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Optimal Design of Tube Banks in Crossflow Using Entropy Generation Minimization Method

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Outline

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- Modeling
- Optimization Problem
- Results and Comparisons
- Conclusions
- Acknowledgments

Introduction

- Industrial applications:
  - Heat exchanger devices (like automobile radiator, oil cooler, pre-heater, air-cooled steam condenser)
  - Process industry
  - Air conditioning and refrigeration industry

- Primary interest of mechanical engineers:
  - Optimal design of tube bank

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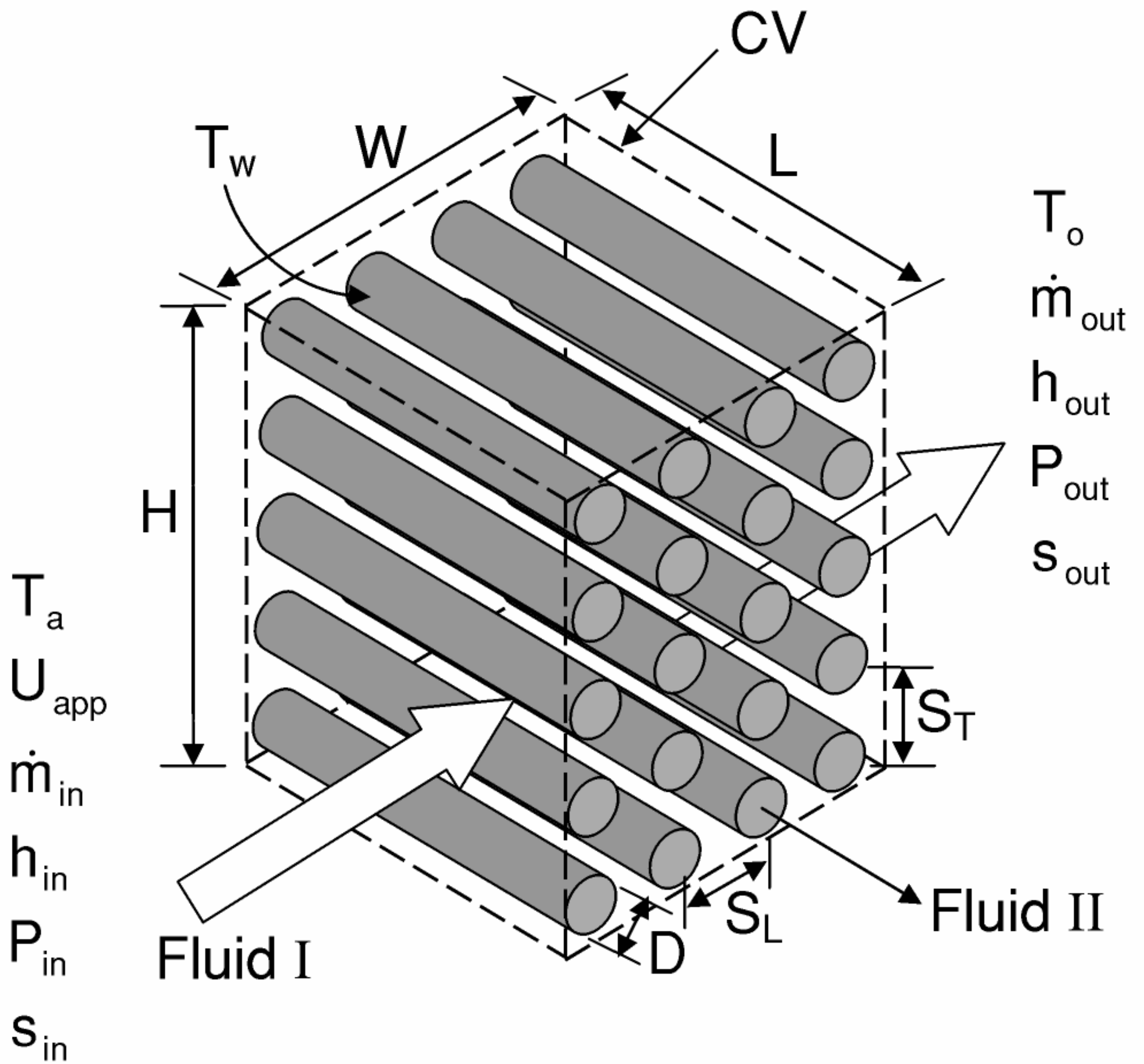
Assumptions

- Tube bank is insulated from surroundings.
- Tubes are plain.
- Flow is 2-D, steady, laminar.
- Fluid is Newtonian and incompressible.
- Thermo-fluid properties are constant.
- Conduction along tube wall is negligible.
- Radiation heat transfer is negligible.
- Potential and kinetic energy changes are negligible.

Modeling (Control Volume for S_{gen})



Entropy Generation Rate

$$\dot{S}_{gen} = \left(\frac{Q^2}{T_a T_w}\right) R_{tube} + \frac{\dot{m}\Delta P}{\rho T_a}$$

$$\begin{aligned} R_{tube} &= \frac{\Delta T}{Q} = \frac{1}{h_{avg} A} \\ \dot{m} &= \rho U_{app} N_T S_T L \\ \Delta P &= N_L f \left(\frac{1}{2}\rho U_{max}^2\right) \end{aligned}$$

Heat Transfer Coefficient

$$Nu_D = \frac{h_{avg} D}{k_f} = C_1 Re_D^{1/2} Pr^{1/3}$$

From AIAA 2005-958:

$$C_1 = \begin{cases} [0.25 + \exp(-0.55\mathcal{S}_L)]\mathcal{S}_T{}^{0.285}\mathcal{S}_L{}^{0.212} & \text{In-Line Arrangement} \\ \dfrac{0.61\,\mathcal{S}_T{}^{0.091}\mathcal{S}_L{}^{0.053}}{[1 - 2\exp(-1.09\,\mathcal{S}_L)]} & \text{Staggered Arrangement} \end{cases}$$

$$U_{max} = max\left\{\frac{\mathcal{S}_T}{\mathcal{S}_T - 1}U_{app}, \frac{\mathcal{S}_T}{\mathcal{S}_\mathcal{D} - 1}U_{app}\right\}$$

Friction Factors

From Zukauskas Experimental Data:

$$f = \begin{cases} K_1 \left[0.233 + 45.78/(\mathcal{S}_T - 1)^{1.1} Re_D\right] & \text{In-Line Arrangement} \\ K_1 \left[378.6/\mathcal{S}_T^{13.1/\mathcal{S}_T}\right] / Re_D^{0.68/\mathcal{S}_T^{1.29}} & \text{Staggered Arrangement} \end{cases}$$

$$K_1 = \begin{cases} 1.009 \left(\dfrac{\mathcal{S}_T - 1}{\mathcal{S}_L - 1}\right)^{1.09/Re_D^{0.0553}} & \text{In-Line Arrangement} \\ 1.175(\mathcal{S}_L/\mathcal{S}_T Re_D^{0.3124}) + 0.5 Re_D^{0.0807} & \text{Staggered Arrangement} \end{cases}$$

Entropy Generation Rate

$$\dot{S}_{gen} = \frac{\mathcal{Q}^2/T_a T_w}{C_1 N \pi L k_f Re_D^{1/2} Pr^{1/3}} + \frac{N f \rho U_{max}^3 (\mathcal{S}_T - 1) L}{2 T_a}$$

$$N_s = \frac{T_a/T_w}{C_1 N \pi \gamma Re_D^{3/2} Pr^{1/3}} + \frac{1}{2} f N \gamma B Re_D^2 (\mathcal{S}_T - 1)$$

$$B = \rho \nu^3 k_f T_a/\mathcal{Q}^2 \qquad Re_D = \frac{D U_{max}}{\nu} \qquad \gamma = \frac{L}{D}$$

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Optimization Problem

$$minimize \;\; f(\mathbf{x}) = N_s(\mathbf{x})$$

subject to $\quad g_j(\mathbf{x}) = 0, \qquad j = 1, 2, ..., m$

$$l_j(\mathbf{x}) \geq 0, \qquad j = m+1, ..., n$$

inequality constraints

$$D\;(mm) \geq 10$$

$$1.25 \leq S_L \leq 3$$

$$1.25 \leq S_T \leq 3$$

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Assumed Parameter Values

Quantity	Dimension/Data
Cross-Sectional Area (mm^2)	235×235
Length of Tubes (mm)	1000
Tube Diameter (mm)	12
Heat Load (kW)	20
Ambient Temperature (K)	300
Tube Wall Temperature (K)	365

Optimized Results

(In-Line Arrangement)

Dimensionless Pitch Ratio $\mathcal{S}_T \times \mathcal{S}_L$	Tube Diameter (mm)	Optimum Approach Velocity (m/s)	Number of Tubes $N_T \times N_L$	Nu_D	ΔP (Pa)	$N_s \times 10^{10}$
	12	3.4	15×15	88.4	590.2	0.180
1.25×1.25	14	3.8	13×13	100.9	621.6	0.191
	16	4.2	11×11	113.1	650.3	0.201
	12	5.7	13×13	88.5	480.4	0.251
1.5×1.5	14	6.4	11×11	101.0	507.3	0.266
	16	7.0	10×10	112.9	532.8	0.281
	12	9.6	10×10	91.5	452.7	0.365
2.0×2.0	14	10.6	8×8	104.0	477.7	0.389
	16	11.6	7×7	116.0	499.9	0.410

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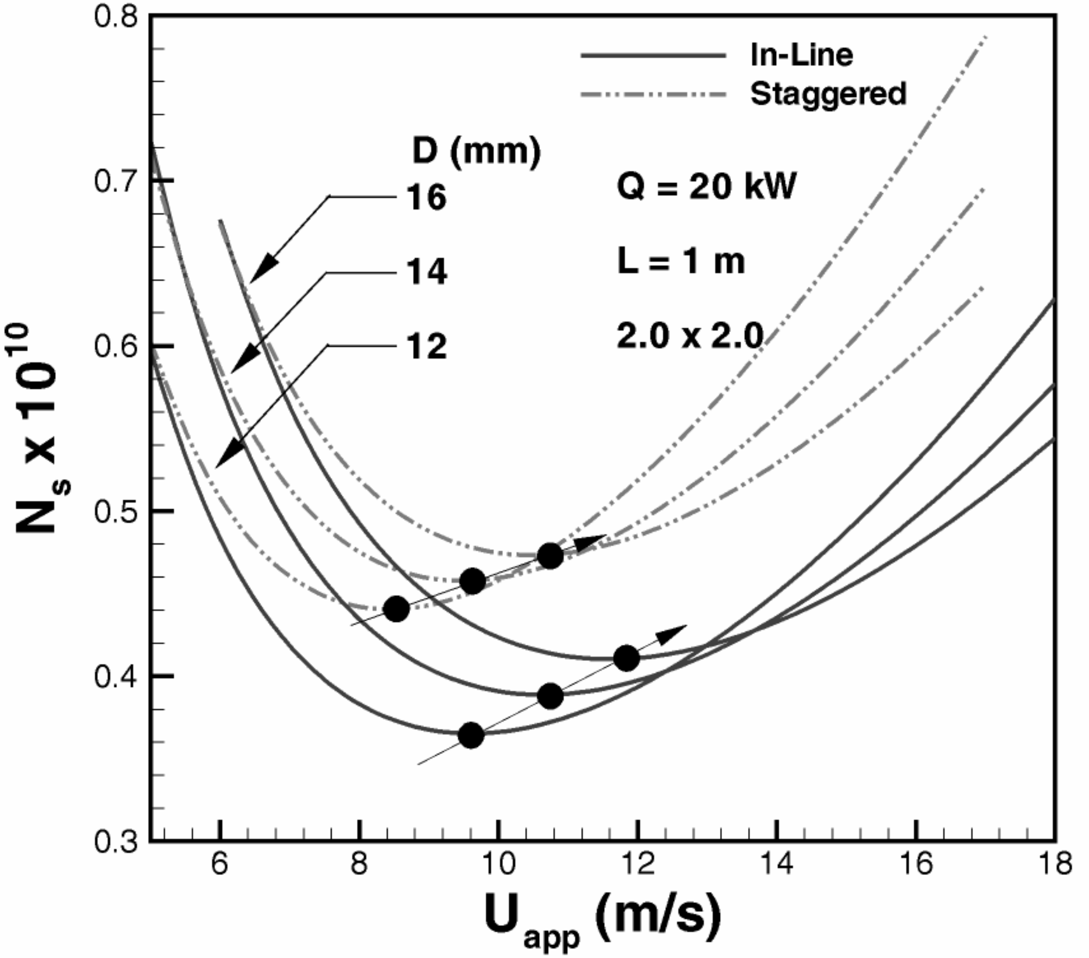
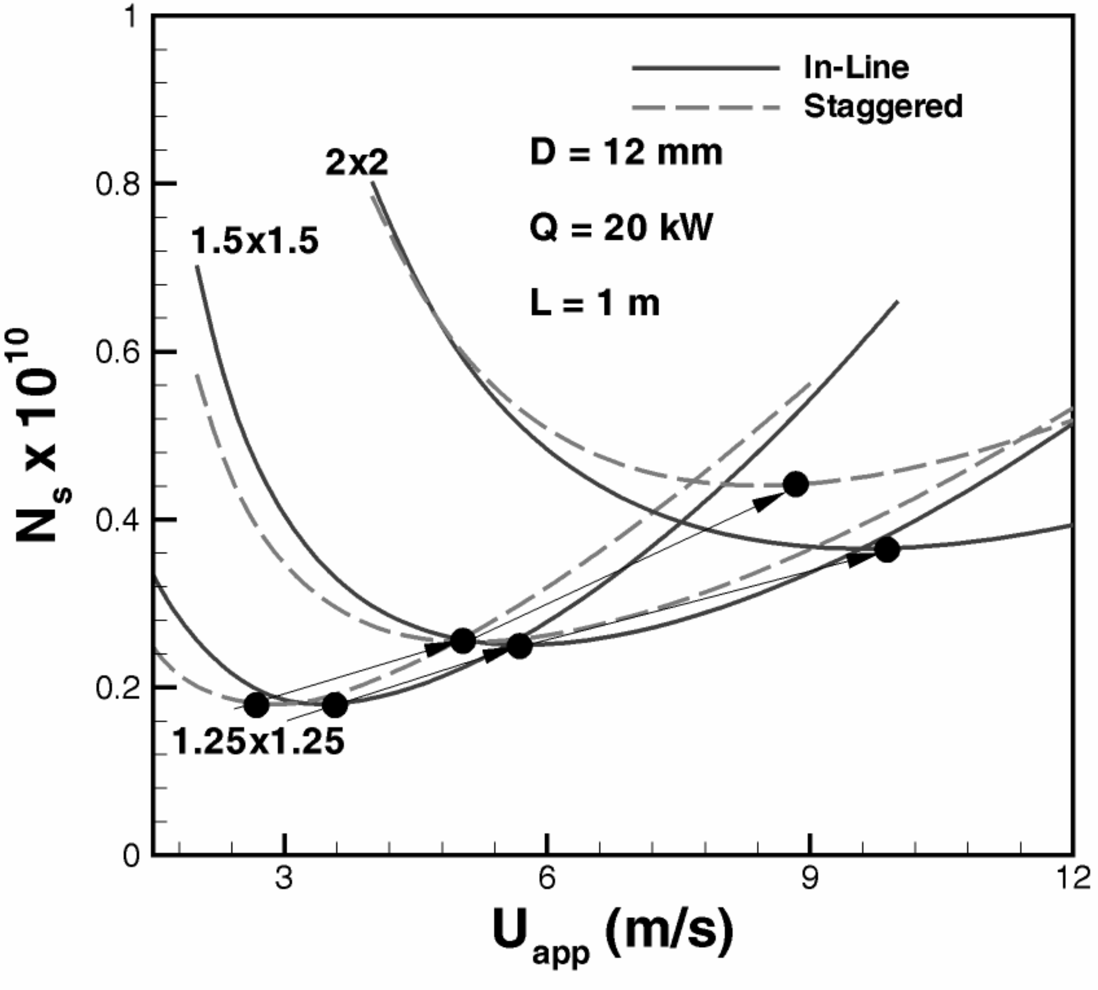
Optimized Results
(Staggered Arrangement)

Dimensionless Pitch Ratio $\mathcal{S}_T \times \mathcal{S}_L$	Tube Diameter (mm)	Optimum Approach Velocity (m/s)	Number of Tubes $N_T \times N_L$	Nu_D	ΔP (Pa)	$N_s \times 10^{10}$
	12	2.8	15×15	122.2	657.6	0.179
1.25×1.25	14	3.2	13×13	142.0	660.8	0.180
	16	3.6	11×11	161.7	664.5	0.181
	12	5.1	13×13	105.5	535.1	0.254
1.5×1.5	14	5.8	11×11	121.8	544.1	0.259
	16	6.6	10×10	137.9	553.1	0.265
	12	8.4	10×10	90.7	580.3	0.441
2.0×2.0	14	9.4	8×8	104.0	597.6	0.458
	16	10.5	7×7	116.8	612.5	0.473

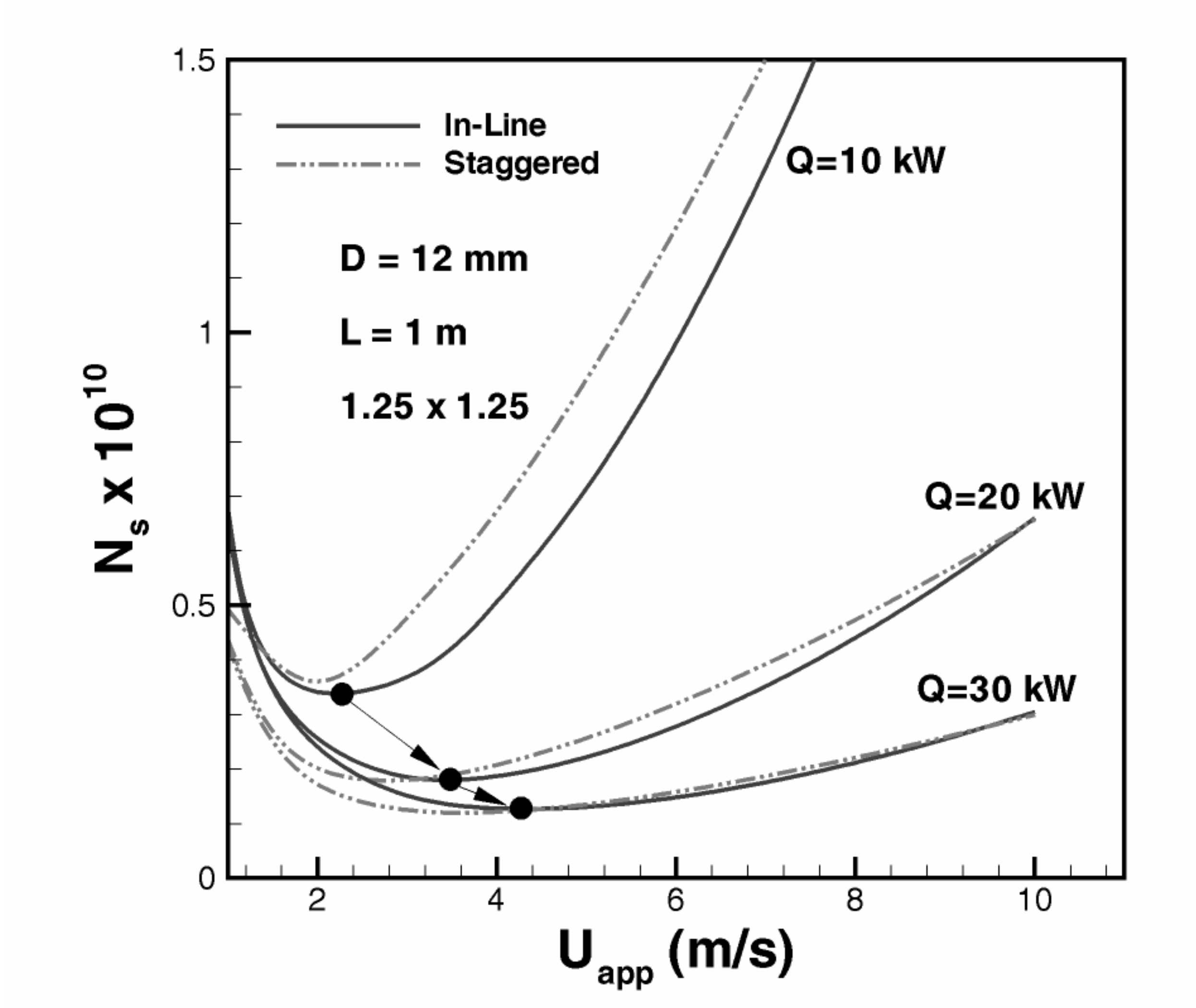
Effect of Tube Diameter



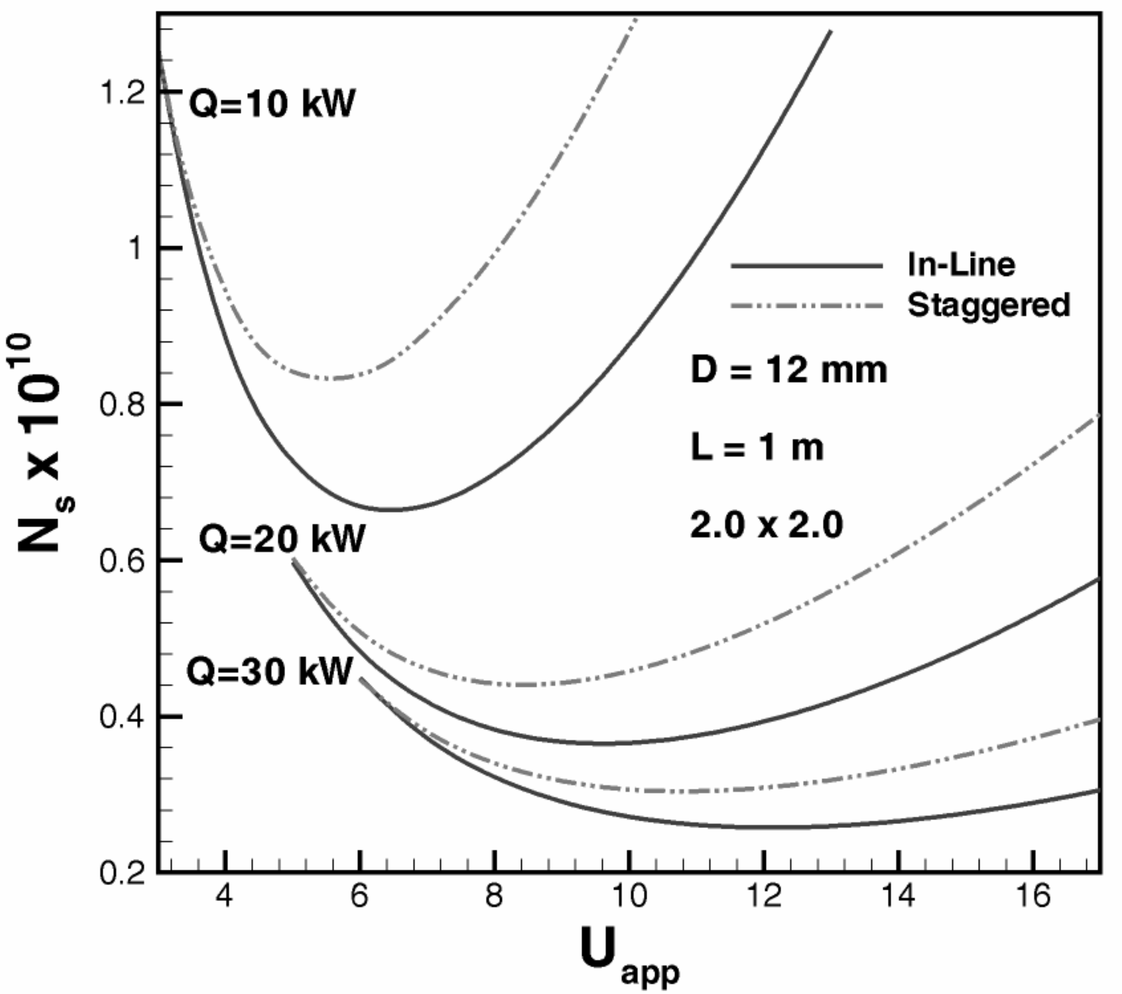
Effect of Pitch Ratio



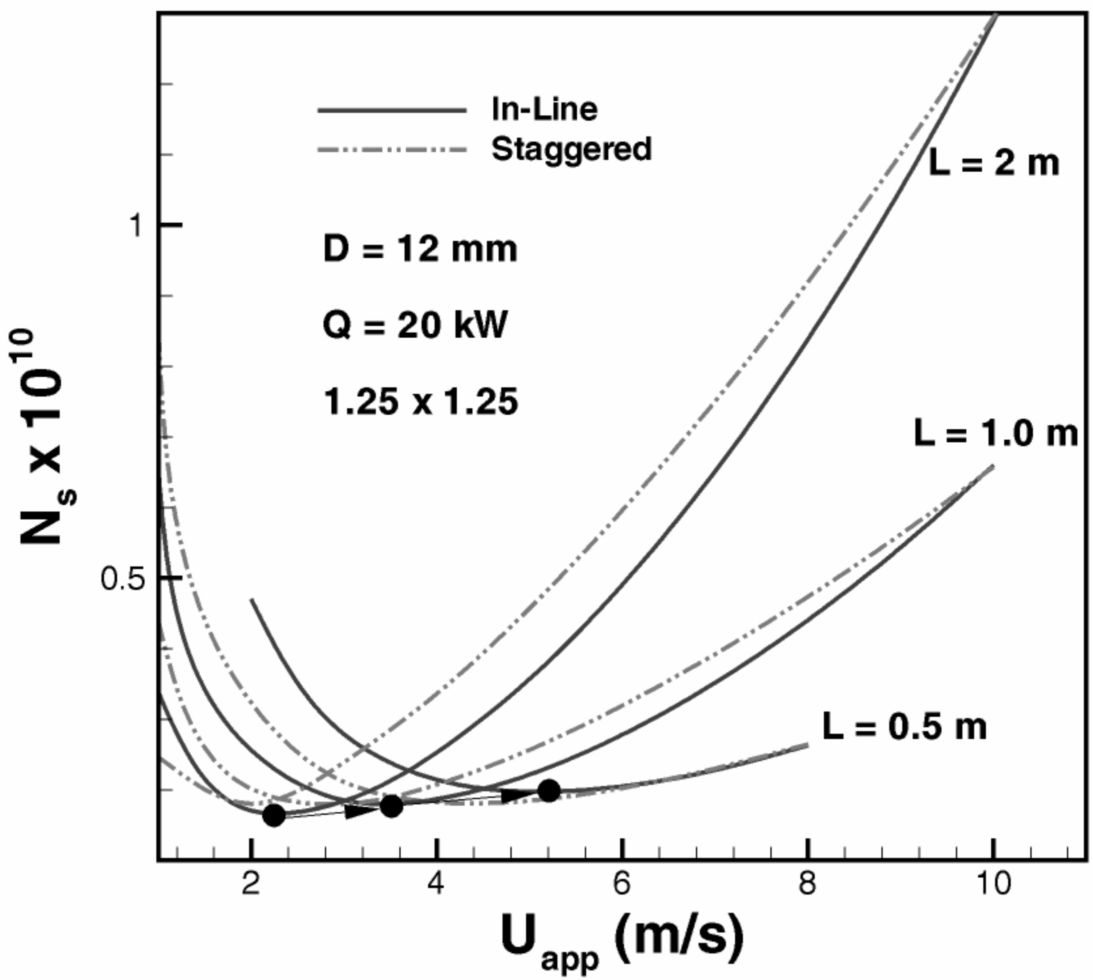
Effect of Heat Load (Compact Bank)



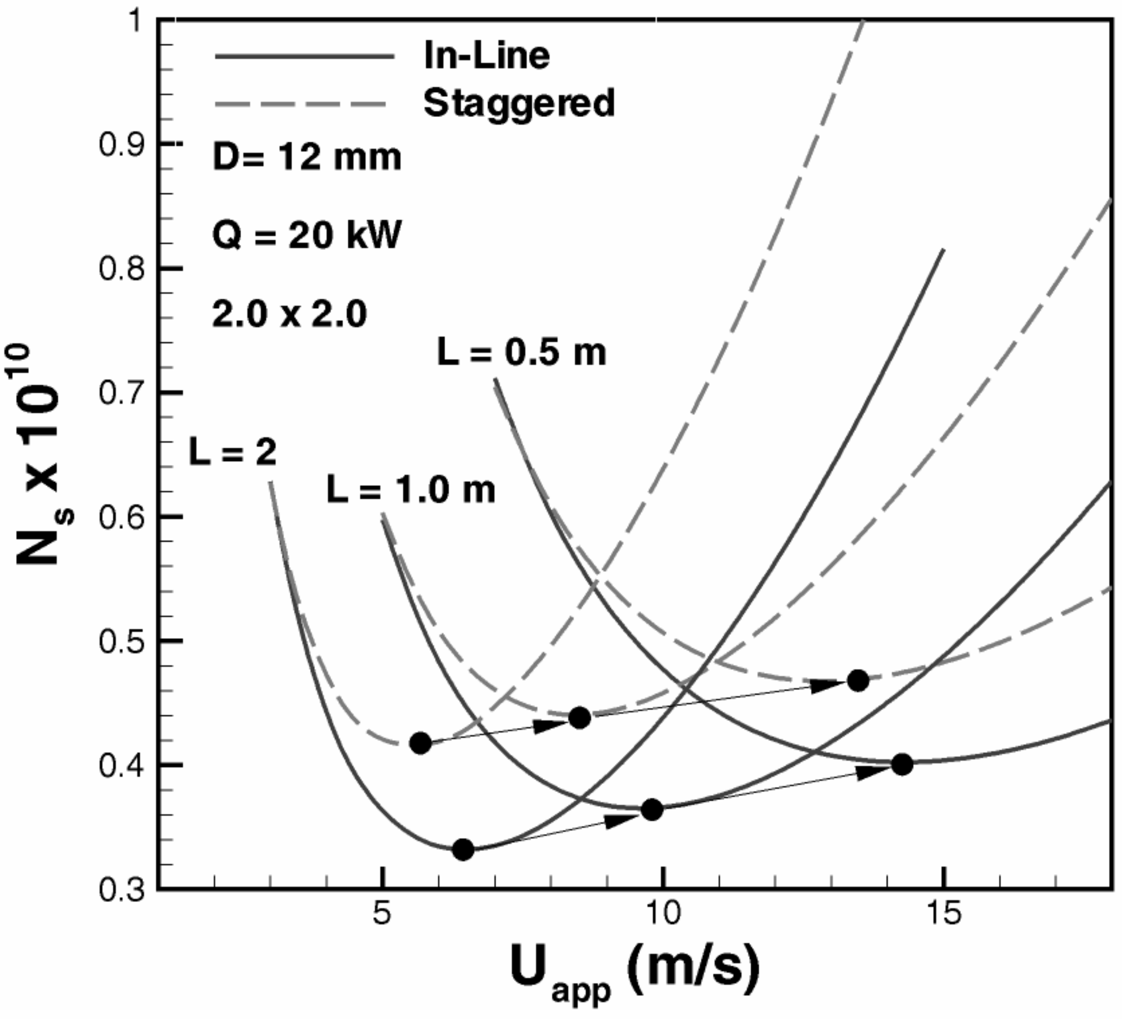
Effect of Heat Load (Widely Spaced)



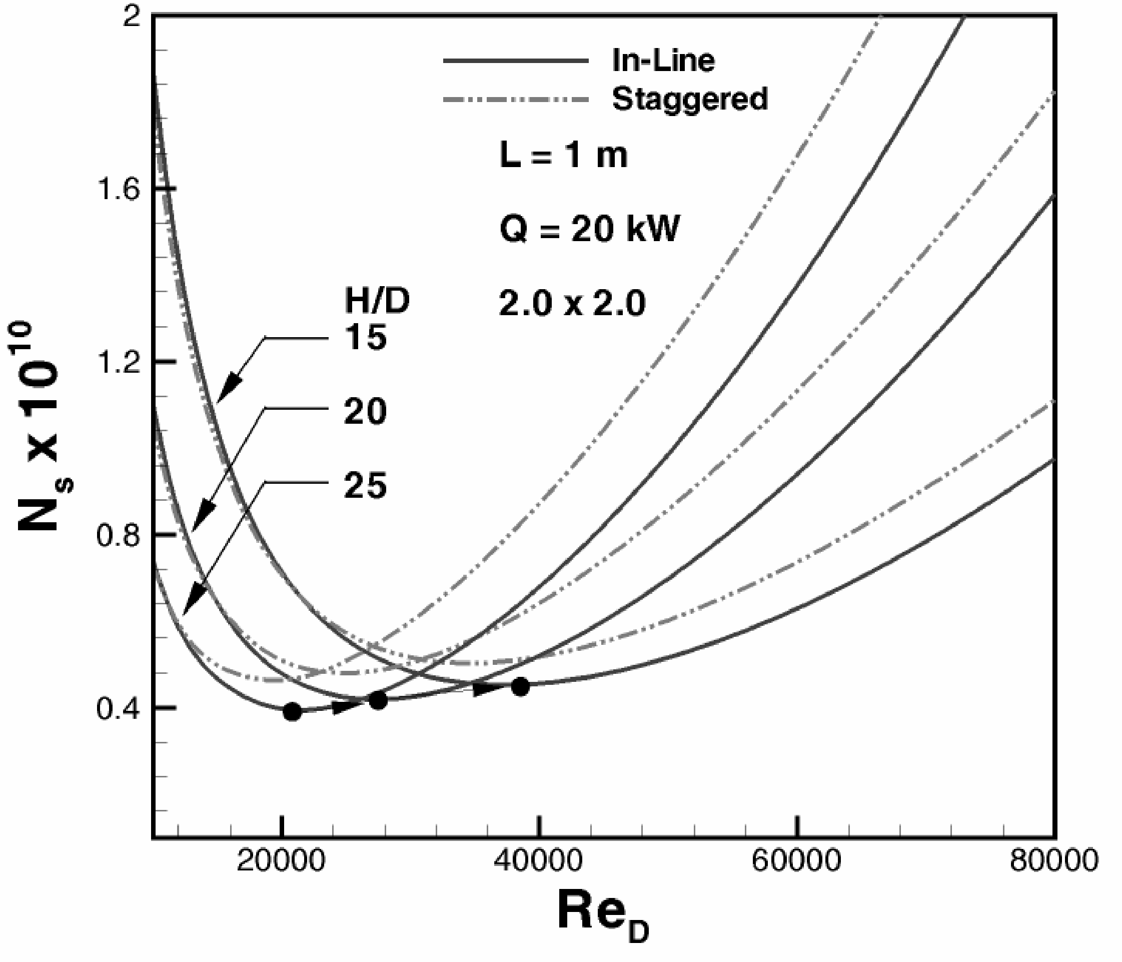
Effect of Tube Length (Compact)



Effect of Tube Length (Widely Spaced)



Effect of Reynolds Number



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Conclusions

- Staggered arrangement gives better performance for lower approach velocities and longer tubes.
- In-line arrangement performs better for higher approach velocities and larger pitch ratios.
- Compact tube banks perform better for both arrangements and for smaller tube diameters.

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