A Scale Analysis Approach to Thermal Contact Resistance

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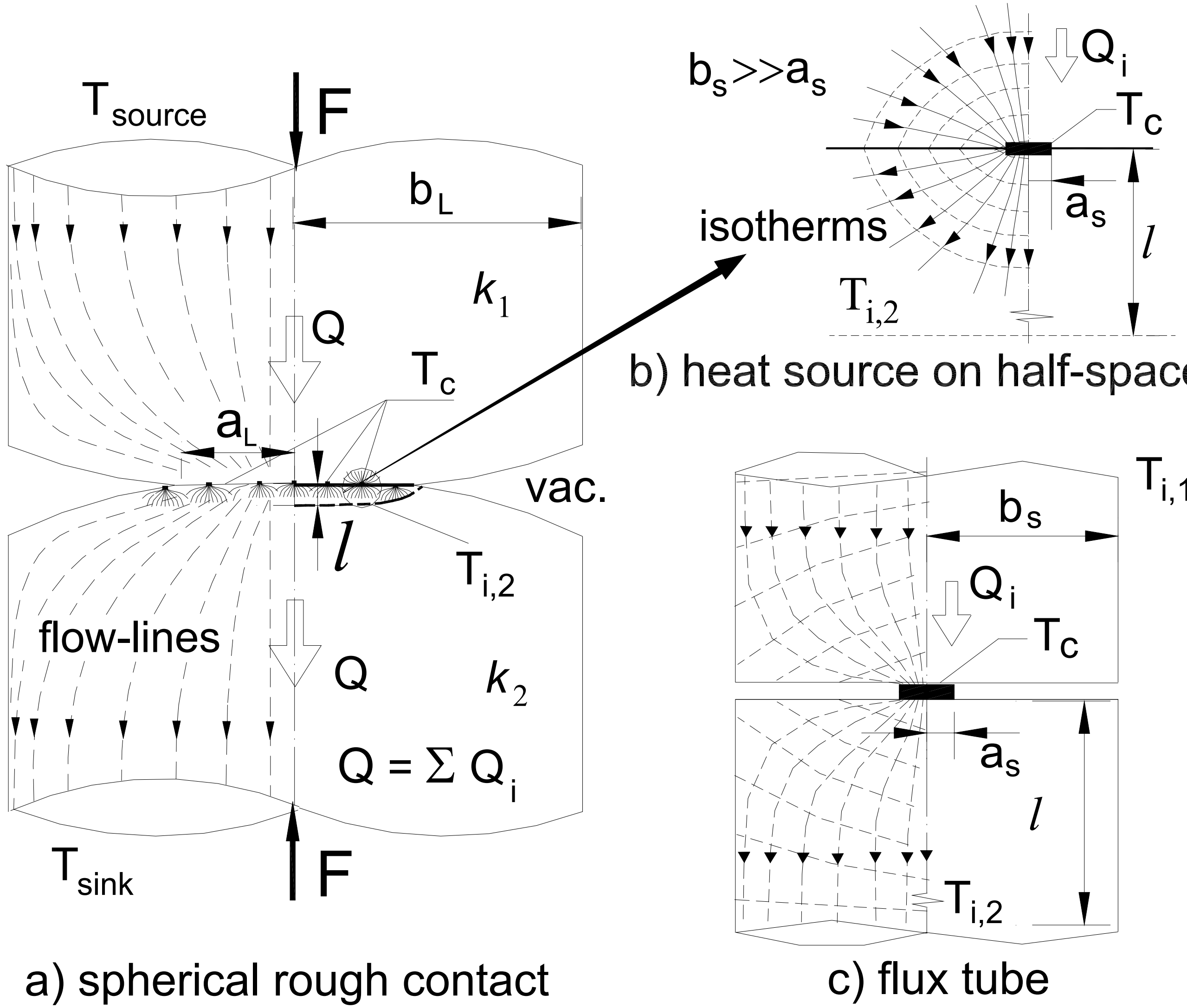
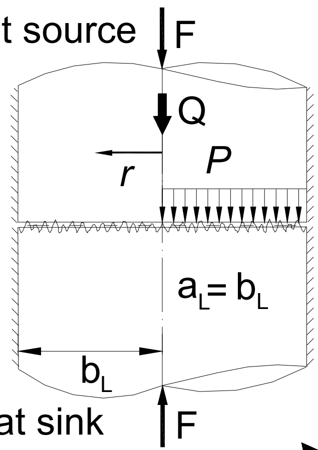
AGENDA

- introduction
- macro and micro thermal resistances
- brief review
- present model assumptions
- scale relations between microcontact and roughness
- general model: conforming rough contacts
- general model: non-conforming rough contacts
- comparison with experimental data
- summary and conclusions
- acknowledgements

NON-CONFORMING ROUGH CONTACT

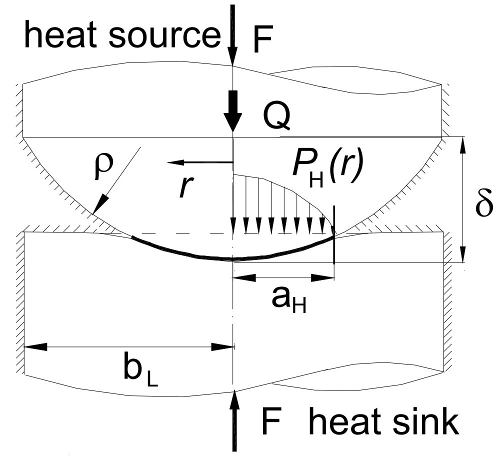


MACRO & MICRO THERMAL RESISTANCES



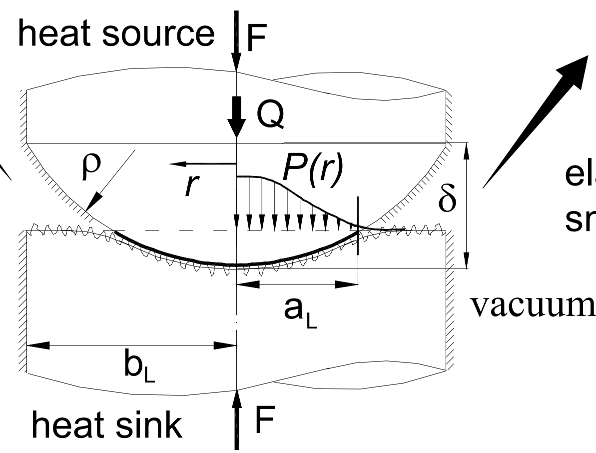
$$R_j = R_s$$

conforming rough limit



$$R_j = R_L$$

elastoconstriction limit
smooth sphere-flat contacts



$$R_j = R_s + R_L$$

general contact, transition region

MICROHARDNESS

Hegazy (1985)

• effective microhardness is significantly greater than bulk hardness

• microhardness decreases with increasing depth of indentation until bulk hardness level

$$H_v \quad c_1 \; d_v \quad {}^{c_2} \qquad\qquad d_v \quad d_v / d_0$$

Sridhar and Yovanovich (1994)

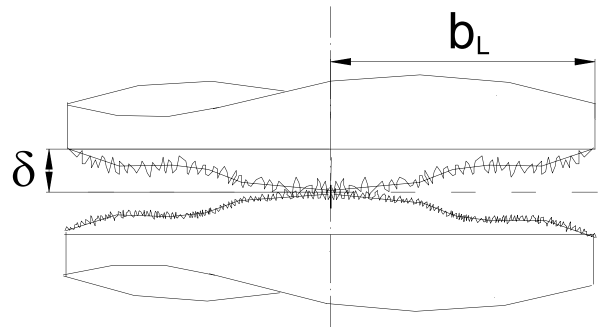
• suggested empirical relations to estimate Vickers microhardness coefficients using bulk hardness

$$c_1 = 3.178\left(4.0 - 5.77\kappa + 4.0\kappa^2 - 0.61\kappa^3\right)$$

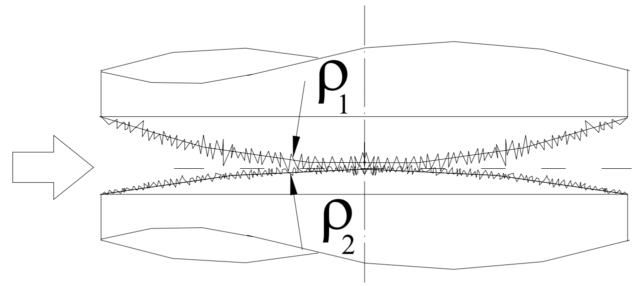
$$c_2 = -0.57 + 0.82\kappa - 0.41\kappa^2 + 0.06\kappa^3$$

$$\kappa = H_B / 3.178 \qquad\qquad 0.41 \le \kappa \le 2.39$$

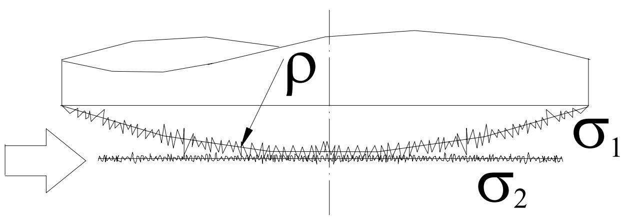
GEOMETRICAL MODELING



a) contact of non-conforming rough surfaces

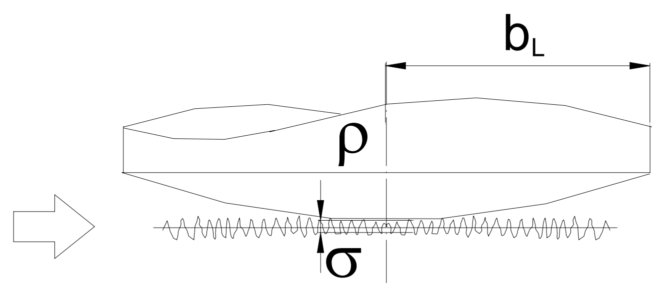


b) contact of two rough spherical segments



c) rough sphere-flat contact, effective radius of curvature

$$\rho = \frac{b_L^2}{2\delta}$$



d) equivalent sphere-flat contact, effective radius and roughness

$$\frac{1}{\ } \qquad \frac{1}{\ _1} \qquad \frac{1}{\ _2}$$

$$\sqrt{\ _1^2 \qquad \ _2^2} \quad \text{and} \quad m \qquad \sqrt{m_1^2 \quad m_2^2}$$

PRESENT MODEL ASSUMPTIONS

- surfaces have Gaussian roughness distribution
- steady-state heat transfer
- radiation heat transfer is negligible
- contact is static and in a vacuum
- first-time contact and clean surfaces

PRESENT MODEL ASSUMPTIONS

- elastic bulk deformation
- microcontacts deform plastically
- contacting solids are thick relative to roughness
- microcontacts are circular and isothermal
- surface curvature is spherical

MICROCONTACT AND SURFACE ROUGHNESS

- microcontact radius is proportional to surface roughness

$$a_s \sim \sigma$$

- microcontact radius is inversely proportional to surface slope

$$a_s \sim 1 / m$$

- combining above relations, microcontact radius scale is

$$a_s \sim \sigma / m$$

CONFORMING ROUGH CONTACTS

microhardness
$$H_{mic} \propto H' \equiv c_1 \left(\sigma / m \sigma_0\right)^{c_2}$$

force balance
$$F = A_r H_{mic} \propto \pi n_s \left(\sigma / m\right)^2 H'$$

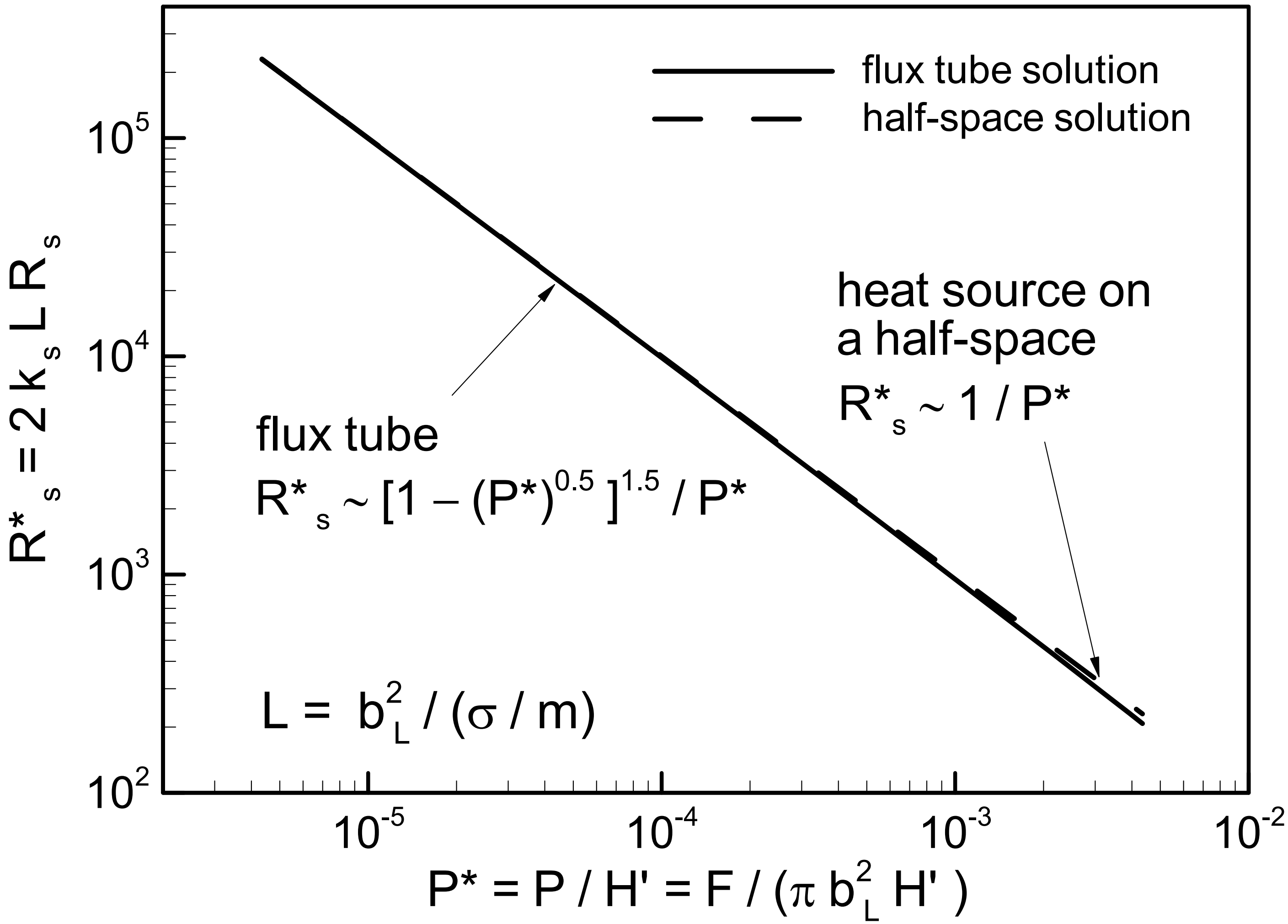
real contact area
$$A_r = \pi n_s a_s^2 \propto \pi n_s \left(\sigma / m\right)^2$$

micro resistance, half-space
$$R_{s,\text{half-space}} = \frac{1}{2 k_s n_s a_s} \propto \frac{1}{2 k_s n_s \left(\sigma / m\right)}$$

micro resistance, flux tube
$$R_{s,\text{flux tube}} = \frac{\psi\left(\varepsilon_s\right)}{2 k_s n_s a_s} \propto \frac{\psi\left(\varepsilon_s\right)}{2 k_s n_s \left(\sigma / m\right)}$$

$$\psi\left(\varepsilon_s\right) = \left(1 - \varepsilon_s\right)^{1.5}$$

FLUX TUBE and HALF-SPACE SOLUTIONS



SCALE RELATION CONSTANT

- using isothermal heat source on half-space solution, effective micro thermal resistance becomes

$$R_s^* = \frac{c}{P^*}$$

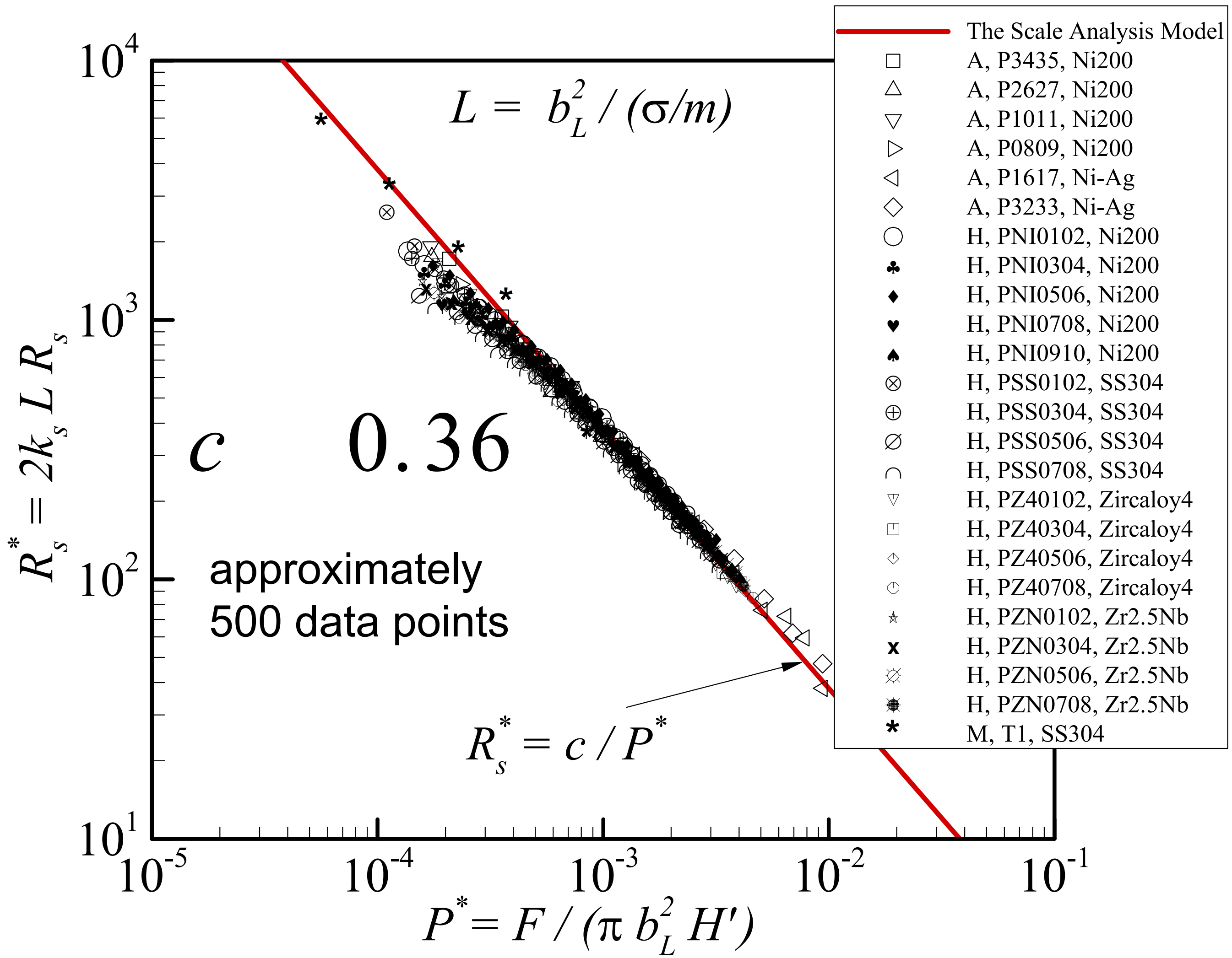
- in dimensional form,

$$R_s = \frac{\pi c(\sigma / m) H'}{2 k_s F}$$

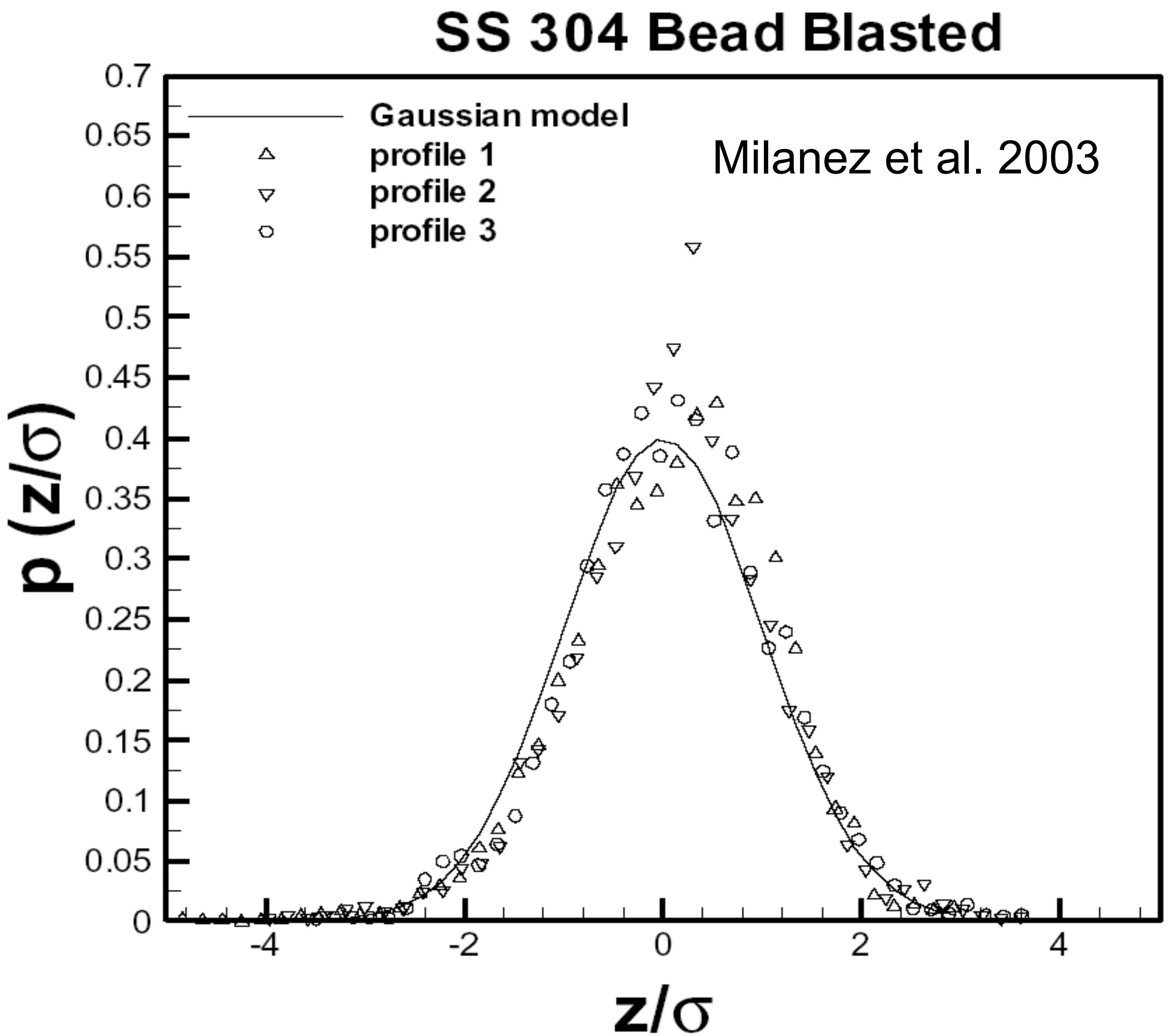
$$h_s = \frac{2}{\pi c} k_s \left(\frac{\sigma}{m}\right) \frac{P}{H'}$$

Ref.	Researcher	Material(s)
A	Antonetti (1983)	Ni200 Ni200-Ag
B	Burde (1977)	SPS 245, CS
CC	Clausing-Chao (1963)	Brass Anaconda Mg AZ 31B SS303
F	Fisher (1985)	Ni 200-Carbon Steel
H	Hegazy (1985)	Ni200 SS304 Zircaloy4 Zr-2.5%wt Nb
K	Kitscha (1982)	Steel 1020-CS
MM	McMillan-Mikic (1970)	SS303
MR	Mikic-Rohsenow (1966)	SS305
M	Milanez et al. (2003)	SS304

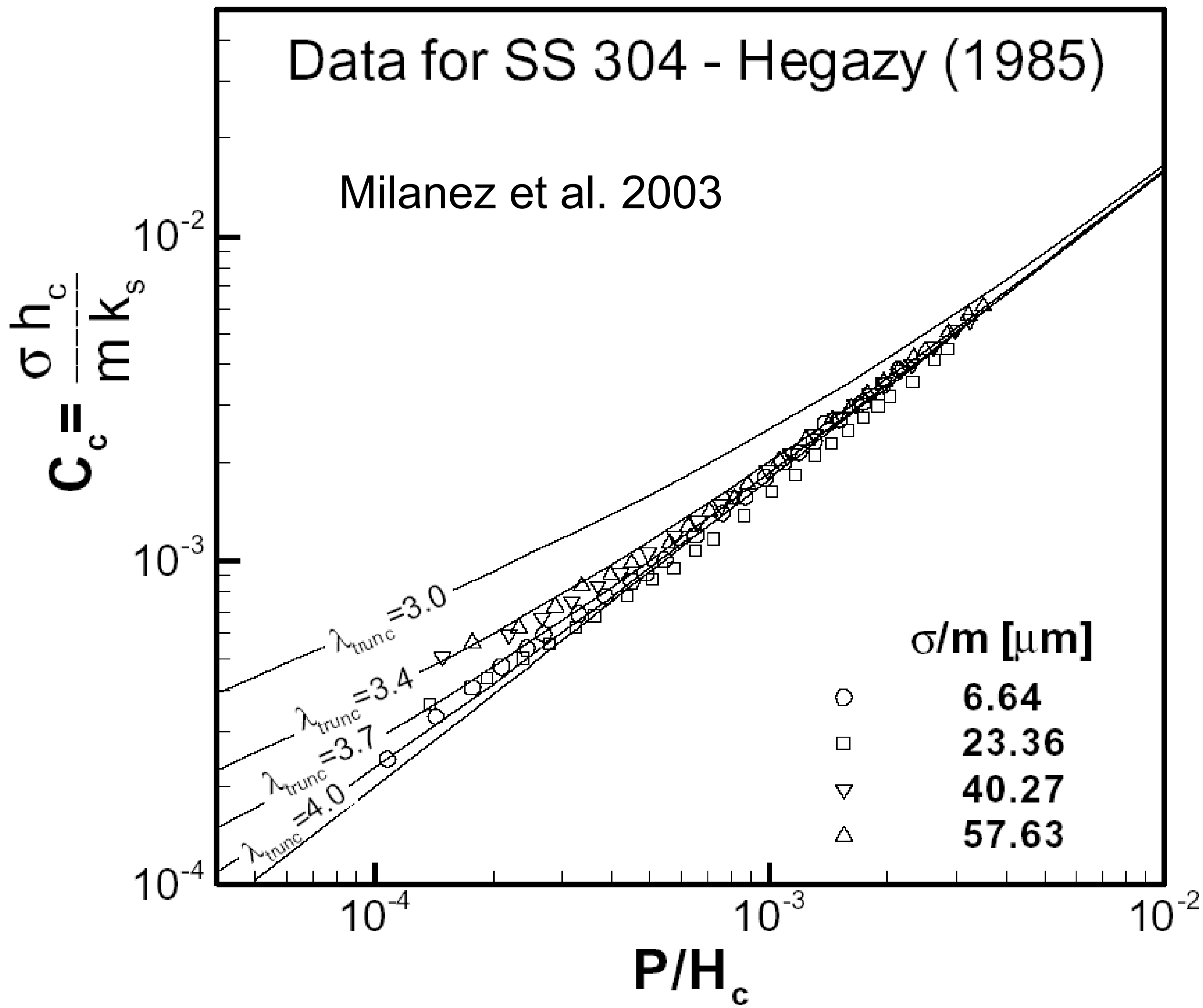
CONFORMING ROUGH CONTACTS



ASPERITY HEIGHTS DISTRIBUTION



TRUNCATION EFFECT



GENERAL MODEL

- radius of macrocontact, Bahrami et al. (2003)

$$a_L = 1.80 a_H \frac{\sqrt{\alpha + 0.31\tau^{0.056}}}{\tau^{0.028}}$$

$/a_H^2$

$/a_H$

- macro resistance $R_L = \left(1 - a_L / b_L\right)^{1.5} / 2k_s a_L$

- energy balance over macrocontact area, vacuum

$$Q = 2\pi \Delta T_s \int_0^{a_L} h_s(r) r dr$$

- effective micro resistance for non-conforming contacts

$$R_s = cH'(\sigma / m) / 4k_s \left[\int_0^{a_L} P(r) r dr \right]^{-1}$$

GENERAL MODEL

- applying a force balance $F = 2\pi \int_0^{a_L} P(r) r dr$

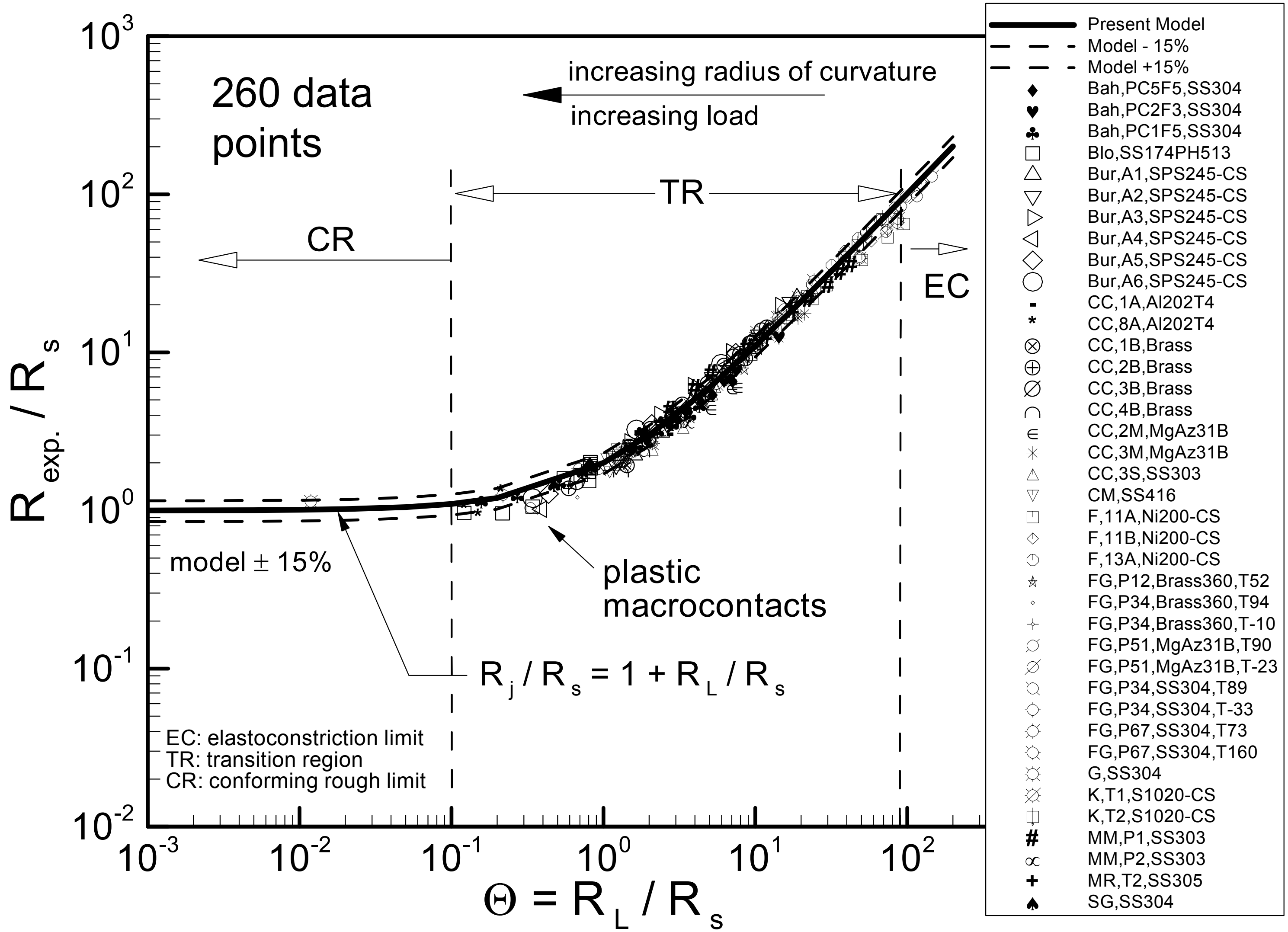
- effective micro thermal resistance for non-conforming simplifies

$$R_s = \frac{\pi c(\sigma / m) H'}{2 k_s F}$$

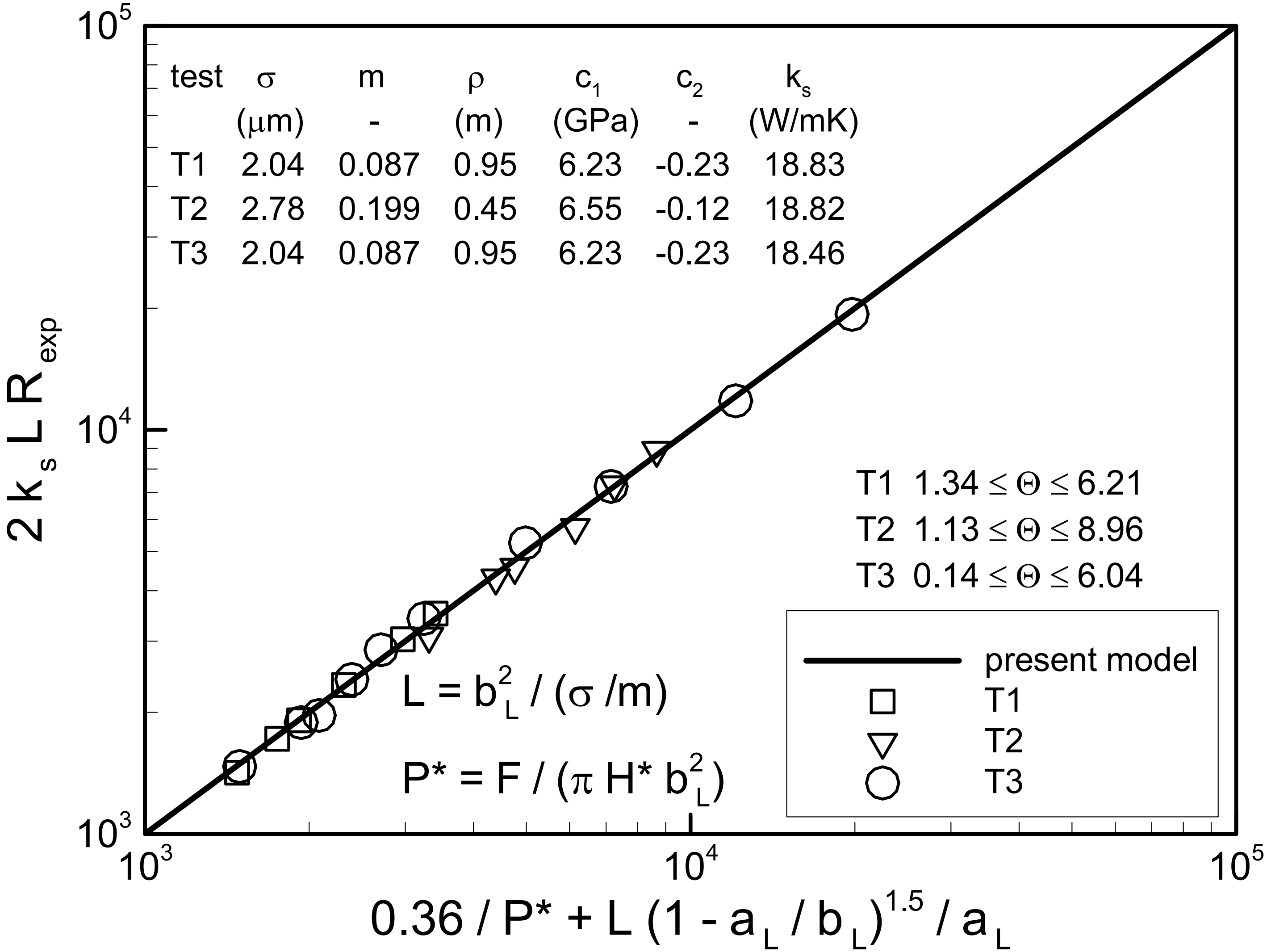
- which is identical to conforming rough, therefore, micro thermal resistance is,
  - ➢ independent of surface curvature
  - ➢ independent of pressure distribution profile

- thermal joint resistance $R_j = \frac{0.36\pi(\sigma / m) H'}{2 k_s F} + \frac{(1 - a_L / b_L)^{1.5}}{2 k_s a_L}$

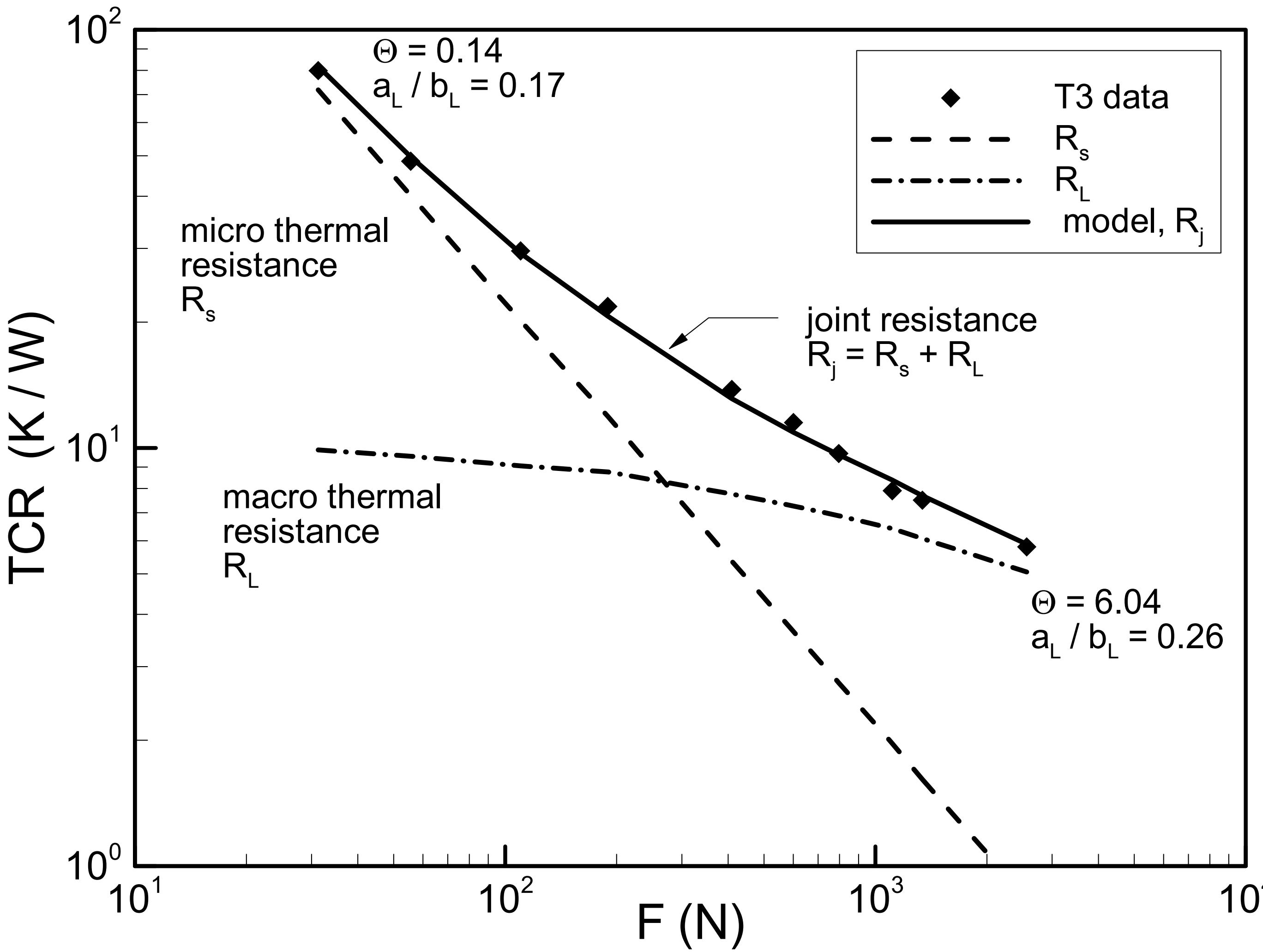
NON-CONFORMING CONTACTS



EXPERIMENTAL PROGRAM



DATA SET T3

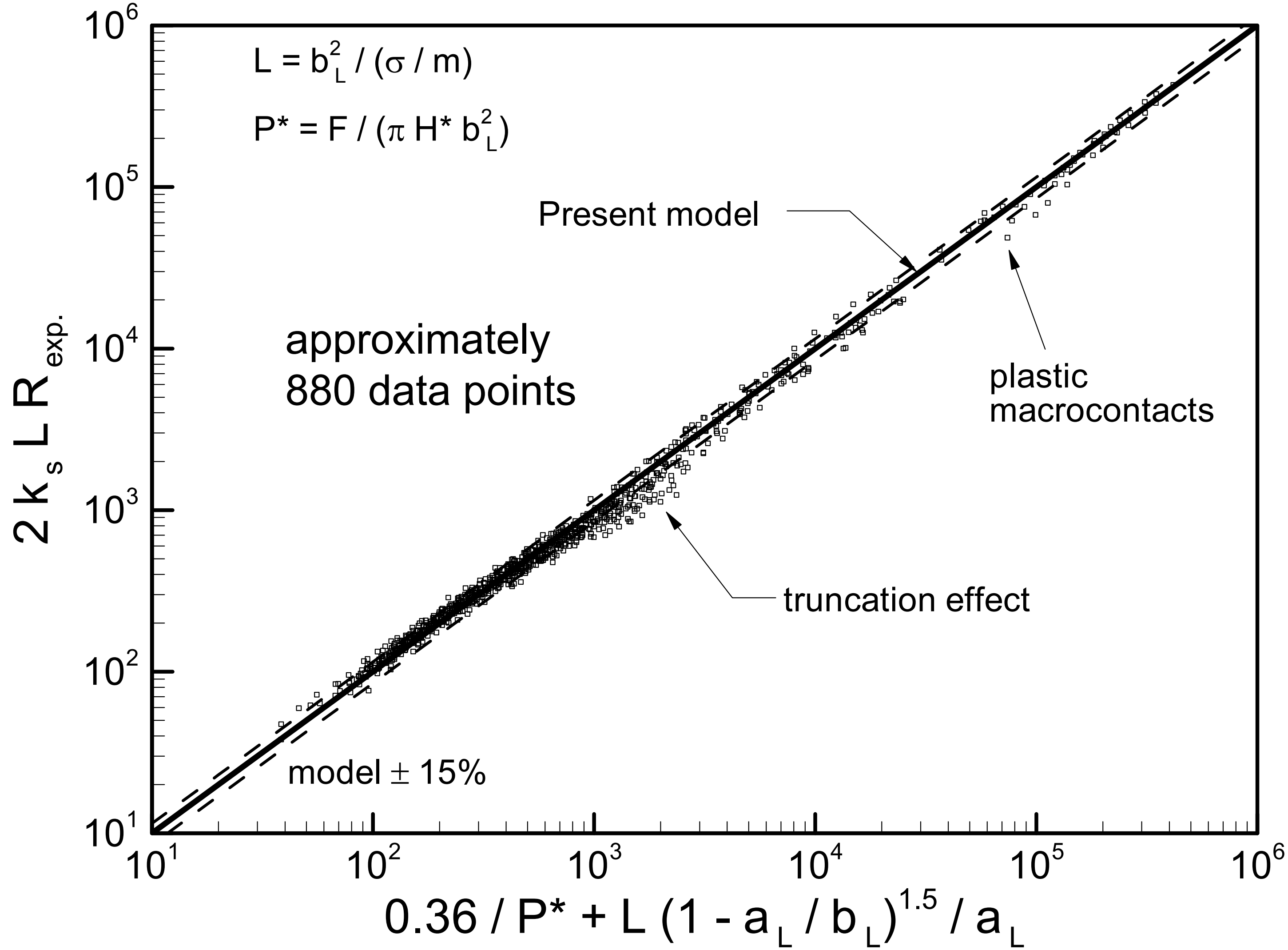


Θ = 0.14
a_L / b_L = 0.17
T3 data
R_s
R_L
model, R_j
micro thermal resistance R_s
joint resistance
$R_j = R_s + R_L$
macro thermal resistance R_L
Θ = 6.04
a_L / b_L = 0.26
TCR (K / W)
10^2
10^1
10^0
10^1
10^2
10^3
10^4
F (N)

RANGE OF INPUT PARAMETERS IN COMPARISONS

Parameter		
7.15	b_L	14.28 mm
25.64	E	114.0 GPa
7.72	F	16763.9 N
16.6	k_s	227.2 W/mK
0.04	m	0.34
0.12		13.94 μm
60	$\overline{T_c}$	195 C
0.0127		120 m

COMPARISON WITH DATA



SUMMARY AND CONCLUSIONS

- joint resistance is superposition of macro and micro thermal resistances in vacuum
- three regions were distinguished for TCR, conforming rough limit, elasto-constriction limit and transition region
- it is shown that heat source on half-space assumption for geometry of microcontacts is justifiable
- using scale analysis methods a new analytical TCR model was developed for conforming rough contacts. Constant of scale relationship was found through comparison with data

SUMMARY AND CONCLUSIONS

- micro thermal resistance is not a function of surface curvature
- contact pressure distribution does not affect micro thermal resistance
- a simple compact model was proposed that covers entire TCR ranging from conforming to spherical smooth joints in vacuum
- experimental program designed and three sets of data obtained

SUMMARY AND CONCLUSION

- excellent agreement between obtained data and model, maximum difference 6.8% with relative RMS difference less than 4%

- present model was compared with 75 data sets more than 880 TCR data points collected by many researchers

- model shows good agreement with other data, RMS relative difference 13.8%.

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