COMPACT ANALYTICAL MODELS FOR EFFECTIVE THERMAL CONDUCTIVITY OF ROUGH SPHEROID PACKED BEDS

Majid Bahrami

M. M. Yovanovich

J. R. Culham

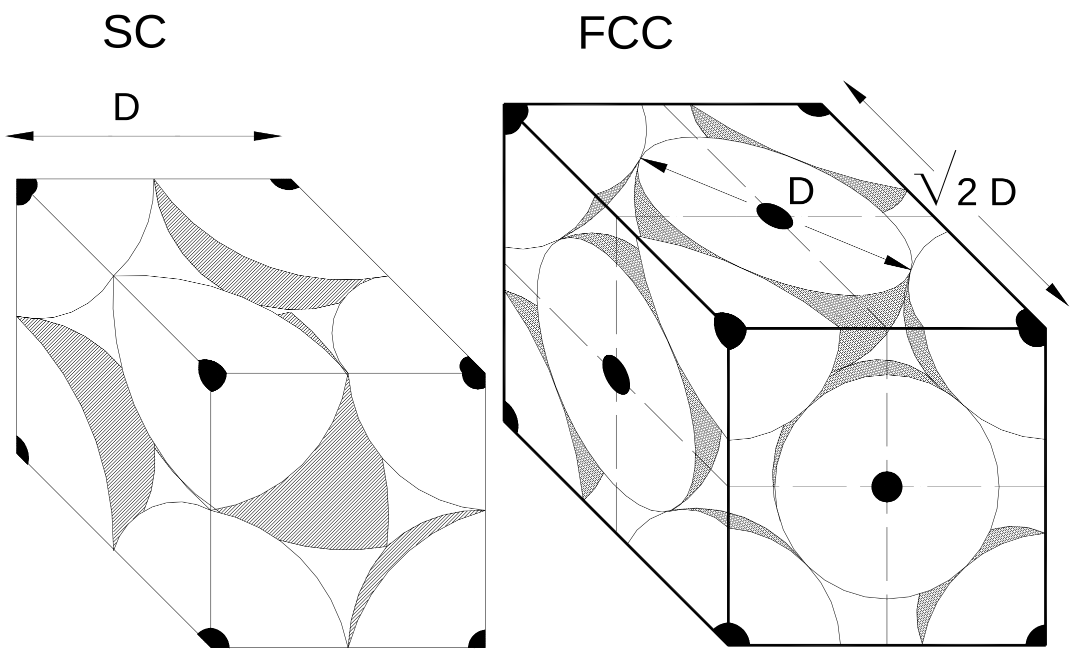
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Ontario, Canada

OVERVIEW

- Introduction
- Motivations and Objectives
- Conduction Through Contact Spots
- Conduction Through Interstitial Gas
- Present Model
- Comparison with Experimental Data
- Conclusions

INTRODUCTION

- high ratio of solid surface area to volume.
- packed beds applications:
  – Catalytic reactors, heat recovery systems, heat exchangers, heat storage systems, and insulators
- regular packing: Simple Cubic (SC), Body Center Close (BCC), and Face Center Close (FCC)



MOTIVATIONS AND OBJECTIVES

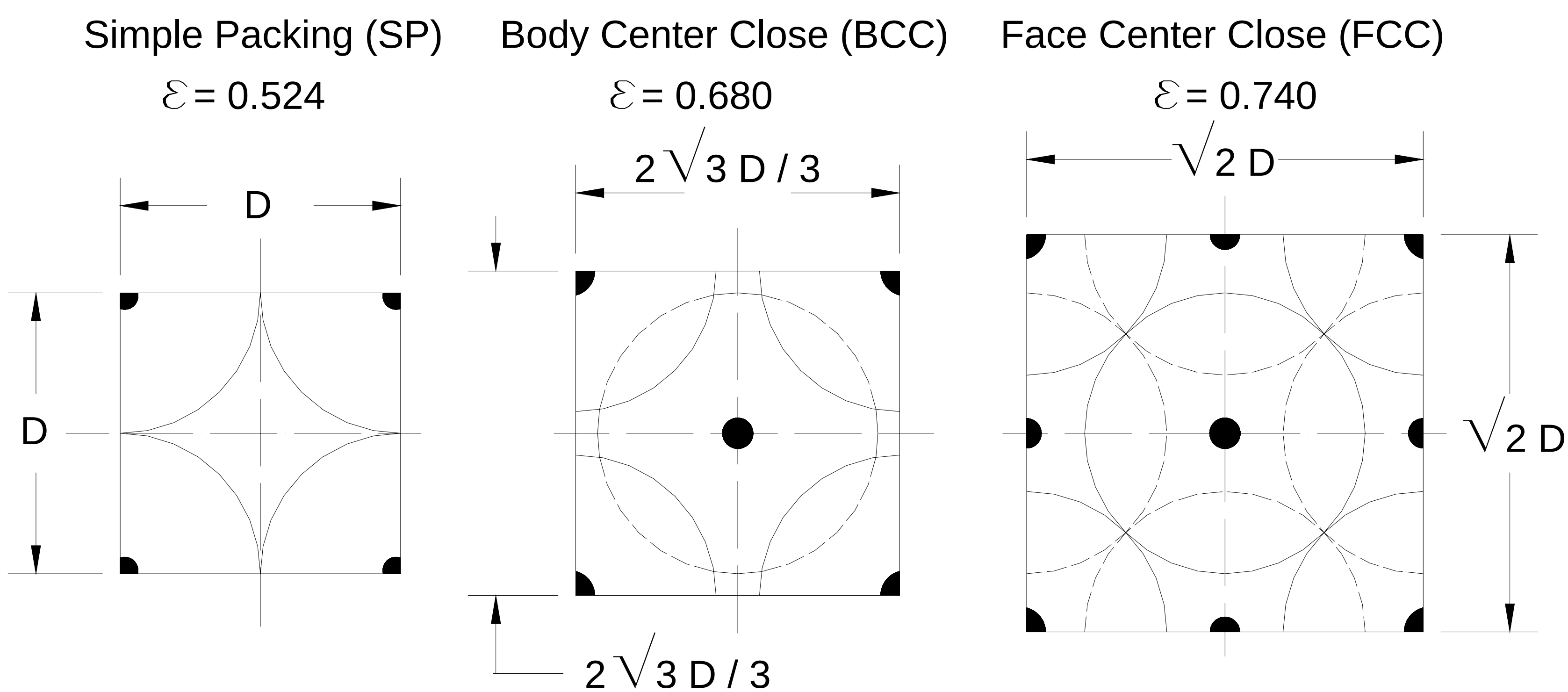
- existing models can be categorized into:
  - numerical (FEM) models:
    - Buonanno et al. : time consuming, B.C. must be fed into the code for thermal contact resistance
  - analytical models:
    - Slavin et al. : a point contact between spheres assumed
    - Ogniewicz & Yovanovich and Turyk & Yovanovich: limited to smooth spheres
- develop compact models for determining effective thermal conductivity that account for:
  - roughness
  - gas rarefaction effect
  - contact load
  - gas temperature and pressure

REGULAR PACKED BED ARRANGMENTS

- solid fraction is defined

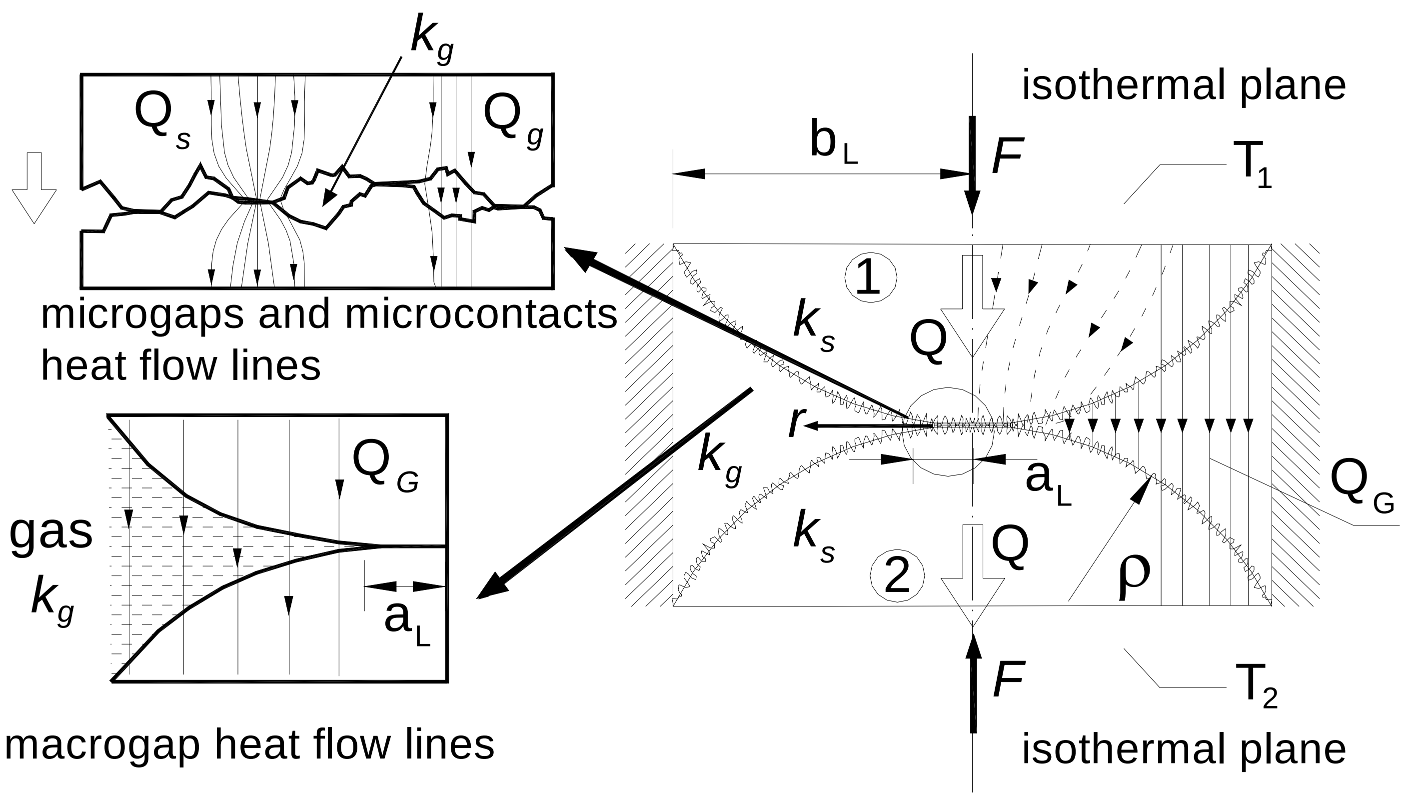
$$\varepsilon = V_s / V$$

Packing	Solid fraction
SC	0.524
BCC	0.680
FCC	0.740



two main paths for transferring thermal energy in packed beds are:

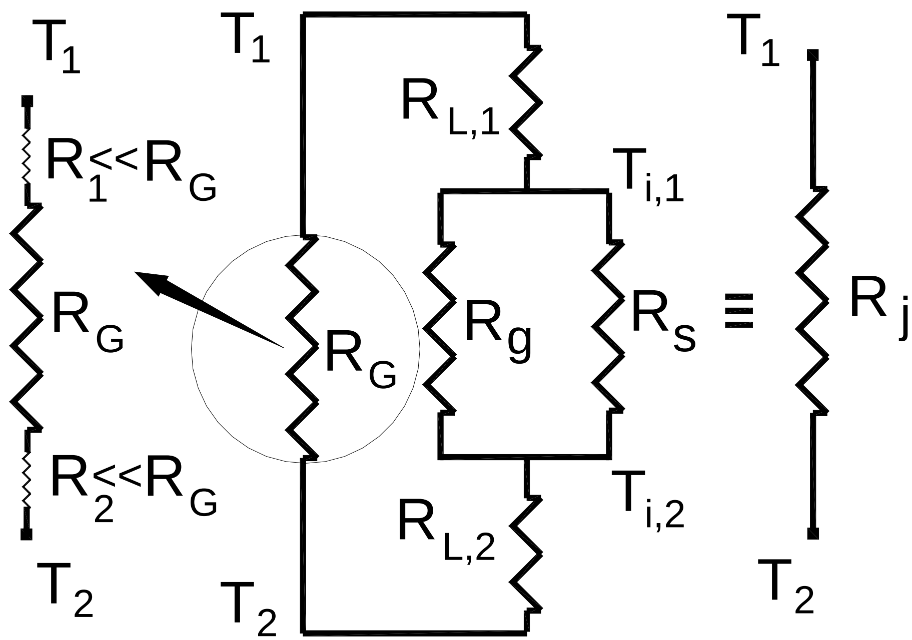
- conduction through microcontacts
- heat transfer through interstitial gas



THERMAL RESISTANCE NETWORK

thermal joint resistance network components:

- macro-constriction, $\mathbf{R_L}$
- micro-constriction, $\mathbf{R_s}$
- microgap resistance, $\mathbf{R_g}$
- macrogap, $\mathbf{R_G}$



$$R_j = \left[\frac{1}{\left(1/R_s + 1/R_g\right)^{-1} + R_L} + \frac{1}{R_G} \right]^{-1}$$

CONDUCTION THROUGH CONTACT SPOTS

- macro-constriction resistance,
  R, Bahrami et al. [17]

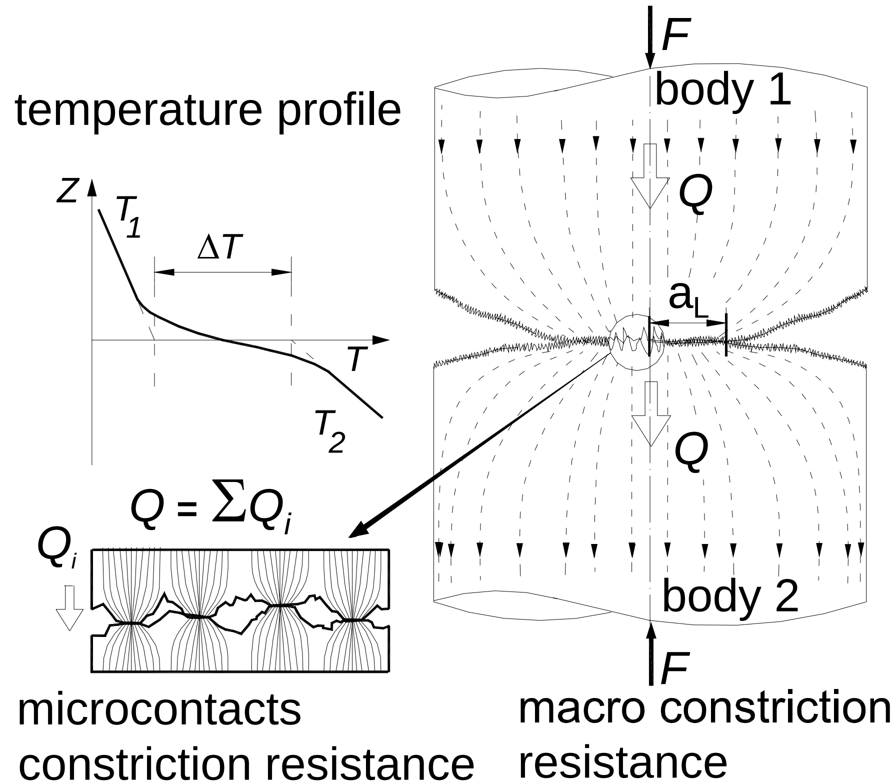
$$P(\xi) = P_0\left(1-\xi^2\right)^{\gamma}$$

$$P_0^{'} = \frac{P_0}{P_{0,H}} = \frac{1}{1+1.37\alpha\,\tau^{-0.075}}$$

$$\alpha = \sigma\rho / a_H^2 \quad \text{and} \quad \tau = \rho / a_H$$

$$\frac{a_L^{'}}{a_H} = \begin{cases} 1.605/\sqrt{P_0^{'}} & 0.01 \le P_0^{'} \le 0.47 \\ 3.51 - 2.51P_0^{'} & 0.47 \le P_0^{'} \le 1 \end{cases}$$

$$R_L = \frac{1}{2k_s\, a_L}$$



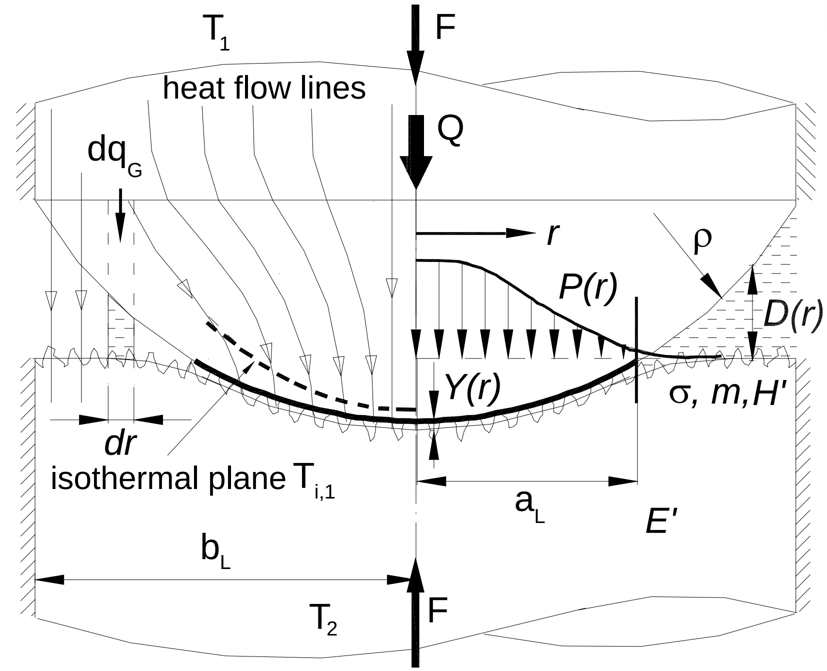
- micro-constriction resistance, R_s
Bahrami et al. [14]

$$R_s = \frac{0.565H^{*}(\sigma/m)}{k_s F}$$

CONDUCTION THROUGH INTERSTITIAL GAS

conduction regimes in a gas layer between two parallel plates:

- continuum
- temperature-jump or slip
- transition
- free-molecular



<u>microgap resistance</u>, R_g, Bahrami et al. [26]

<u>macrogap resistance</u>, R_G, Bahrami et al. [25]

$$R_g = \frac{\sqrt{2}\,\sigma\, a_2}{\pi k_g a_L^2 \ln\left(1 + \frac{a_2}{a_1 + M/\sqrt{2}\,\sigma}\right)}$$

$$a_1 = \text{erfc}^{-1}\left(\frac{2P_0}{H'}\right) \quad \text{and} \quad a_2 = \text{erfc}^{-1}\left(\frac{0.03P_0}{H'}\right) - a_1$$

$$2\pi k_g R_G = \frac{1}{S \ln\left(\frac{S-B}{S-A}\right) + B - A}$$

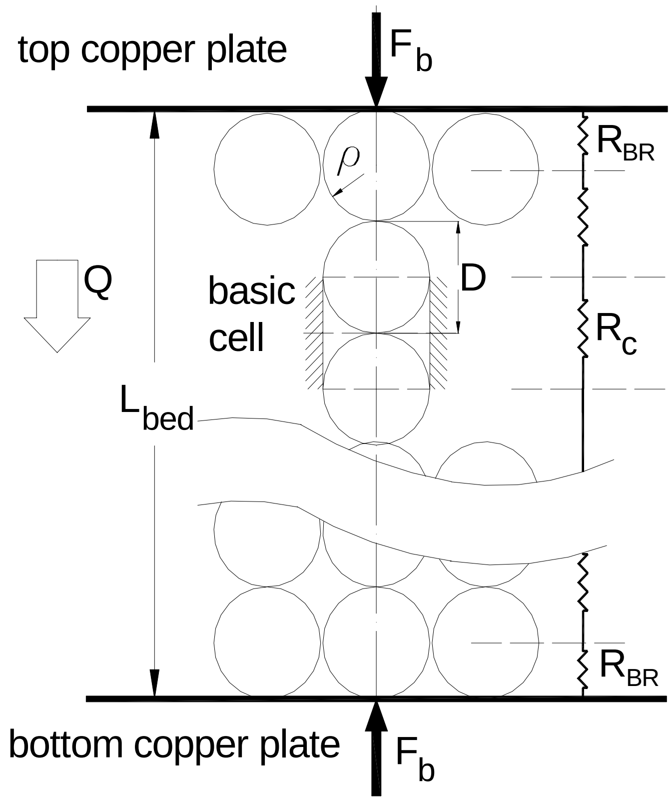
$$A = \sqrt{\rho^2 - a_L^2} \quad B = \sqrt{\rho^2 - b_L^2} \quad S = \rho - \omega_0 + M$$

CONDUCTION IN BASIC CELLS

steps to determine the bed conductivity:

- calculate the relation between the apparent load and contact load
- break up the unit cell into contact regions
- calculate the thermal joint resistance of a contact region
- determine the effective conductivity

$$k_c = \frac{L_c}{R_c A_c}$$



Buonanno et al. [3, 5] experimental apparatus
100Cr6 s. steel spheres
Length of the bed = 15 cm
D = 19.05 mm
spheres H = 8.32 GPa
spheres E = 200 GPa
spheres k = 60 W/mK

air at 20 C, 1 atm.
air k = 0.027 W/mK

copper k = 398 W/mK
copper E = 117 GPa

SC packing is shown.

COMPARISON WITH EXPERIMENTAL DATA

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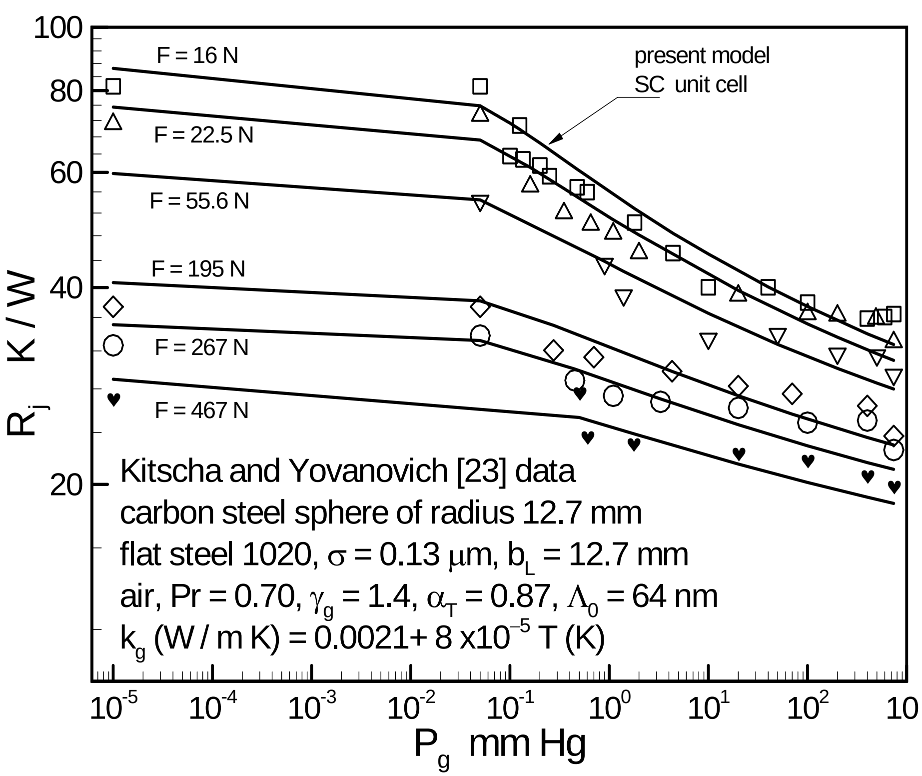
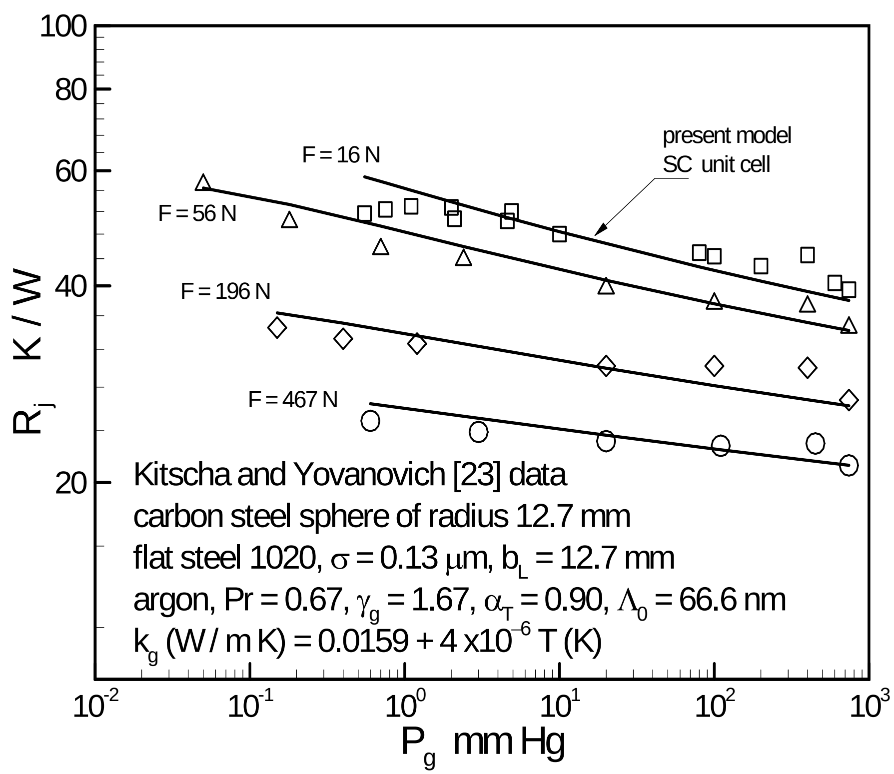
Kitscha and Yovanovich (1974) SC basic cell data



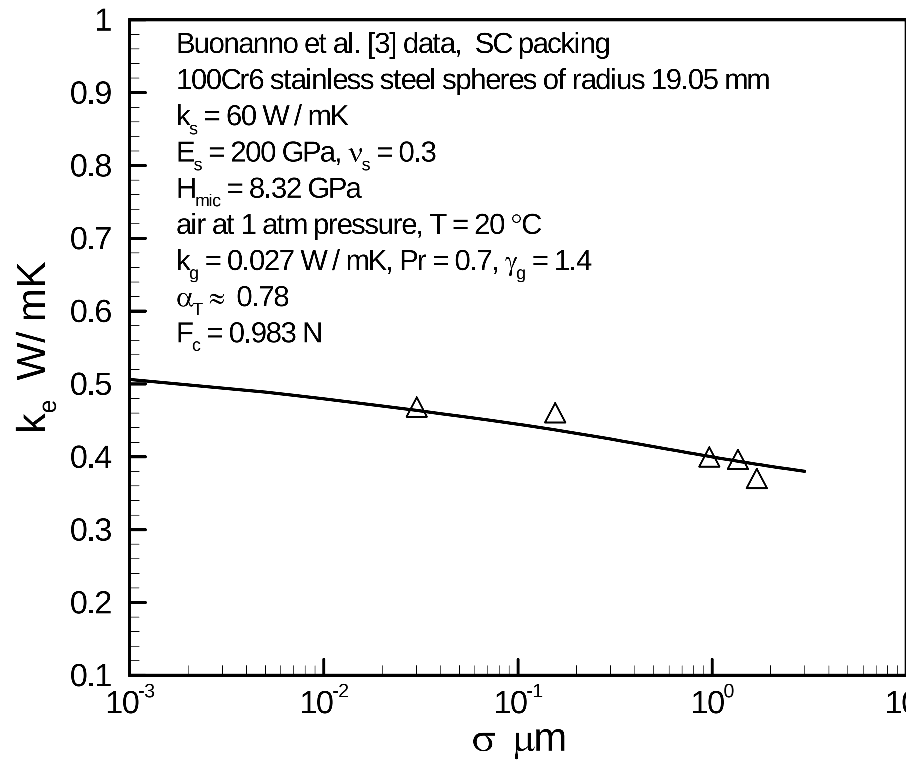
air

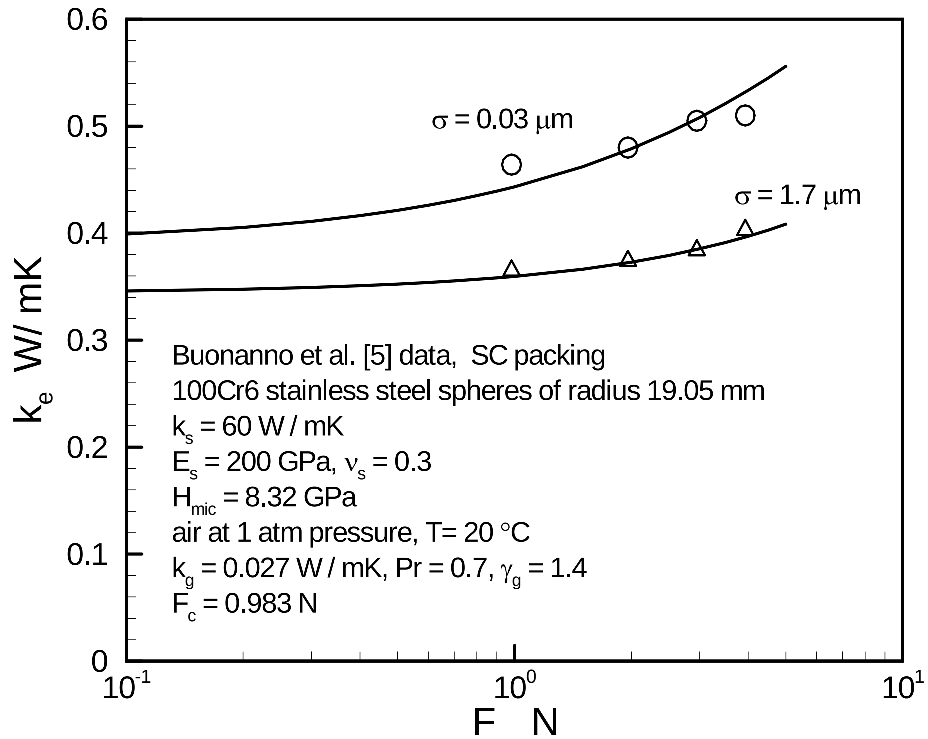


argon

COMPARISON WITH EXPERIMENTAL DATA

SC packed bed, Buonanno et al. (2003) data





atmospheric air

SUMMARY AND CONCLUSIONS

- compact models are proposed for determining effective thermal conductivity in regularly packed beds, SC and FCC arrangements
- present model accounts for thermophysical properties of spheres and gas, load, roughness, gas temperature and pressure, and gas rarefaction effects
- the present model is compared against experimental data, both SC and FCC, over a variety of packed bed conditions and good agreement is observed

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