Thermal-Mechanical Models for Non-Conforming Surface Contacts

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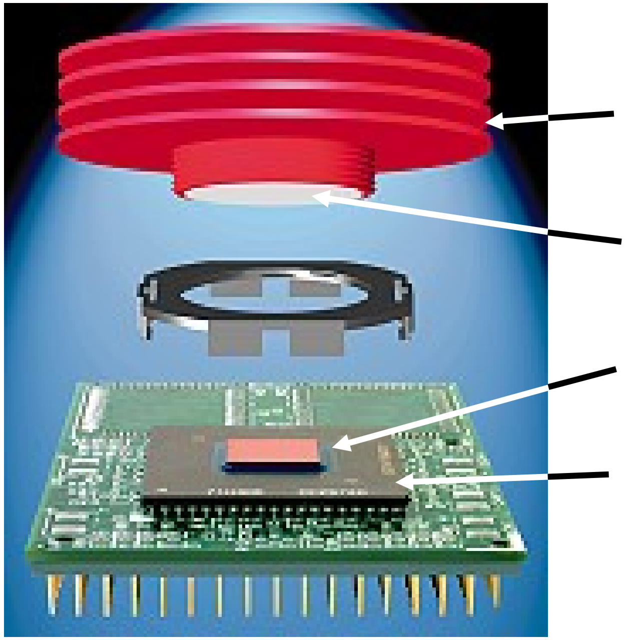
Outline

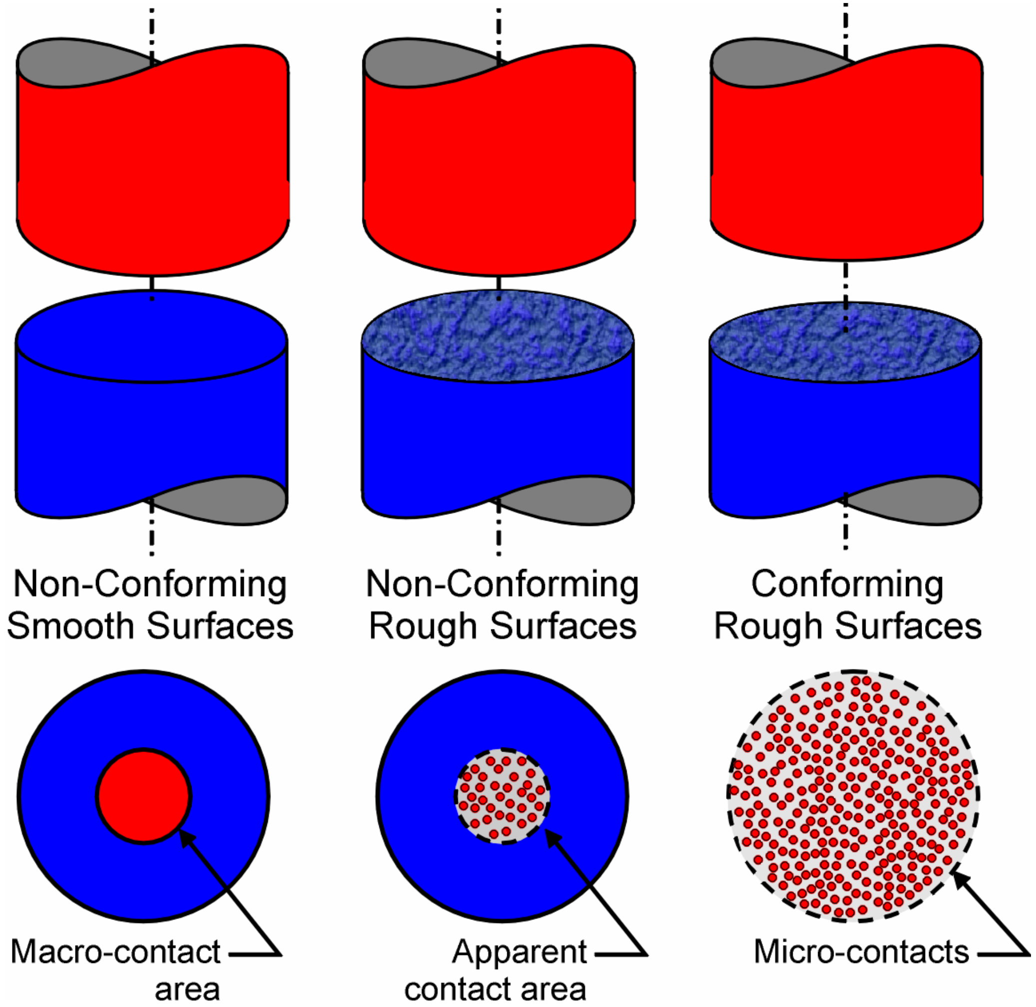
- Introduction
  - types of contacting surfaces
  - types of non-flat surface contacts
- Objectives
- Model development
  - spreading resistance
  - gap resistance
- Sample calculation of joint resistance
- Summary

Introduction: Problem Description

- Steady heat transfer at interface formed by contact between silicon chip and metallic heat sink
- Mechanical contact between two non-conforming, rough surfaces

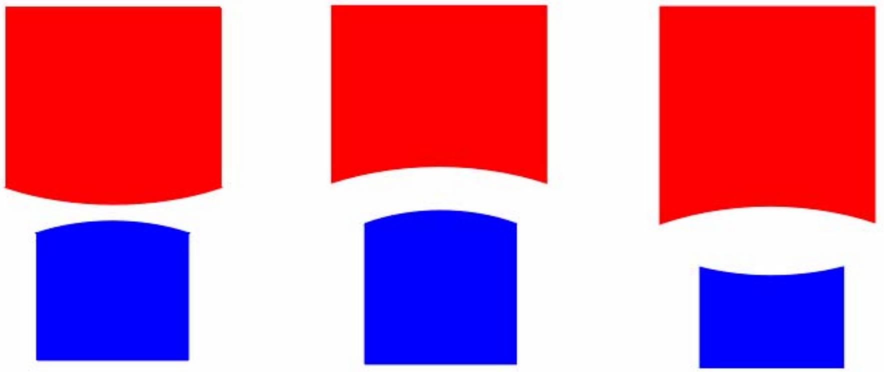


Introduction: Types of Contacts



Introduction: Non-Flat Surfaces

- Contacting surfaces can be concave or convex
- Three possible non-flat surface combinations:



- Large variations in contact areas, gaps and thermal joint resistance models

Objectives

- Develop analytical joint resistance model for metallic heat sink on silicon chip
  - non-conforming (convex/convex), smooth surfaces
  - elastic contact
  - gap filled with gas or liquid
- Assumptions:
  - heat sink modeled as baseplate with effective uniform heat transfer coefficient
  - all edges adiabatic
  - negligible radiative heat transfer across interface

Model Development

- Joint resistance

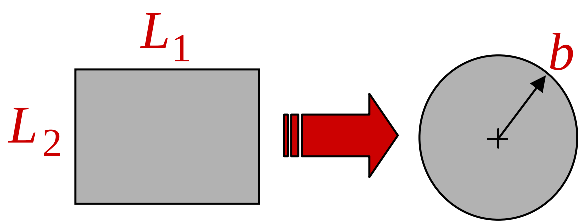
$$\frac{1}{R_j} = \frac{1}{R_s} + \frac{1}{R_g}$$

R_s - spreading/constriction resistance

R_g - gap resistance

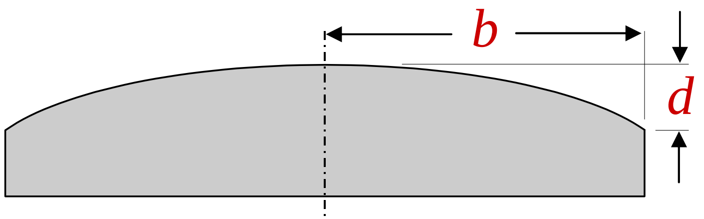
- Rectangular cross sections transformed to equivalent circular disks:

$$b = \sqrt{\frac{L_1 \, L_2}{\pi}}$$



- Radii of curvature for convex contacting surfaces:

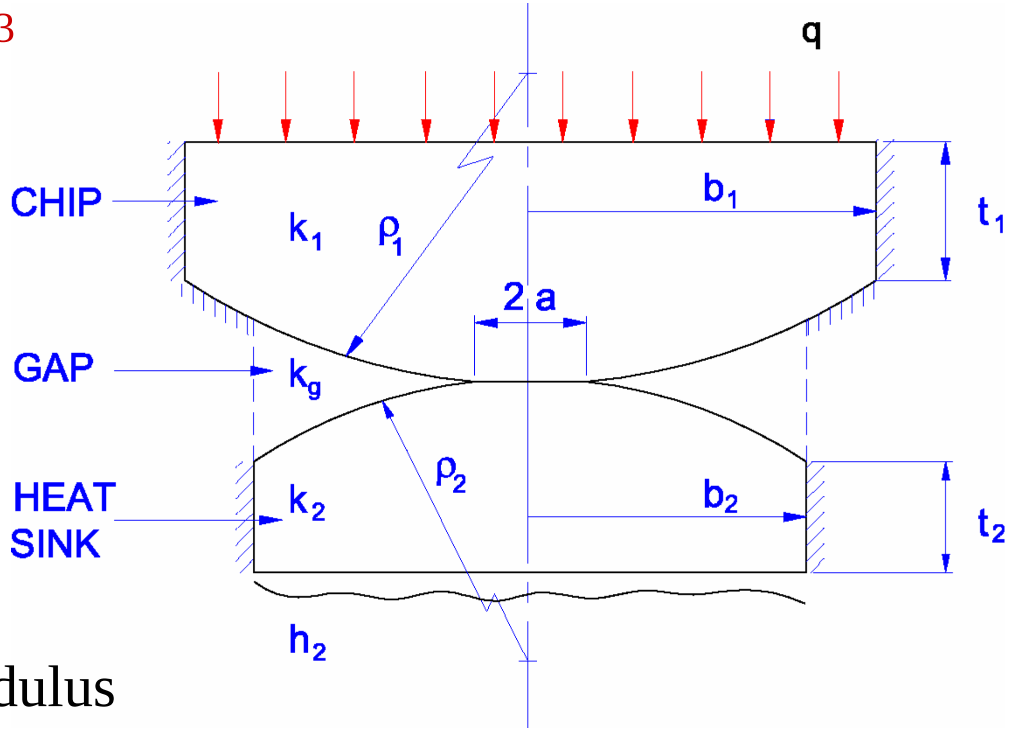
$$\rho = \frac{b^2}{2\,d} = \frac{L_1 \, L_2}{2\,\pi\,d}$$



Model Development

- Contact radius from Hertz elastic contact model:

$$a = \left[\frac{3}{4} F \frac{\rho_1 \rho_2}{\rho_1 + \rho_2} \Delta \right]^{1/3}$$



where:

$$\Delta = \frac{1-\upsilon_1^2}{E_1} + \frac{1-\upsilon_2^2}{E_2}$$

F - load

E_1, E_2 - Young's modulus

υ_1, υ_2 - Poisson's ratios

Spreading Resistance

- Total spreading (constriction) resistance:

$$R_s = R_{s,1} + R_{s,2} = \frac{\psi_1}{4 k_1 a} + \frac{\psi_2}{4 k_2 a}$$

where: $\psi_1 = 4 k_1 a R_{s,1}$

$\psi_2 = 4 k_2 a R_{s,2}$

- Dimensionless spreading resistance:

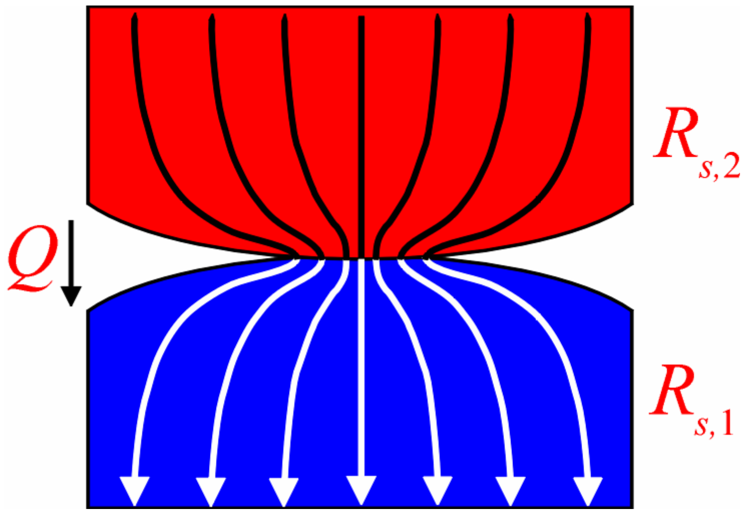
$$\psi = f(\varepsilon, \tau, Bi)$$

$\varepsilon = a/b$ - relative contact size

$\tau = t/b$ - relative thickness

$Bi = h\, b/k$ - Biot number

$$h = \frac{Q_{heat\ sink}}{A_{interface} \cdot \Delta T_{heat\ sink}}$$



Spreading Resistance

- Function of boundary condition at contact area:

isoflux

$$\psi = \frac{16}{\pi\,\varepsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\delta_n\,\varepsilon)}{\delta_n^3\; J_0^2(\delta_n)}\,\phi_n$$

equivalent isothermal

$$\psi = \frac{8}{\pi\,\varepsilon} \sum_{n=1}^{\infty} \frac{J_1(\delta_n\,\varepsilon)\sin(\delta_n\,\varepsilon)}{\delta_n^3\; J_0^2(\delta_n)}\,\phi_n$$

where:

$$\phi_n = \frac{\tanh(\delta_n\,\tau) + \delta_n / Bi}{1 + (\delta_n / Bi)\tanh(\delta_n\,\tau)}$$

$$\delta_n = \frac{\beta_n}{4}\left[1 - \frac{6}{\beta_n^2} + \frac{6}{\beta_n^4} - \frac{4716}{5\,\beta_n^6} + \frac{3902418}{70\,\beta_n^8}\right]$$

$$\beta_n = \pi(4n+1)$$

Spreading Resistance -Calculations

- Three calculation alternatives for spreading resistance:
  - solve general equations with computer algebra system, such as Maple or Mathematica
  - Web-based caculation tools available at:

    http://www.mhtl.uwaterloo.ca

    http://www.idealanswers.com
  - Correlation for infinitely long flux tube limit, $\tau > 0.72$

$$\psi = C_0 + C_1\,\varepsilon + C_3\,\varepsilon^3 + C_5\,\varepsilon^5 + C_7\,\varepsilon^7$$

Gap Resistance

- Elemental flux tube:

$$d\,Q_g = \frac{k_g\,\Delta T_g(r)}{t + M}\,2\,\pi\,r\,dr$$

M - gas rarefaction parameter

k_g - gap thermal conductivity

- Local gap thickness for small deformations:

$$t = t_1 + t_2 = \frac{r^2}{2\,\rho_1} + \frac{r^2}{2\,\rho_2} = (d_1 + d_2)\frac{r^2}{b^2}, \quad 0 \le r \le b$$

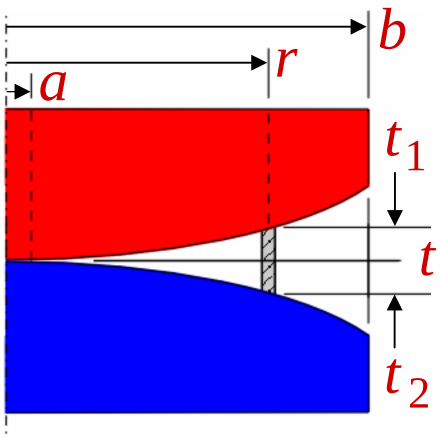
- Gas parameter:

$$M = \alpha\,\beta\,\Lambda$$

α - accommodation parameter

β - gas parameter

Λ - molecular mean-free-path



Gap Resistance

- Total gap heat transfer:

$$Q_g = \iint_{A_g} d\,Q_g = 2\pi k_g \int_a^b \frac{\Delta T_g(r)\, r\, dr}{t + M}$$

- Average gap conductance:

$$h_g = \frac{Q_g}{A_g\,\Delta T_j} = \frac{2 k_g}{d_1 + d_2} \int_a^b \frac{f(r)\, r\, dr}{r^2 + M\, b^2 / (d_1 + d_2)}$$

- Local gap temperature drop:

$$0 \le f(r) = \frac{\Delta T_g(r)}{\Delta T_j} \le 1, \quad a \le r \le b$$

Gap Resistance - Solutions

- For light contact loads where $a << b$, assuming $f(r) = 1$ gives simple, closed form solution:

$$R_g = \frac{d_1 + d_2}{\pi k_g b^2 \ln\left(\frac{b^2 + K}{a^2 + K}\right)}, \quad K = \frac{M b^2}{d_1 + d_2}$$

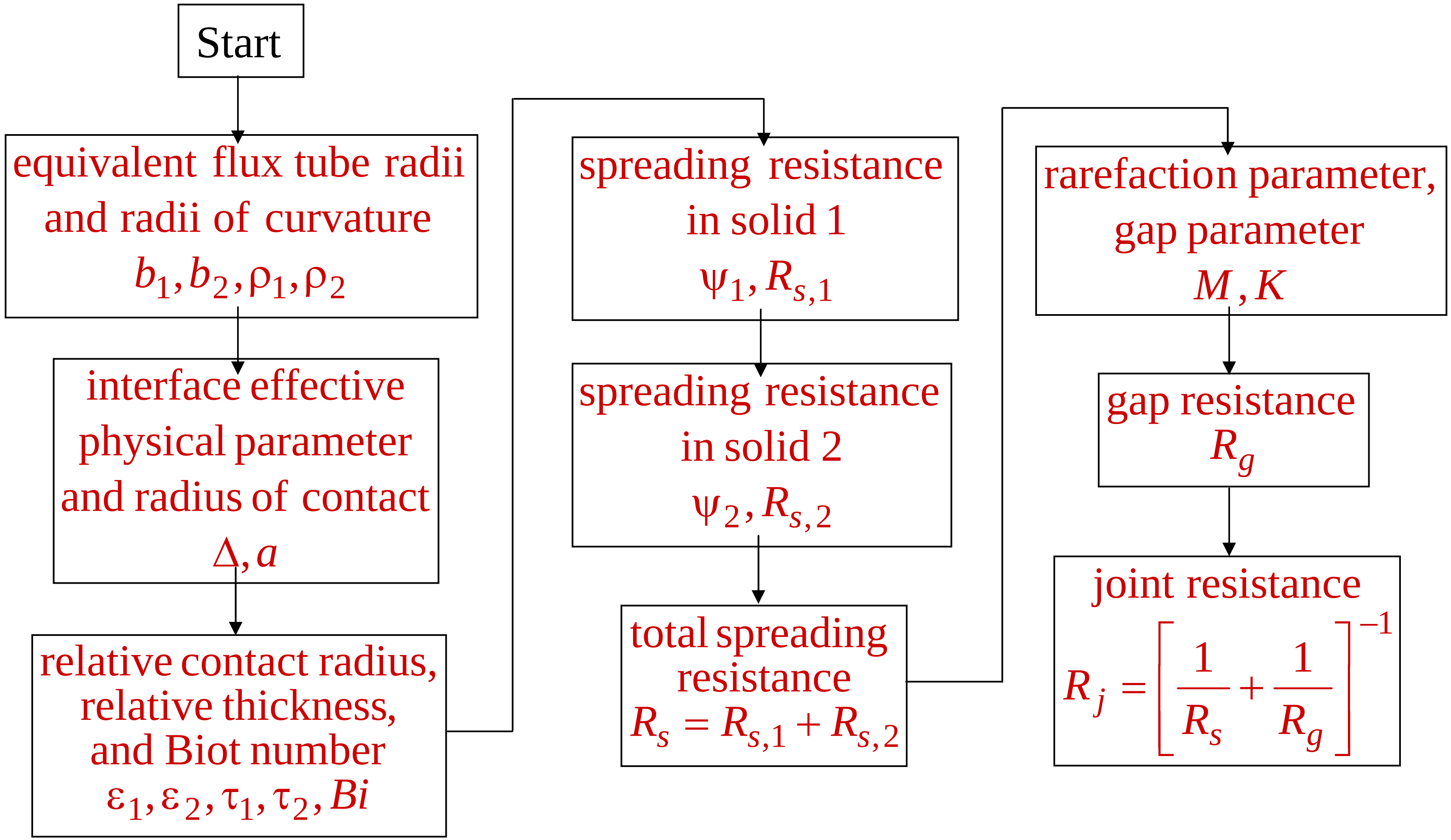
- For liquid or grease in gap:

$$K = M = 0$$

and:

$$R_g = \frac{d_1 + d_2}{\pi k_g b^2 \ln(b/a)}$$

Computation Procedure



Summary

- Models developed for thermal joint resistance for non-conforming, smooth hemispherical surfaces
- Joint resistance consists of spreading and gap resistances connected in parallel
- Gap resistance model developed for various gases, liquids or greases
- Example provided of contact between a silicon chip and an aluminum heat sink

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