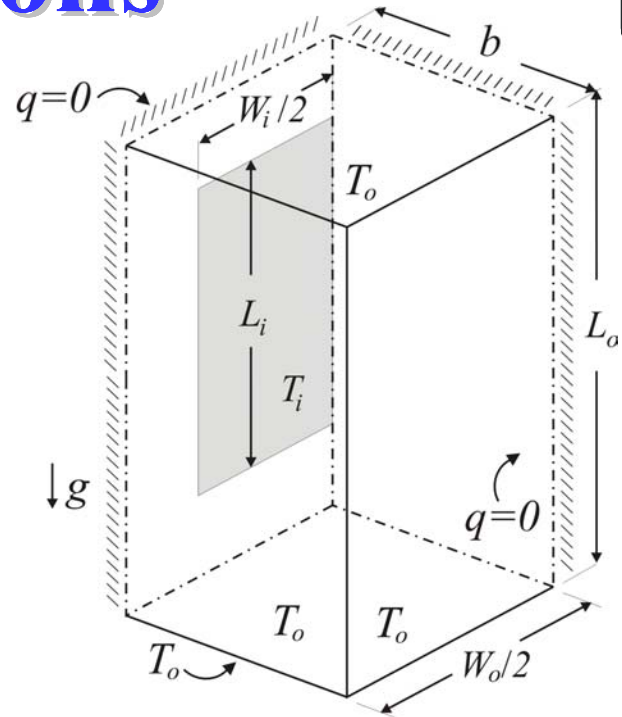
- Conduction shape factor
  - Flotherm simulations of 56 enclosure geometries

$$T_i = 40^{\circ}C \quad T_o = 20^{\circ}C$$

$$L_o/L_i = 1.05, 1.2, 1.6, 2$$

$$L_i/W_i = L_o/W_o = 0.5, 1, 2$$

$$b/L_o = 1 \rightarrow 0.05$$



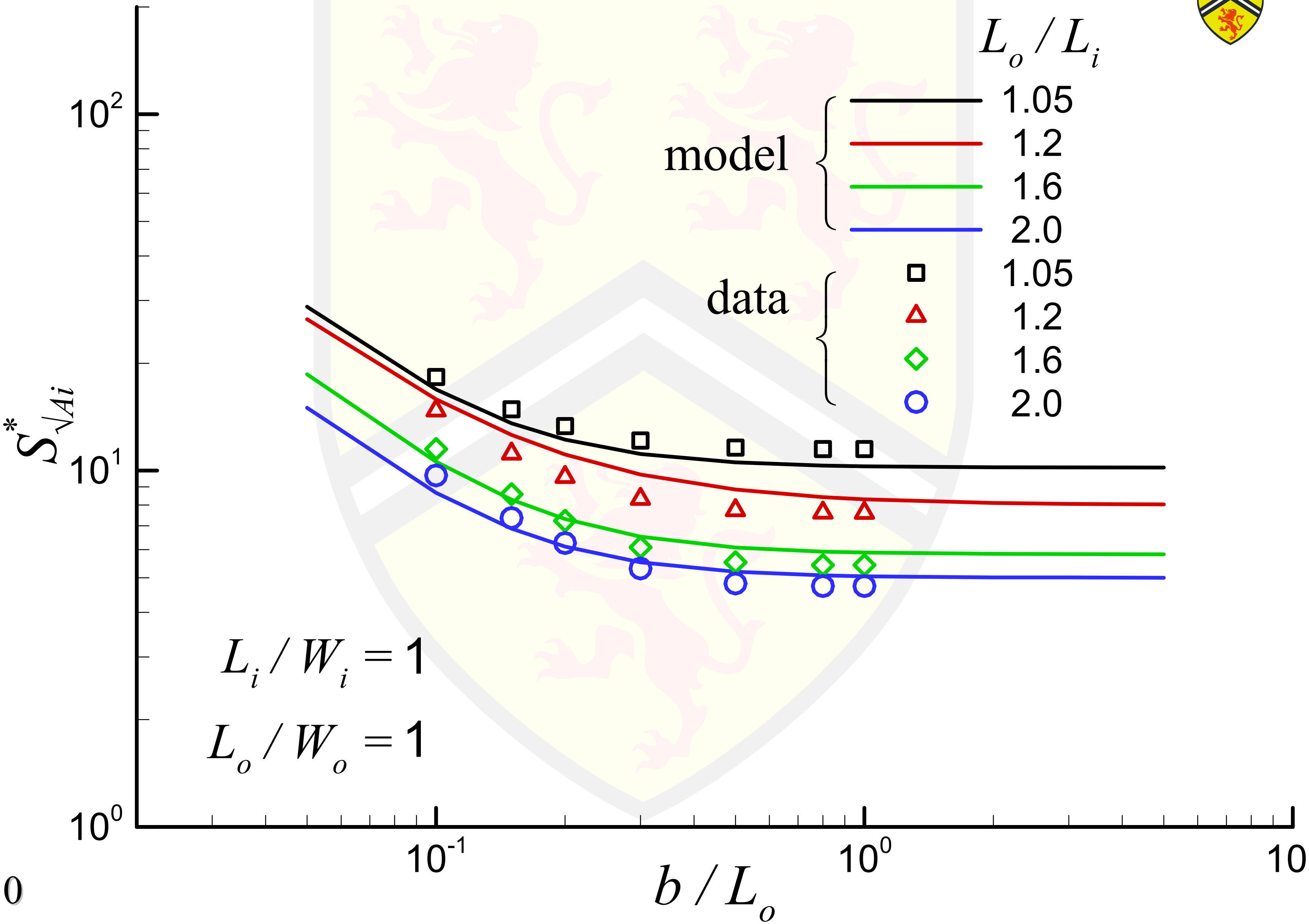
- Natural convection
  - Icepak simulations of 42 enclosure geometries

$$L_o/L_i = 1.05, 1.2, 1.6, 2$$

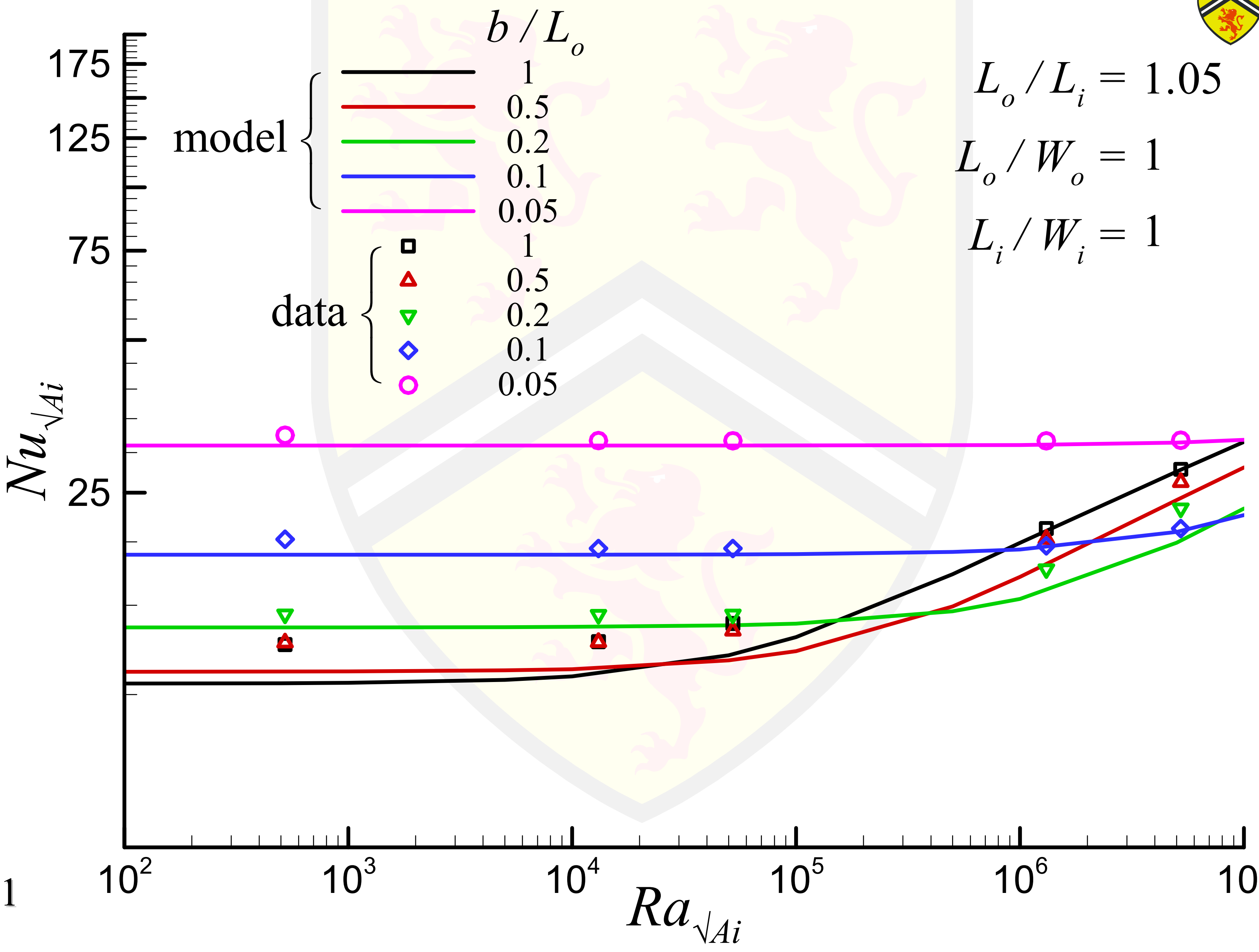
$$L_i/W_i = L_o/W_o = 1, 2$$

$$b/L_o = 1 \rightarrow 0.05$$

Conduction Model Validation



Convection Model Validation



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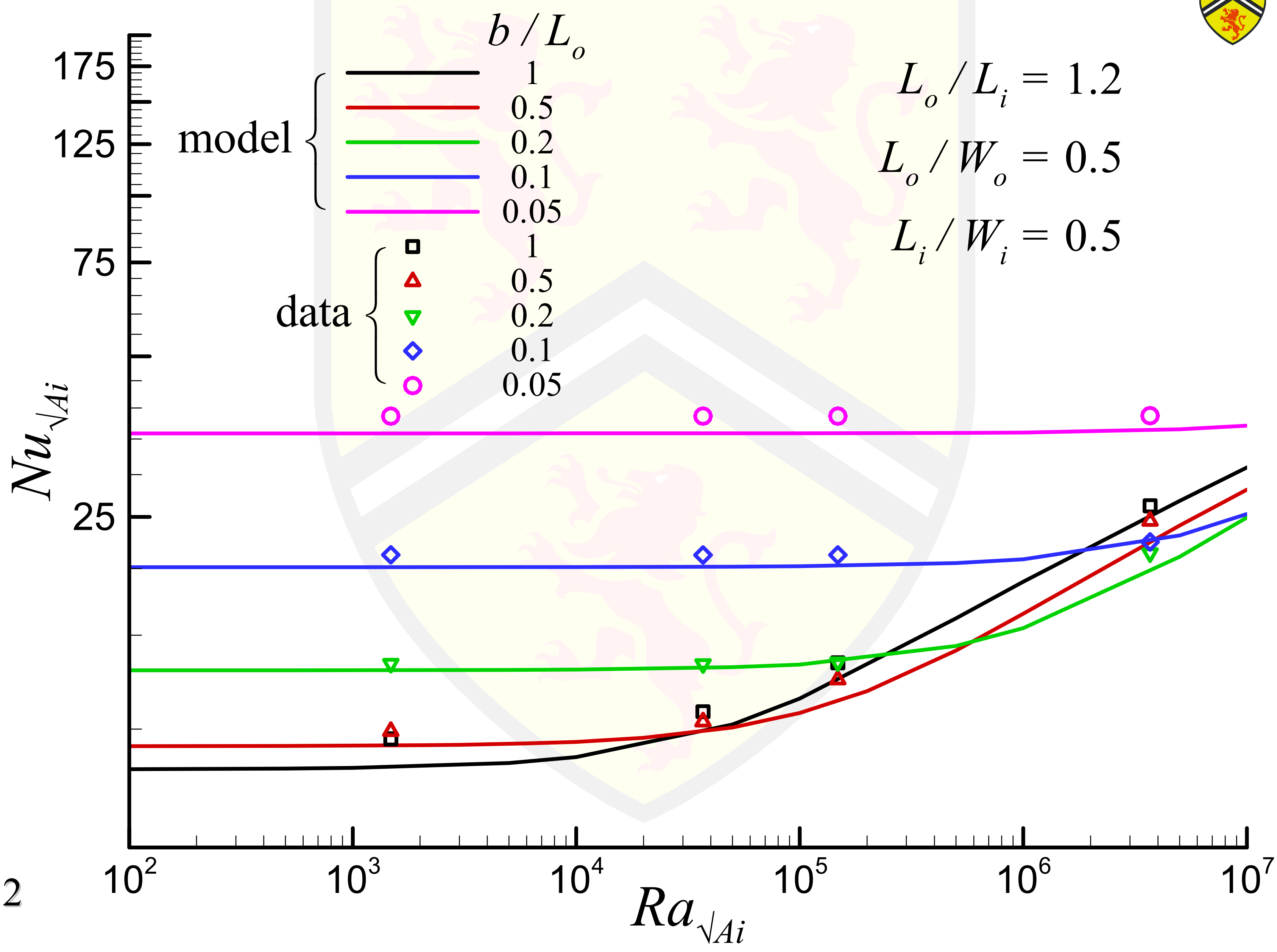

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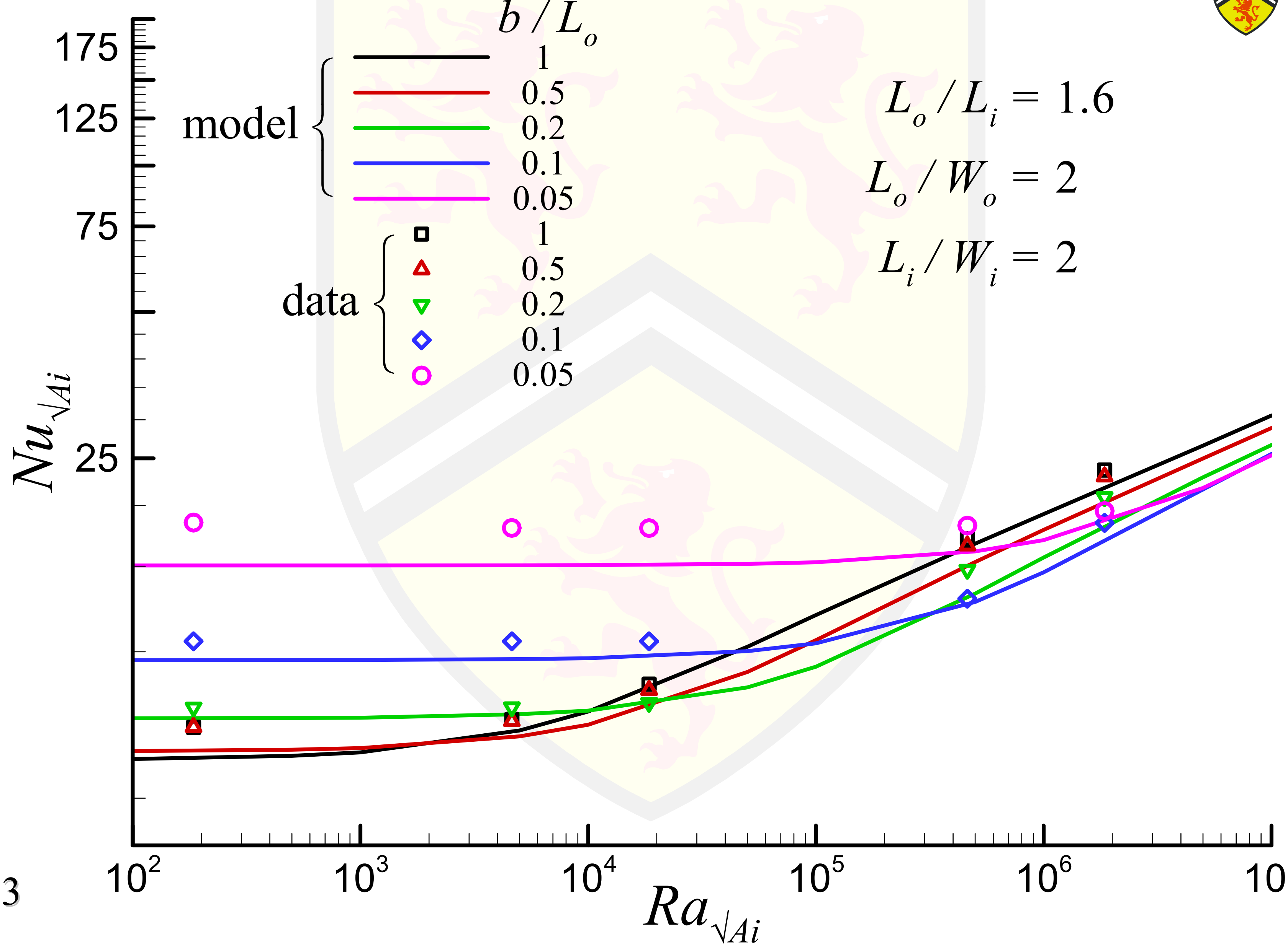
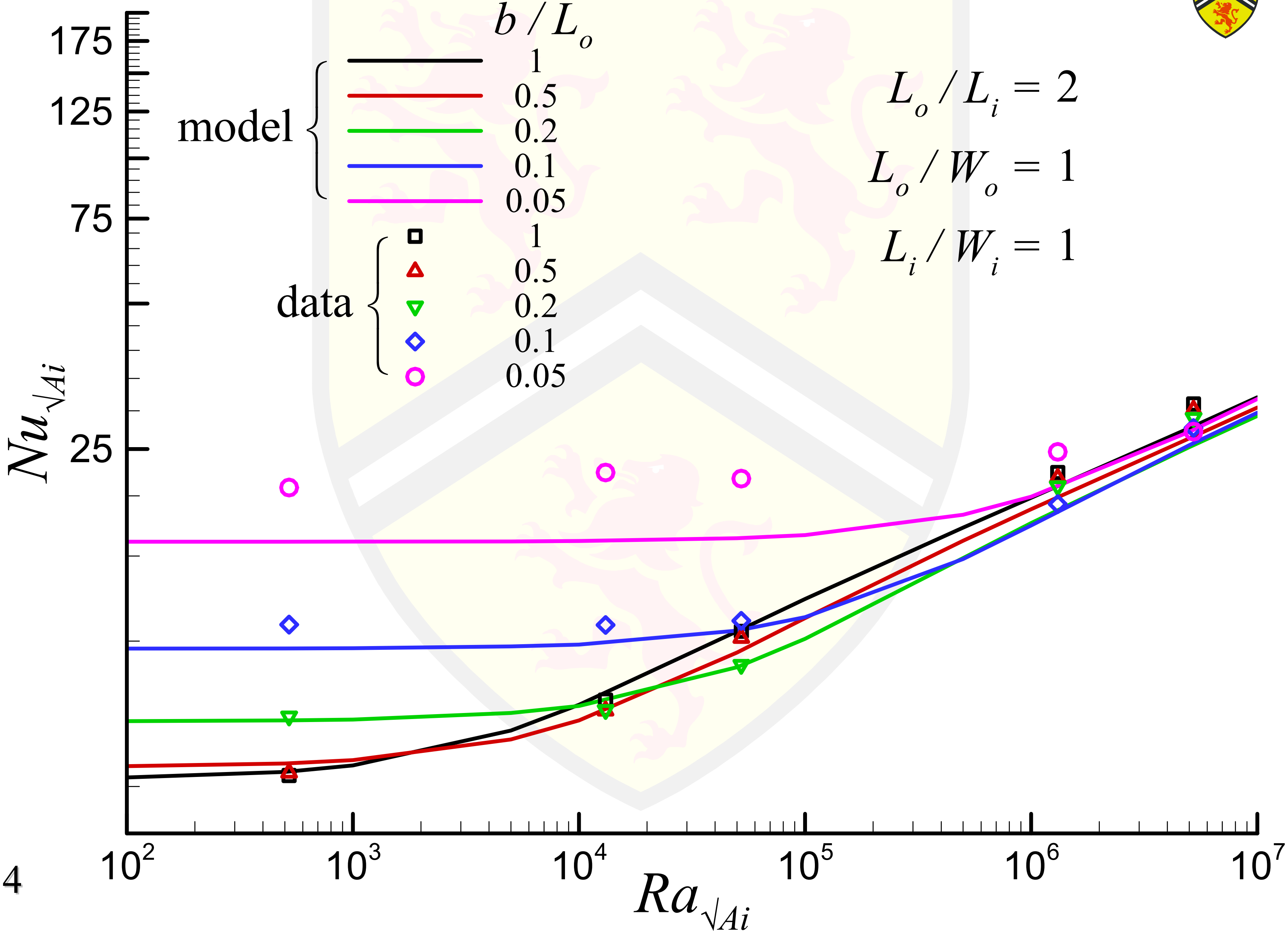


Convection Model Validation



Summary

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- Analytical model developed for natural convection for a vertical plate in a sealed, cuboid shaped enclosure
- Validated with data from CFD simulations
  - 10 % average difference with numerical data
- Demonstrates trends in data as function of geometry and Rayleigh number
- Future work
  - Isoflux inner boundary condition
  - Array of vertical plates
  - Experimental validation of analytical models

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