# Models and Experiments for Laminar Natural Convection from Heated Bodies in Enclosures

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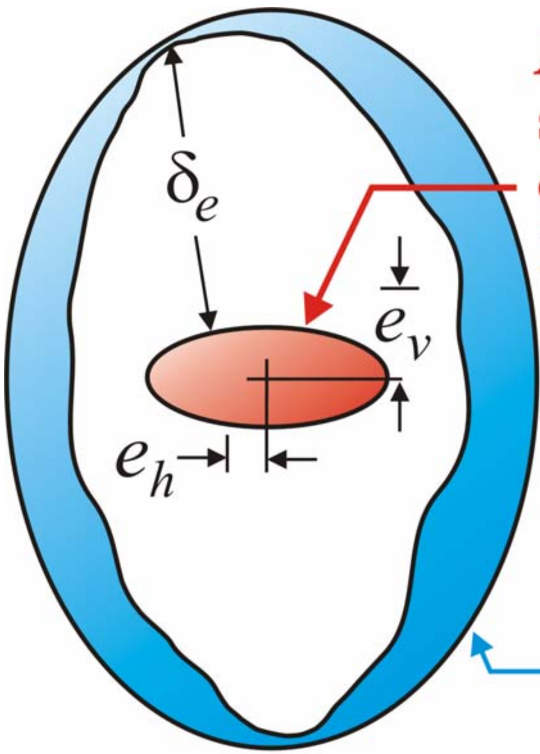
# Outline

- Introduction and problem description
- Literature review and objectives
- Experimental measurements
- Model development and validation
- Summary and conclusions

# Problem Definition

- Steady state, natural convection
- Non-intersecting inner and outer boundaries
- Isothermal boundary conditions, $T_i > T_o$




Geometry:

- Relative boundary size

$$\sqrt{A_o} / \sqrt{A_i} = d_o / d_i \quad \text{(spheres)}$$

- Effective gap spacing

$$\delta_e = (d_o - d_i)/2 \quad \text{(spheres)}$$

- Eccentricity

$$e_h = e_v = 0$$

# Parameter Definitions

- Total heat transfer rate

$$Q = \iint_{A_i} -k \left. \frac{\partial \theta}{\partial \vec{n}} \right|_{A_i} dA_i \ , \quad \theta = T(\vec{r}) - T_b$$

- Non-dimensionalized by Nusselt number

$$Nu_{\sqrt{A_i}} = \frac{Q}{k\sqrt{A_i}\left(T_i - T_o\right)} = S^*_{\sqrt{A_i}} \quad \text{for} \quad Ra \to 0$$

- Rayleigh number

$$Ra_{\sqrt{A_i}} = \frac{g\beta\left(T_i - T_o\right)\left(\sqrt{A_i}\right)^3}{\upsilon\alpha}$$

# Literature Review

## Experimental and Numerical Studies

- Concentric spherical enclosures
  - Experimental data for high Rayleigh number, laminar boundary layer flow only
  - All other data from numerical simulations
- Other enclosure geometries
  - Spheres, cubes, cylinders
  - Experimental and numerical data
- No experimental data for full range of Rayleigh including transition and diffusive limit

# Literature Review

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## Correlations and Models

- Warrington & Powe (1985), Warrington et al. (1988)
  - Correlation of data for variety of inner and outer shapes
  - Effective gap spacing based on equivalent spheres
  - Valid for laminar boundary layer flow only
- Raithby & Hollands (1975, 1985, 1998)
  - Analytically based model for concentric spheres
  - Series combination of resistances of conduction layers at inner and outer boundaries
  - For other geometries, effective gap spacing of Warrington & Powe (1985) recommended

# Objectives

- Experimental measurements:
  - Variety of geometries, spheres, cubes, cylinder, etc.
  - Wide range of $Ra_{\sqrt{A_i}}$
    - Laminar boundary layer convection (atmospheric pressure)
    - Diffusive limit (reduced pressure)
- Analytical modeling:
  - Full range of $Ra_{\sqrt{A_i}}$ from conduction to convection
  - Applicable to wide range of geometries
    - Inner and outer boundary shapes and orientation
    - Relative boundary sizes
  - Physically based analysis

# Experimental Method

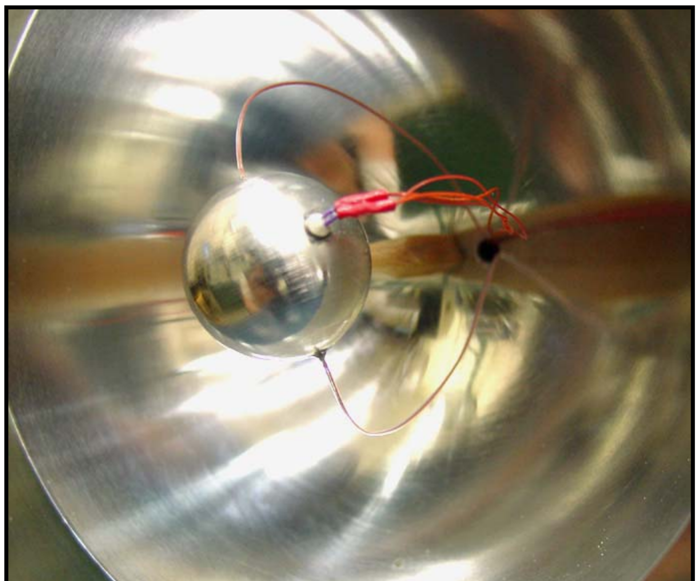
- Wide range of Rayleigh number by of fluid density through reduction in gas pressure (Saunders, 1936, Hollands, 1988)
- Assume ideal gas

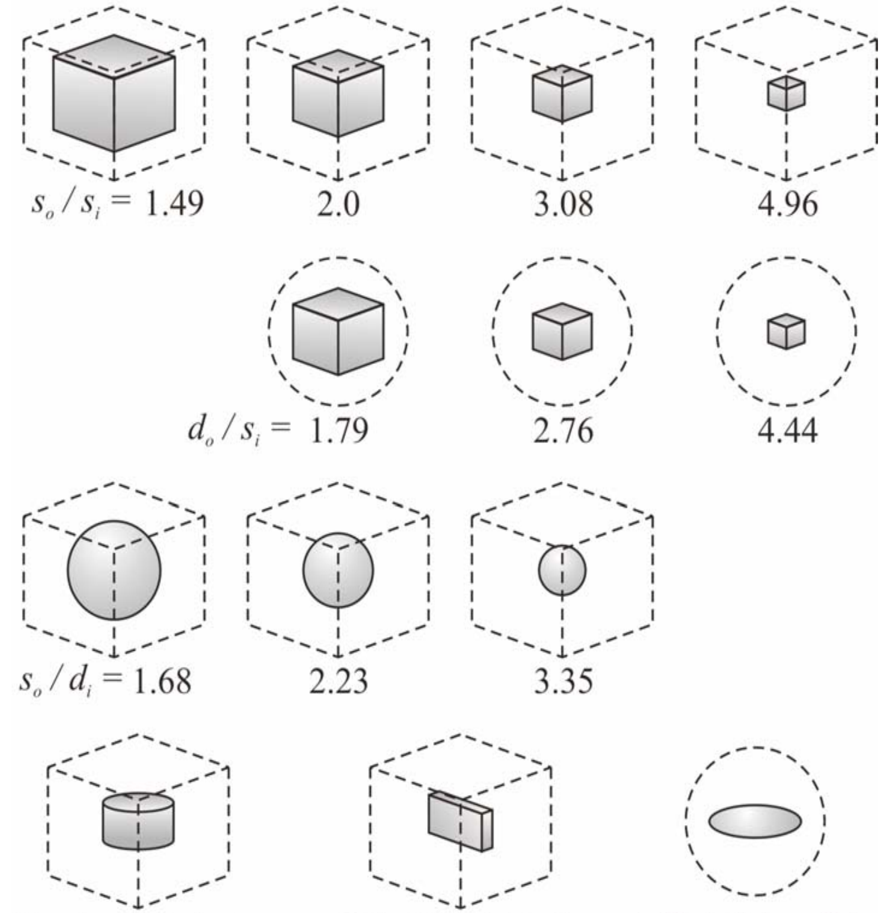
$$\rho = \frac{p}{R\,T_b\,Z} \quad \Rightarrow \quad Ra_{\sqrt{A_i}} = \frac{g\beta\left(T_i - T_o\right)\left(\sqrt{A_i}\right)^3 p^2\,c_p}{R^2\,T_b^2\,k\,\mu Z^2}$$

- Transient test method (Hollands, 1988)
  - Assumes “quasi” steady conditions
  - Fraction of the time required for steady state tests

# Experimental Apparatus

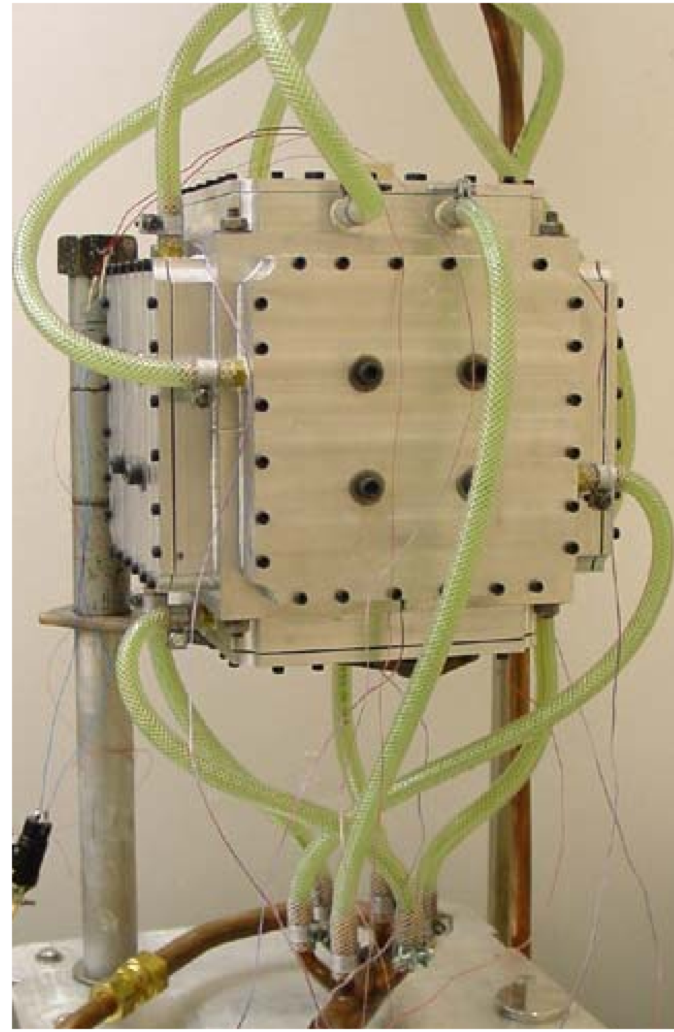
- Spherical and cubical outer geometries
- Eleven different inner bodies
- Temperatures measured using T-type thermocouples





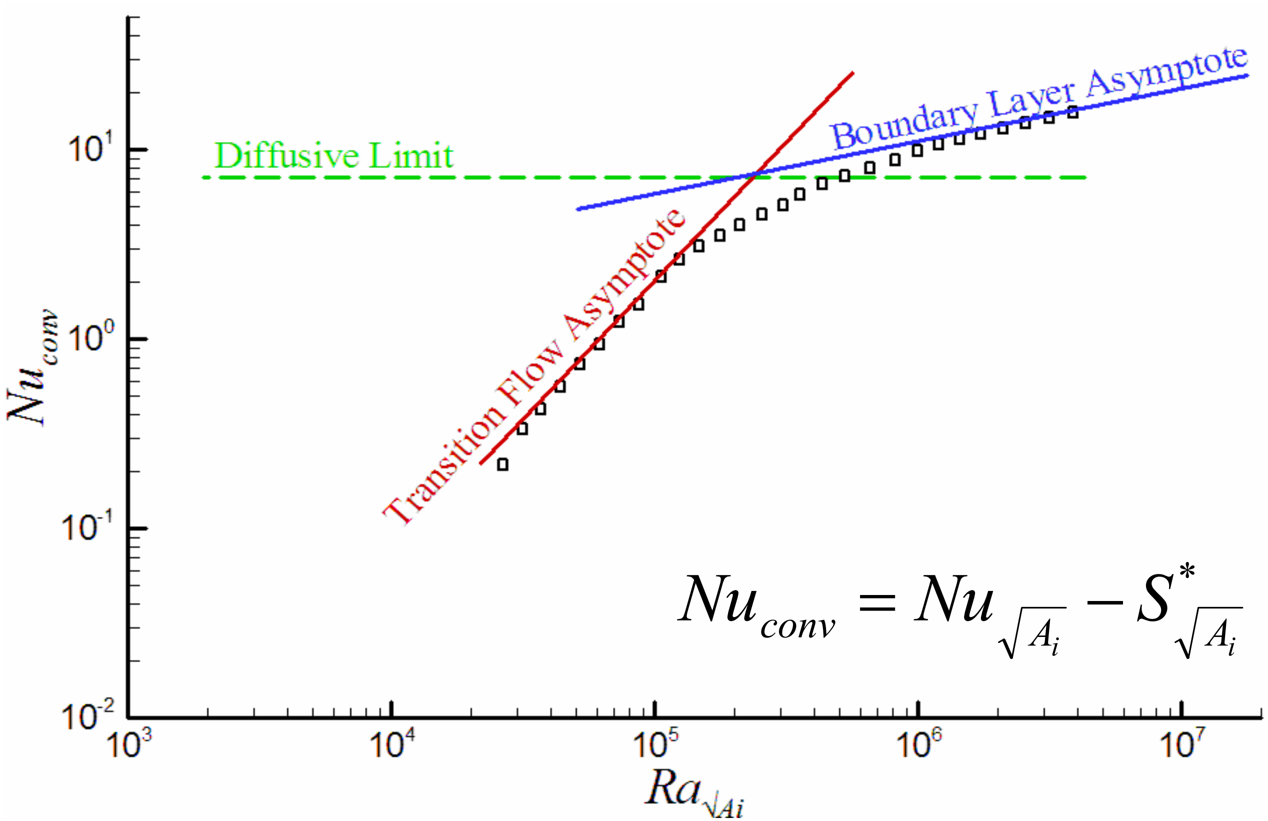
# Experimental Apparatus

- All tests performed in vacuum chamber
- Enclosure walls cooled by cold plates
- Keithley 2700 data acquisition system
- Labview v.5.1 software
  - control of experiment
  - data acquisition and reduction

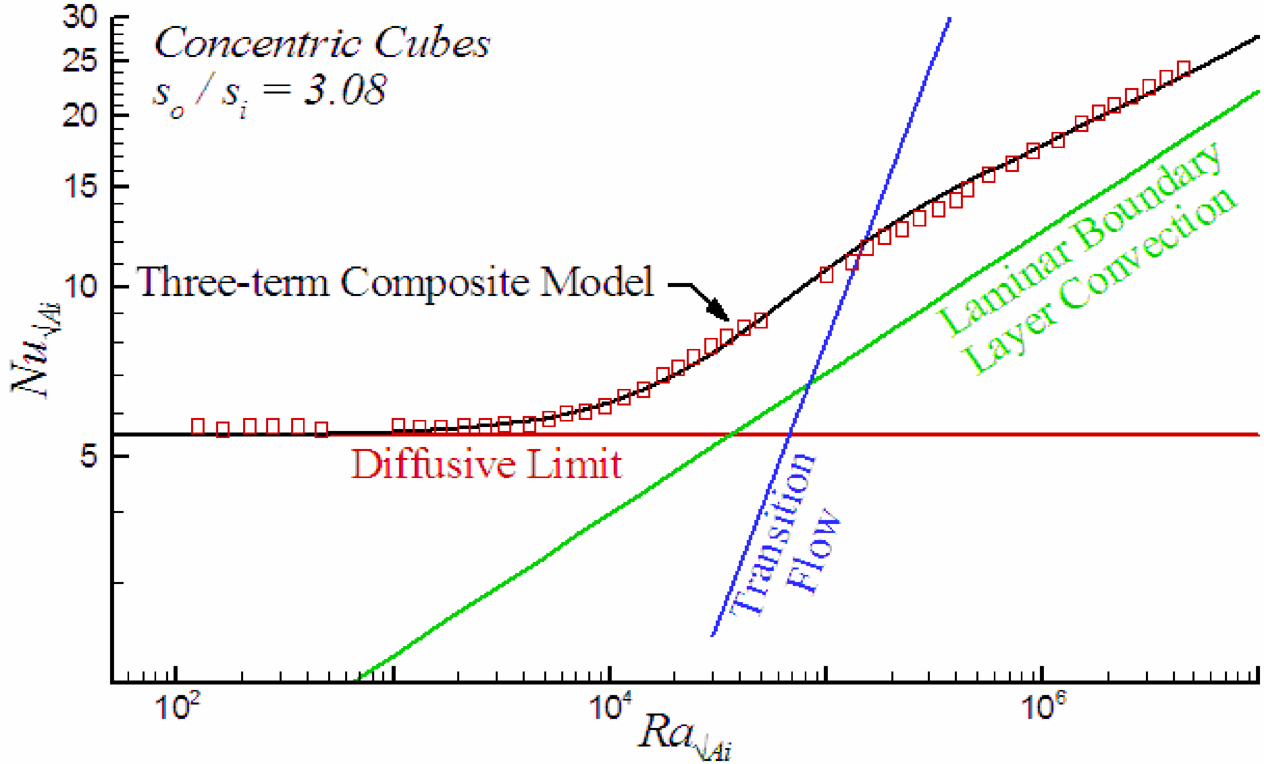


# Model Development

- Assume linear superposition of diffusive and convective limits
- Convection-only data for $s_o / s_i = 2$ concentric cubes



# Model Summary




- Combination of three asymptotic solutions

$$Nu_{\sqrt{A_i}} = S^*_{\sqrt{A_i}} + \left[ \left( \frac{1}{Nu_{tr}} \right)^2 + \left( \frac{1}{Nu_{bl}} \right)^2 \right]^{-1/2}$$

$S^*_{\sqrt{A_i}}$ = conduction shape factor

$Nu_{tr}$ = transition flow convection

$Nu_{bl}$ = laminar boundary layer flow convection

# Conduction Shape Factor

- Linear superposition of two asymptotic solutions

$$S^*_{\sqrt{A_i}} = \frac{\sqrt{A_i}}{\delta_e} + S^*_\infty$$

$S^*_\infty$ = full space diffusive limit

$\sqrt{A_i}/\delta_e$ = 1D planar resistance

- Effective gap spacing from equivalent spherical shell

$$\delta_e = \frac{d_o - d_i}{2}$$

Inner surface area $d_i = \sqrt{A_i/\pi}$

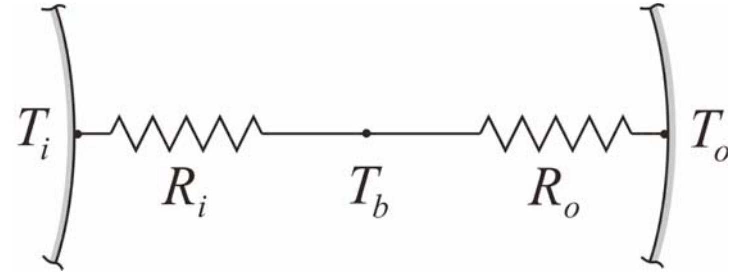
Enclosed volume $d_o = \left[6\left(V + \frac{\pi}{6}d_i^3\right)\Big/\pi\right]^{1/3}$

- Dimensionless conduction shape factor

$$S^*_{\sqrt{A_i}} = \frac{2\sqrt{\pi}}{\left[1 + 6\sqrt{\pi}\left(V^{1/3}\Big/\sqrt{A_i}\right)^3\right]^{1/3} - 1} + S^*_\infty$$

# Boundary Layer Convection

- Assumptions
  - Laminar flow
  - $T_b$ uniform
  - Non-intersecting boundary layers




- Series combination of resistances

$$R_{conv} = R_i + R_o \qquad R_i = \frac{T_i - T_b}{Q} \qquad R_o = \frac{T_b - T_o}{Q}$$

- Non-dimensionalize using Nusselt number

$$Nu_i = \frac{1}{k\sqrt{A_i}}\frac{1}{R_i} \qquad Nu_o = \frac{1}{k\sqrt{A_o}}\frac{1}{R_o} \qquad Nu_{bl} = \frac{Nu_i}{1 + 1/\phi}$$

$$\phi = \frac{T_i - T_b}{T_b - T_o} = \frac{R_i}{R_o} = \frac{\sqrt{A_o}}{\sqrt{A_i}}\frac{Nu_o}{Nu_i}$$

# Boundary Layer Convection

- Convection modeled using Yovanovich [34] and Jafarpur [36]

$$Nu_{\sqrt{A}} = F(\Pr)\, G_{\sqrt{A}}\, Ra_{\sqrt{A}}^{1/4}$$

- Laminar boundary layer convection asymptote

$$Nu_{bl} = \frac{Nu_i}{1 + 1/\phi} = \frac{F(\Pr)\, G_{\sqrt{A_i}}\, Ra_{\sqrt{A_i}}^{1/4}}{\left(1 + 1/\phi\right)^{5/4}}$$

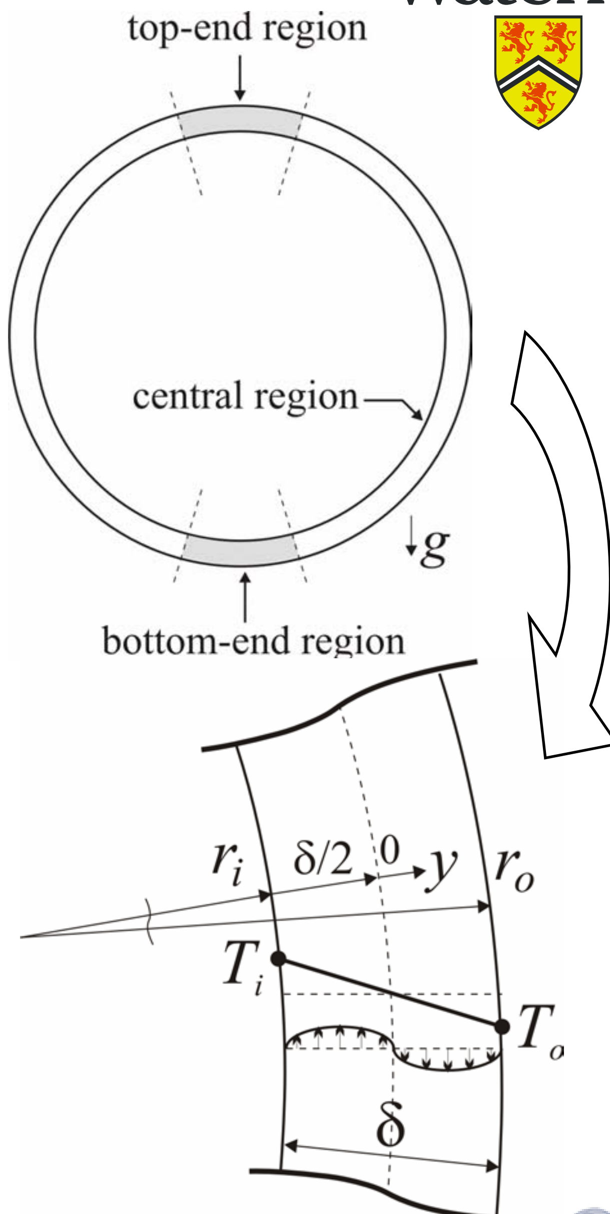
$$Nu_{bl} = \frac{F(\Pr)\, G_{\sqrt{A_i}}\, Ra_{\sqrt{A_i}}^{1/4}}{\left[1 + \left(A_i/A_o\right)^{7/10} \left(G_{\sqrt{A_i}} / G_{\sqrt{A_o}}\right)^{4/5}\right]^{5/4}}$$

# Transition Flow

- Boundary layers merge when $Ra < Ra_{cr}$
- Model as equivalent concentric spheres
- Three distinct regions are formed
- Central region
  - Radial conduction
  - Buoyancy induced flow
- For narrow gap spacing, $\delta << r_i$ , temperature, velocity in central region

$$T - T_b = -\frac{y}{\delta/2}\left(T_i - T_b\right), \quad T_b = \frac{T_i + T_o}{2}$$

$$u = \frac{g_e \beta}{12\upsilon}\left(T_i - T_o\right)\left(\frac{\delta}{2}\right)^2\left[\left(\frac{y}{\delta/2}\right)^3 - \frac{y}{\delta/2}\right]$$



# Transition Flow

- Enthalpy balance in top-end and bottom-end regions

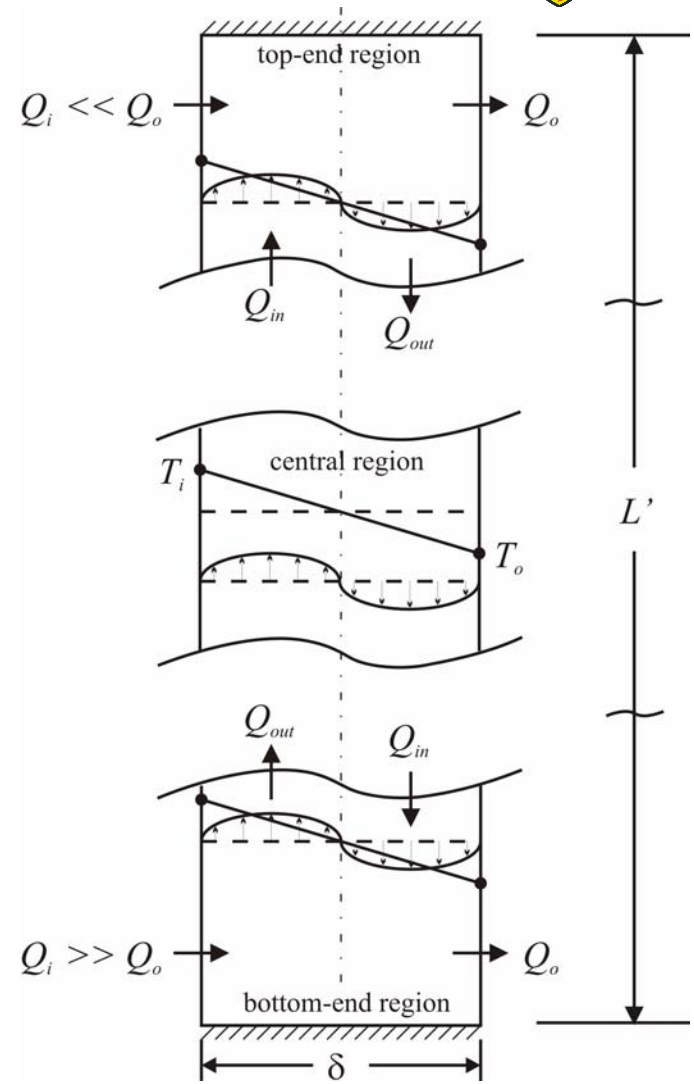
$$Q_{i,o} = \frac{\rho c_p W' g_e \beta \left(T_i - T_o\right)^2 \delta^3}{720 \nu}$$

- Transition flow asymptote

$$Nu_{tr} = \frac{\sqrt{2}}{360} \frac{\sqrt{A_i}}{L'} \left( \frac{\delta_{\text{eff}}}{\sqrt{A_i}} \right)^3 Ra_{\sqrt{A_i}}$$

$\delta_{\text{eff}}$ = gap spacing of equivalent cavity

$L'$ = effective flow length



# Validation: Concentric Cubes

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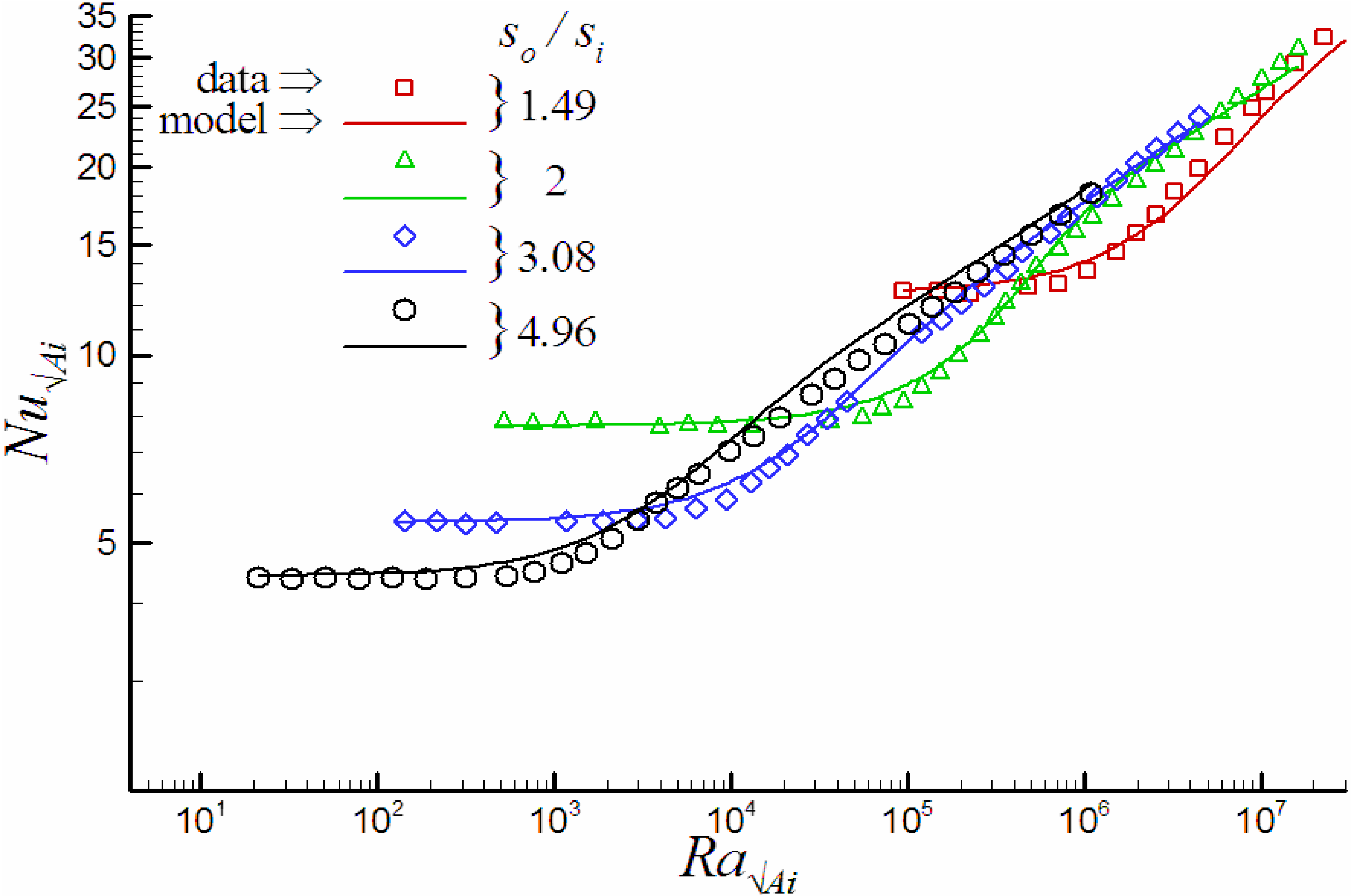




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# Sphere in Cubical Enclosure

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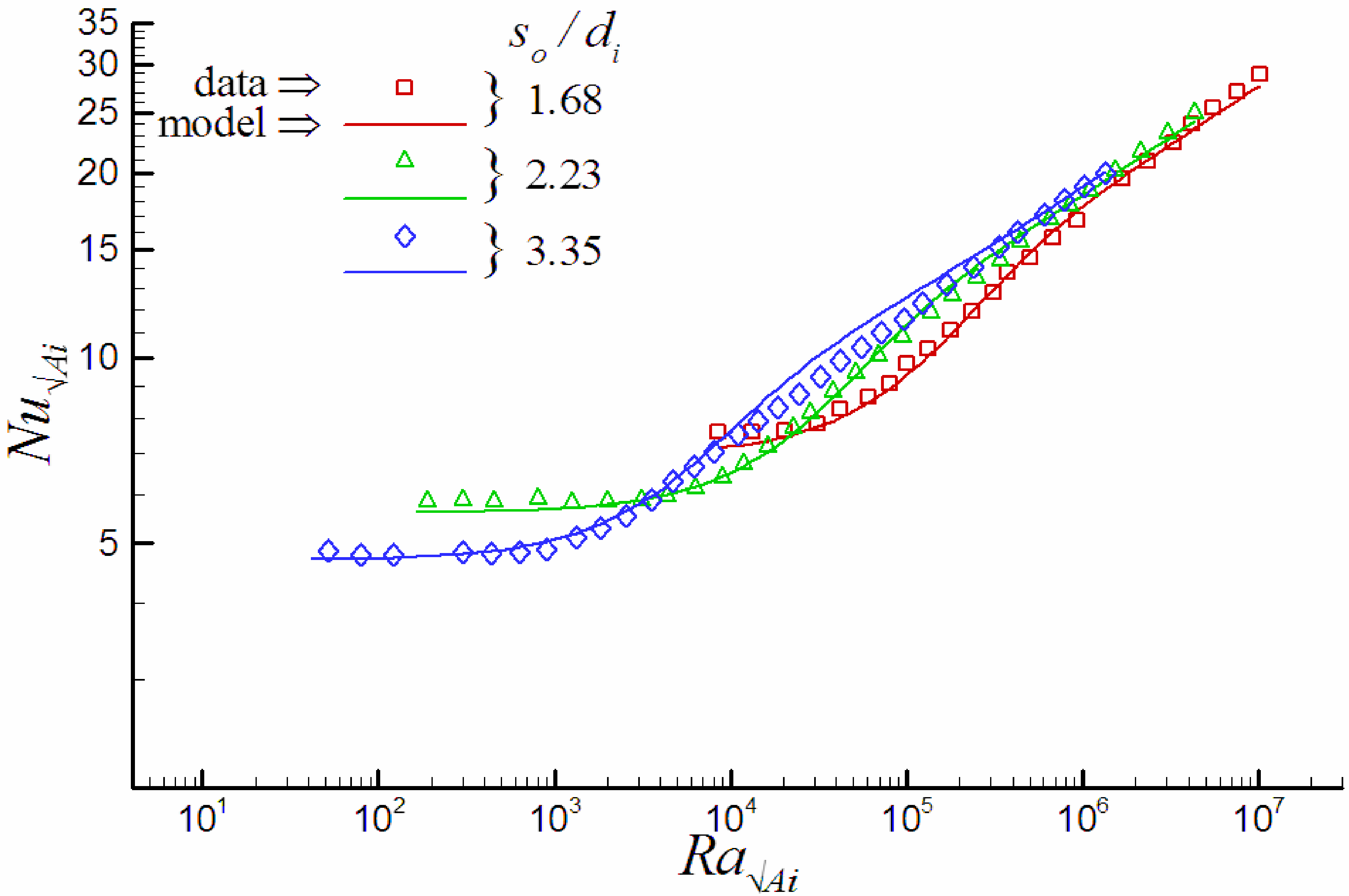




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# Cube in Spherical Enclosure

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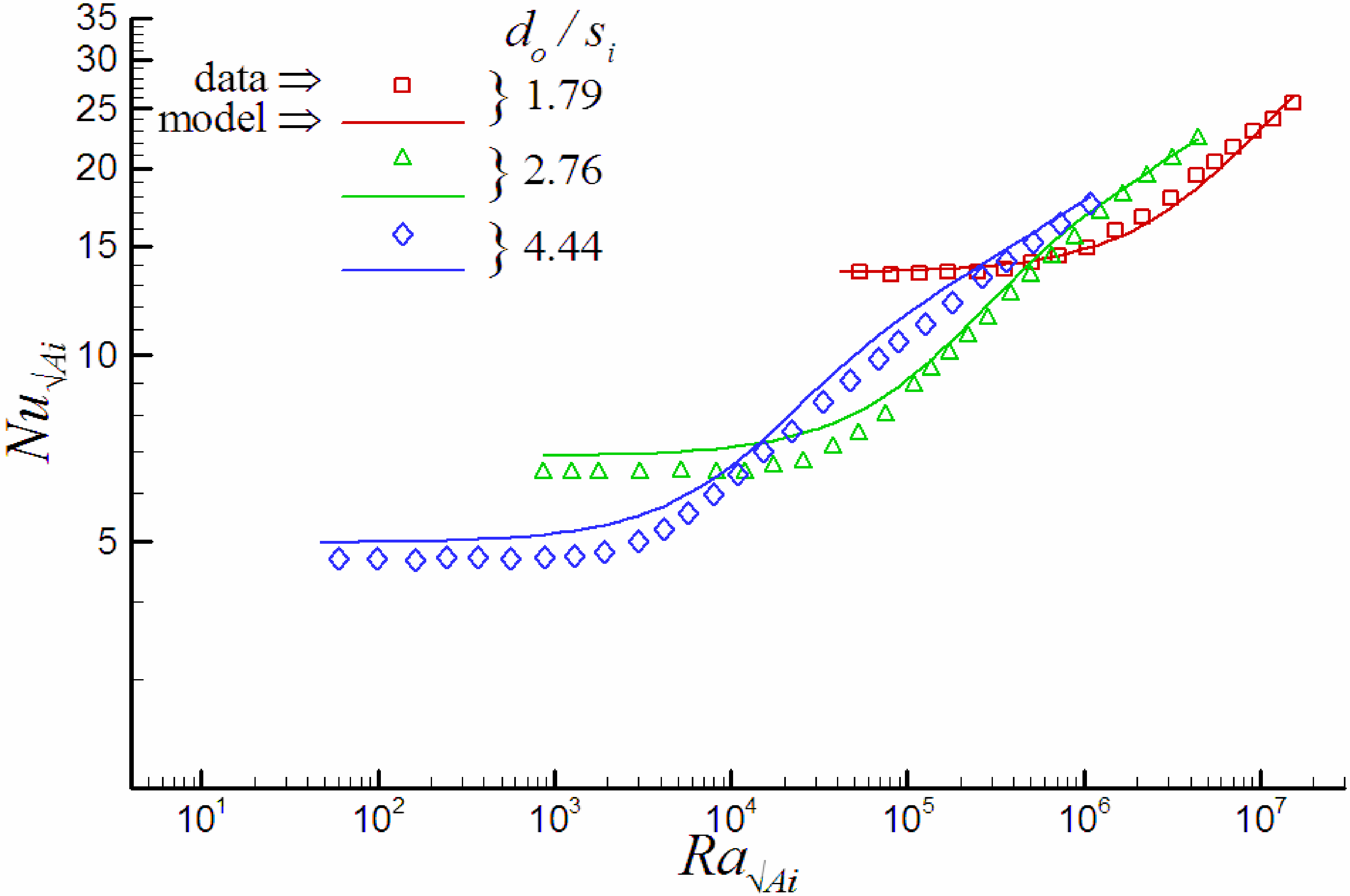




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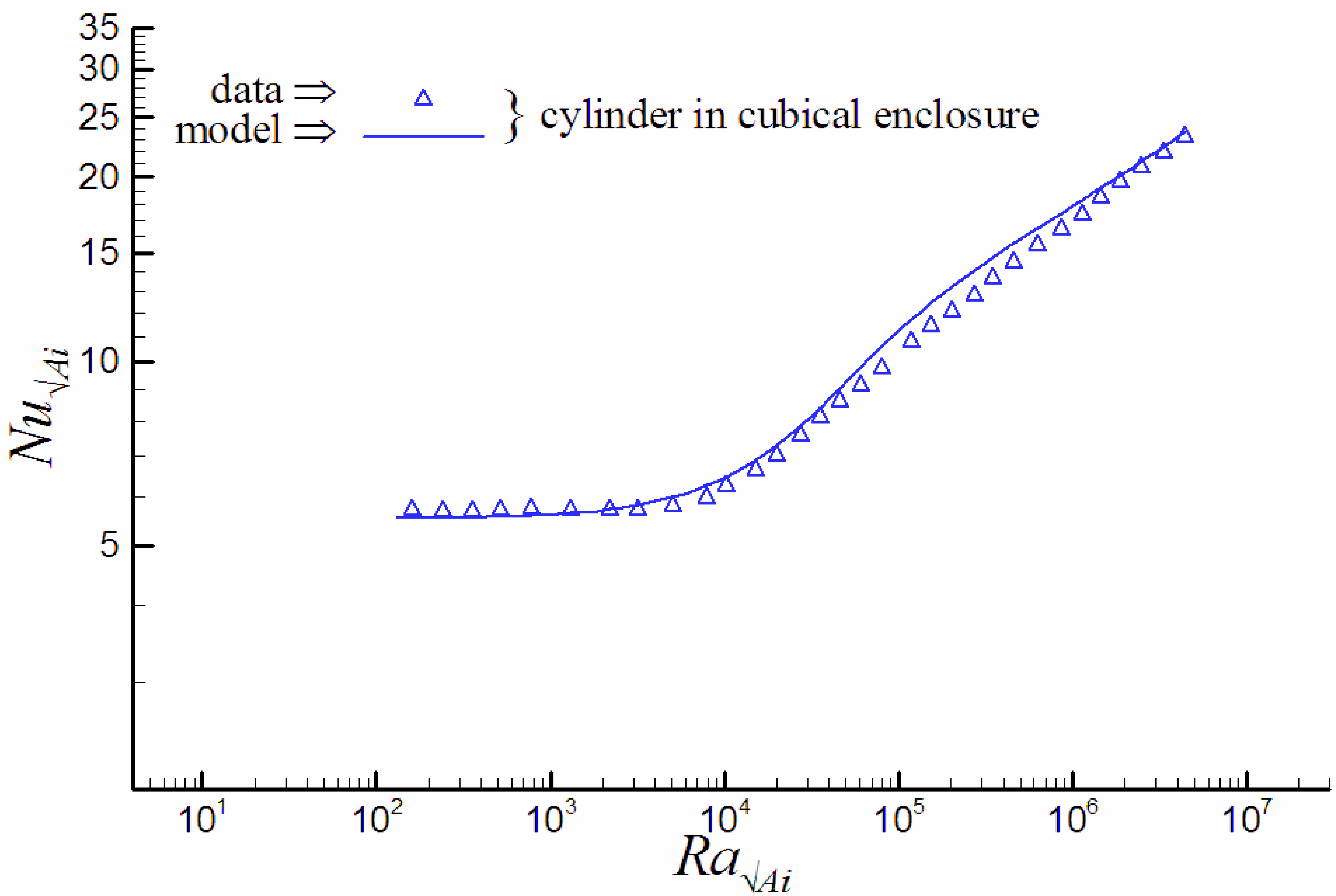
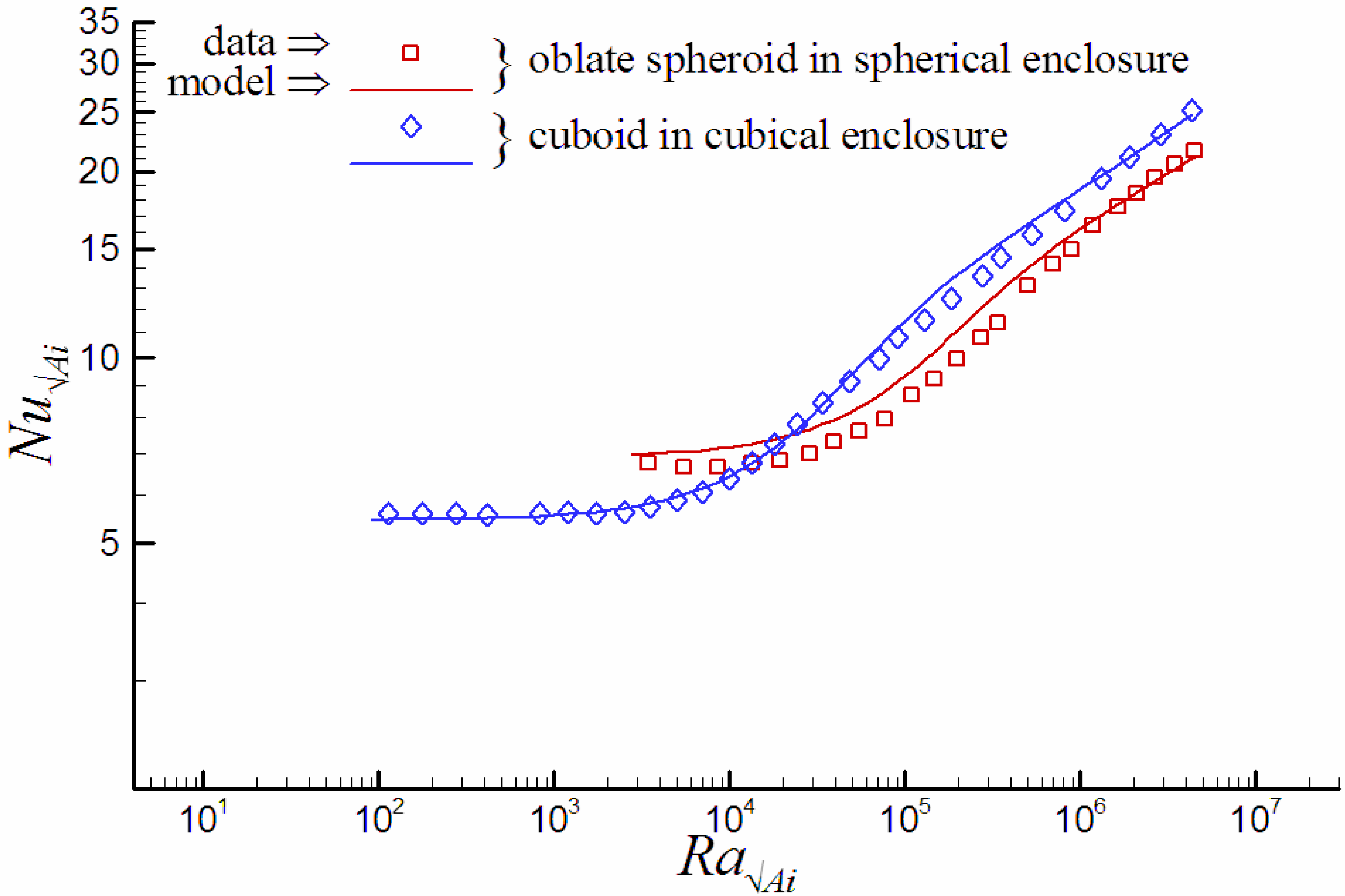
# Cylinder in Cubical Enclosure



# Other Enclosure Geometries



# Summary and Conclusions

- Combined experimental / analytical study of natural convection heat transfer between heated body and cooled enclosure
- Experimental data for variety of enclosure configurations, dimensions
- Model developed based on combination of analytic, asymptotic relationships
  - Diffusive limit
  - Laminary boundary layer convection
  - Transition flow convection
- 2 – 6% RMS, 12% maximum difference between model and data