Thermal Resistances of Gaseous Gap for Conforming Rough Contacts

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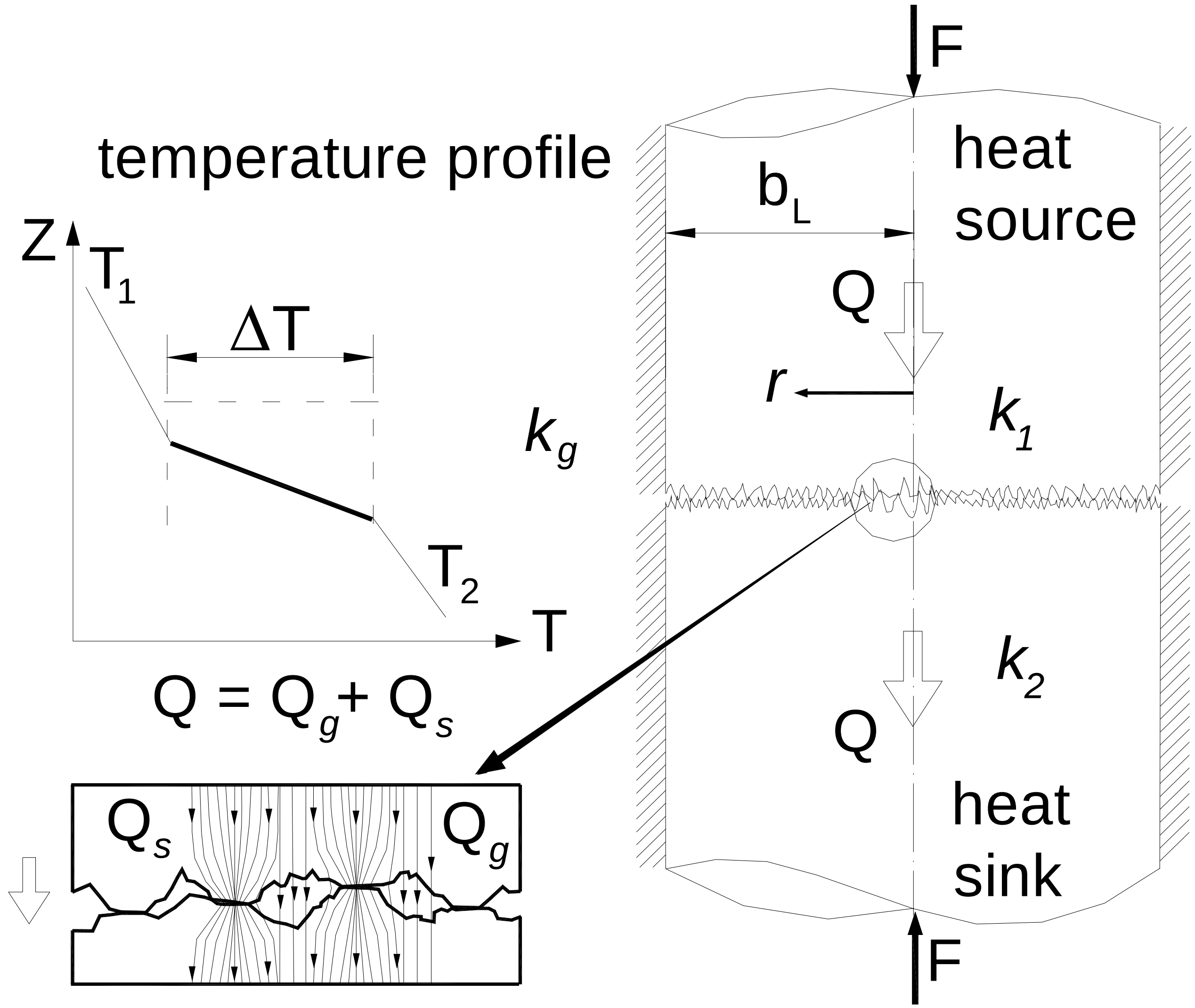
INTRODUCTION

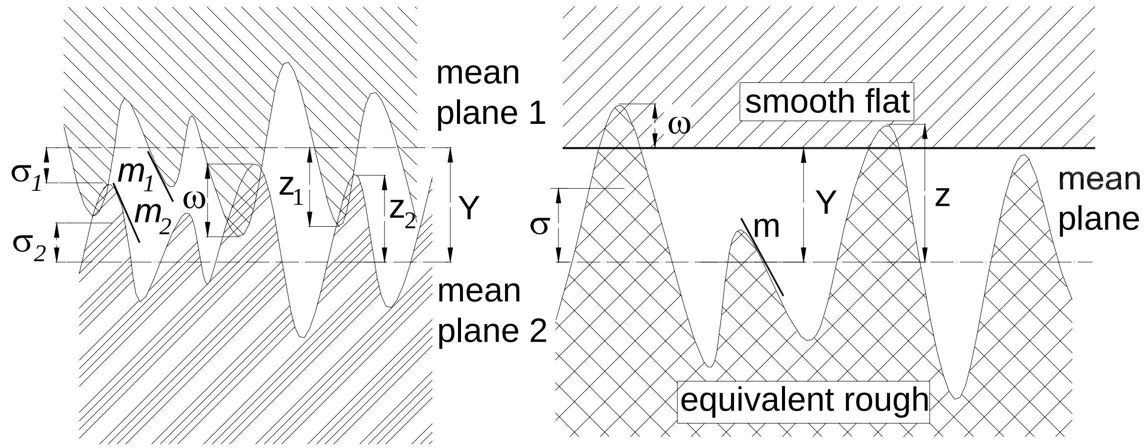


MICROCONTACTS THERMAL RESISTANCE

Bahrami et al. (2003)

$$R_s = \frac{0.565 c_1 (\sigma / m)}{k_s F} \left(\frac{\sigma}{m} \right)^{c_2}$$

$$k_s = \frac{2 k_1 k_2}{k_1 + k_2}$$



a) section through two contacting surfaces

b) corresponding section through equivalent rough - smooth flat

$$\sigma = \sqrt{\sigma_1 + \sigma_2}$$

$$m = \sqrt{m_1 + m_2}$$

CONDUCTION REGIMES IN GASES

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Knudsen number $Kn = \frac{\Lambda}{d}$

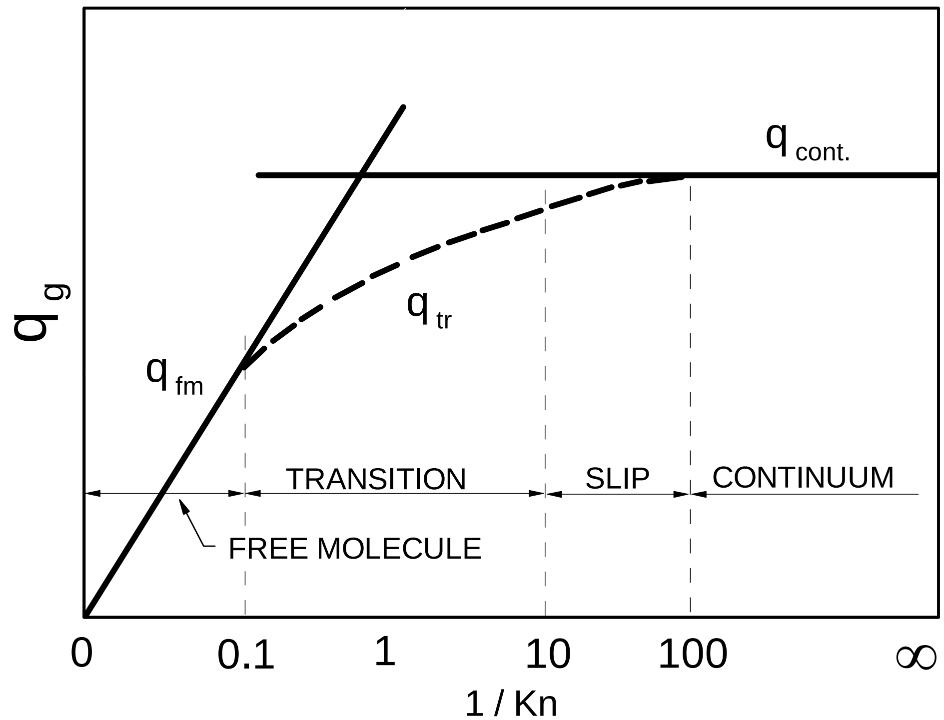
Λ: molecular mean free path

d: separation between two planes

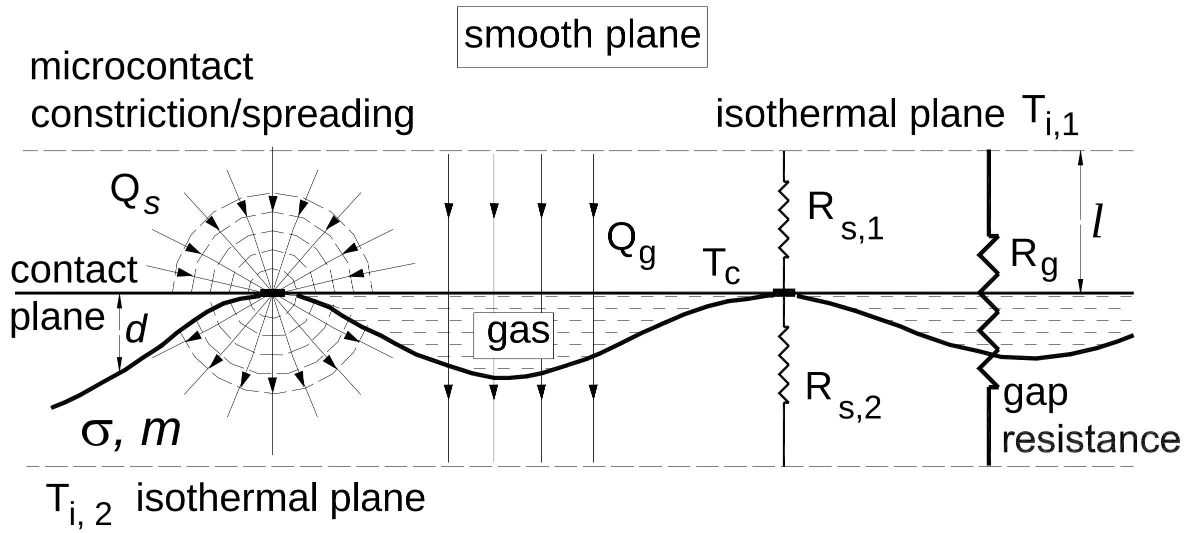
Kennard (1938) and Yovanovich (1982)

$$q_q = \frac{k_g}{d+M}\left(T_1 - T_2\right)$$

$$M = \left(\frac{2-TAC_1}{TAC_1} + \frac{2-TAC_2}{TAC_2}\right)\left(\frac{2\gamma}{1+\gamma}\right)\frac{\Lambda}{\Pr}$$



PRESENT MODEL



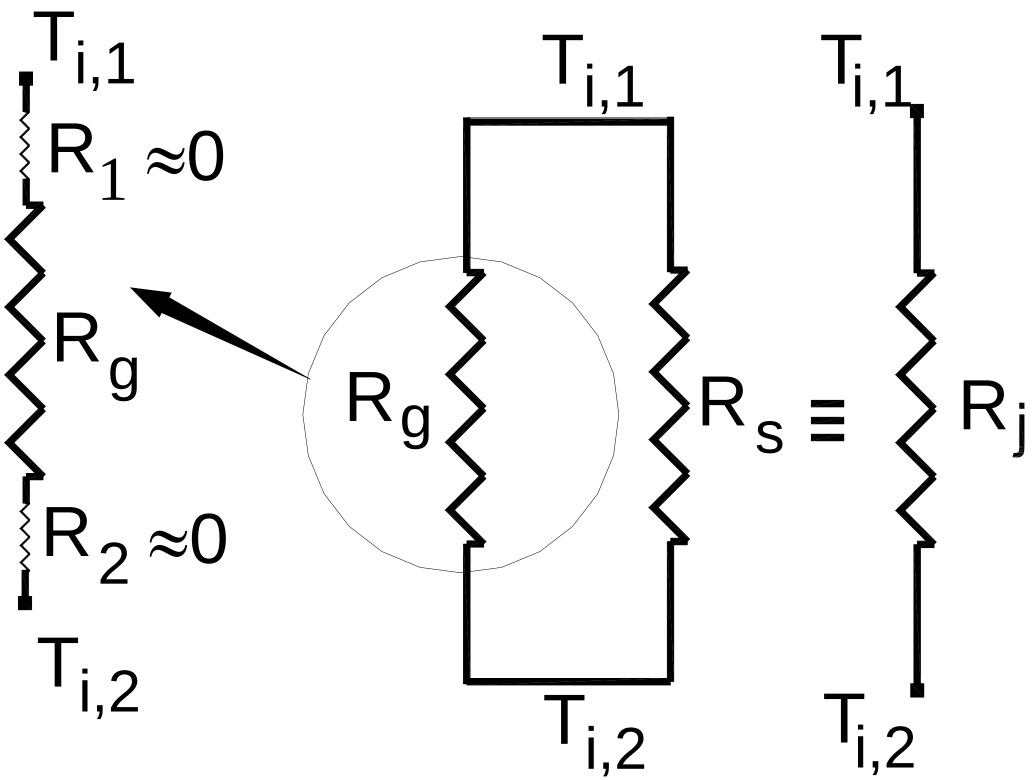
$l \approx 40 a_s$

$a_s \approx 1 \mu m$

equivalent rough surface

$$R_g = \underbrace{\frac{l}{k_{s,1} A_g} + \frac{l}{k_{s,2} A_g}}_{\approx 0} + \frac{d + M}{k_g A_g}, \quad \text{since } k_g / k_s \leq 0.01$$

THERMAL RESISTANCE NETWORK



$$R_j = \left(\frac{1}{R_s} + \frac{1}{R_g} \right)^{-1}$$

GAP THERMAL RESISTANCE

- Gaussian distribution

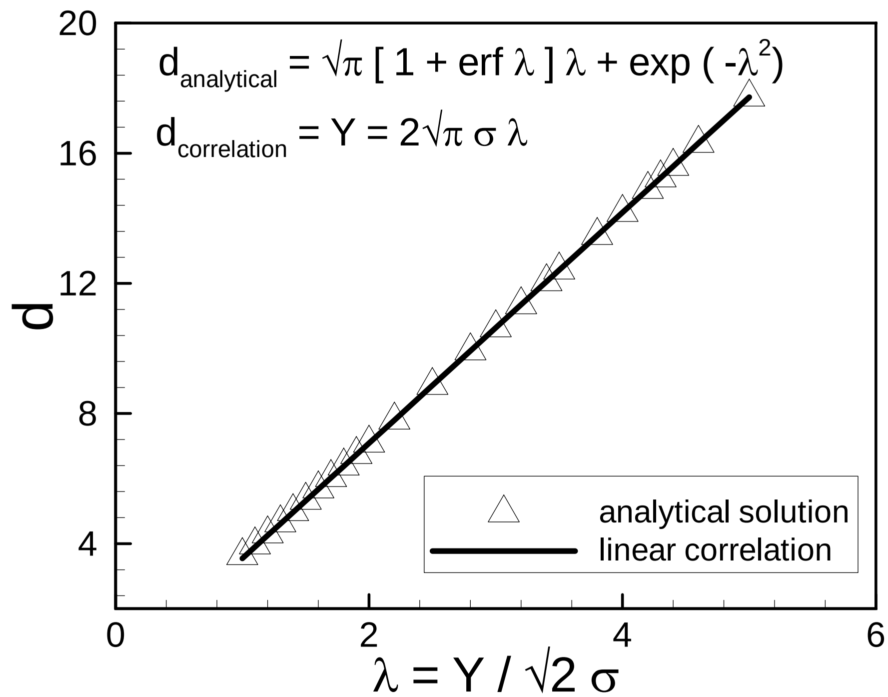
$$\phi(z) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{z^2}{2\sigma^2} \right)$$

- assume $A_g = A_a$

- effective separation, *d:*

$$d = \int_{-\infty}^{Y} (Y - z)\phi(z)\,dz$$

$$d = \frac{\sigma}{\sqrt{2\pi}} \left[\sqrt{\pi}(1 + \text{erf } \lambda)\lambda + \exp(\lambda^{-2}) \right]$$



- correlation for *d:*

$$d = Y$$

GAP THERMAL RESISTANCE

- Cooper et al. (1969) $$\frac{P}{H_{mic}} = \frac{1}{2}\operatorname{erfc}\lambda$$

- Song and Yovanovich (1988)

$$\frac{P}{H_{mic}} = \left(\frac{P}{H'}\right)^{s} \quad \text{where } H' = c_1 (1.62\sigma / m)^{c_2} \text{ and } s = \frac{1}{1 + 0.071 c_2}$$

- assuming s = 1 $$\lambda = \frac{Y}{\sqrt{2}\sigma} = \operatorname{erfc}^{-1}\left(\frac{2P}{H'}\right)$$

- a correlation for erfc^{-1}(x) is proposed, for $10^{-9} < x < 1.9$

- gap thermal resistance $$R_g = \frac{1}{k_g A_a}\left[\mathrm{M} + \underbrace{\sqrt{2}\sigma \operatorname{erfc}^{-1}\left(\frac{2P}{H'}\right)}_{Y}\right]$$

PRESENT MODEL VS INTEGRAL MODEL

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Yovanovich et al. (1982)

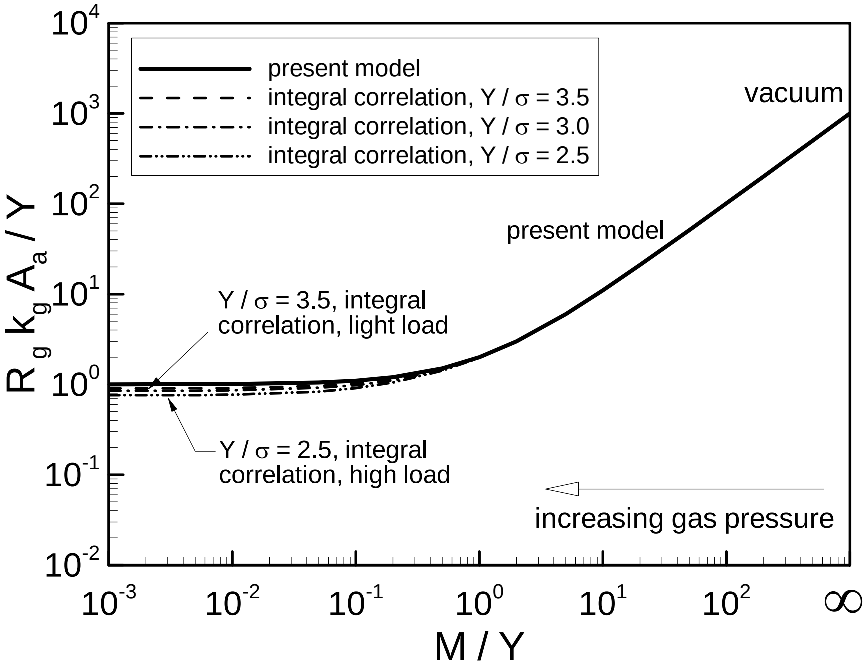
$$R_g = \frac{\sqrt{2\pi} Y}{A_a k_g \int_0^\infty \frac{\exp\left[-\left(Y/\sigma - t/\sigma\right)^2/2\right]}{\left(t/\sigma\right)/\left(Y/\sigma\right) + M/Y} d\left(t/\sigma\right)}$$

Song correlation (1988)

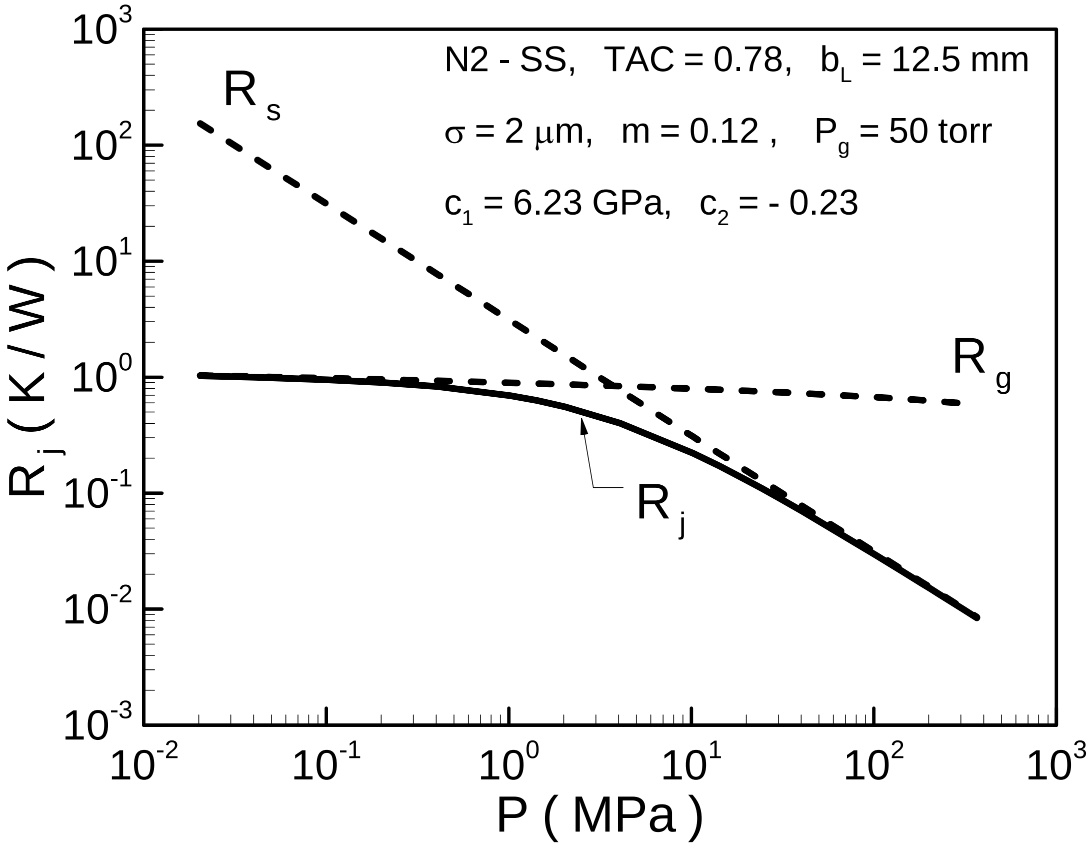
$$\frac{A_g k_g}{Y} R_g = 1 + \frac{0.304\left(\sigma/Y\right)}{\left(1 + M/Y\right)} - \frac{2.29\left(\sigma/Y\right)^2}{\left(1 + M/Y\right)^2} + \frac{M}{Y}$$

present model

$$\frac{A_g k_g}{Y} R_g = 1 + \frac{M}{Y}$$

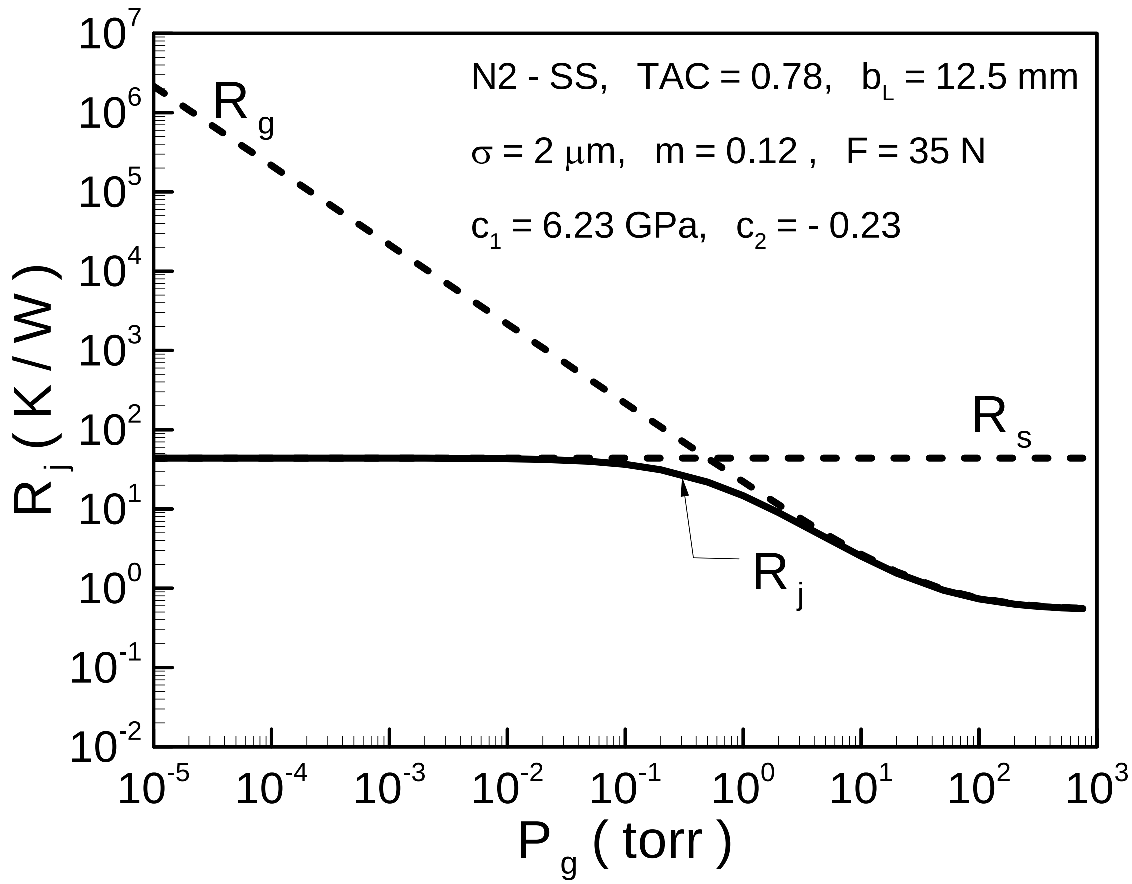


EFFECT OF LOAD ON JOINT RESISTANCE



$$R_s = \frac{0.565 c_1 (\sigma / m)}{k_s F} \left(\frac{\sigma}{m} \right)^{c_2}$$

EFFECT OF GAS PRESSURE ON JOINT RESISTANCE



$$\frac{A_g k_g}{Y} R_g = 1 + \frac{M}{Y}$$

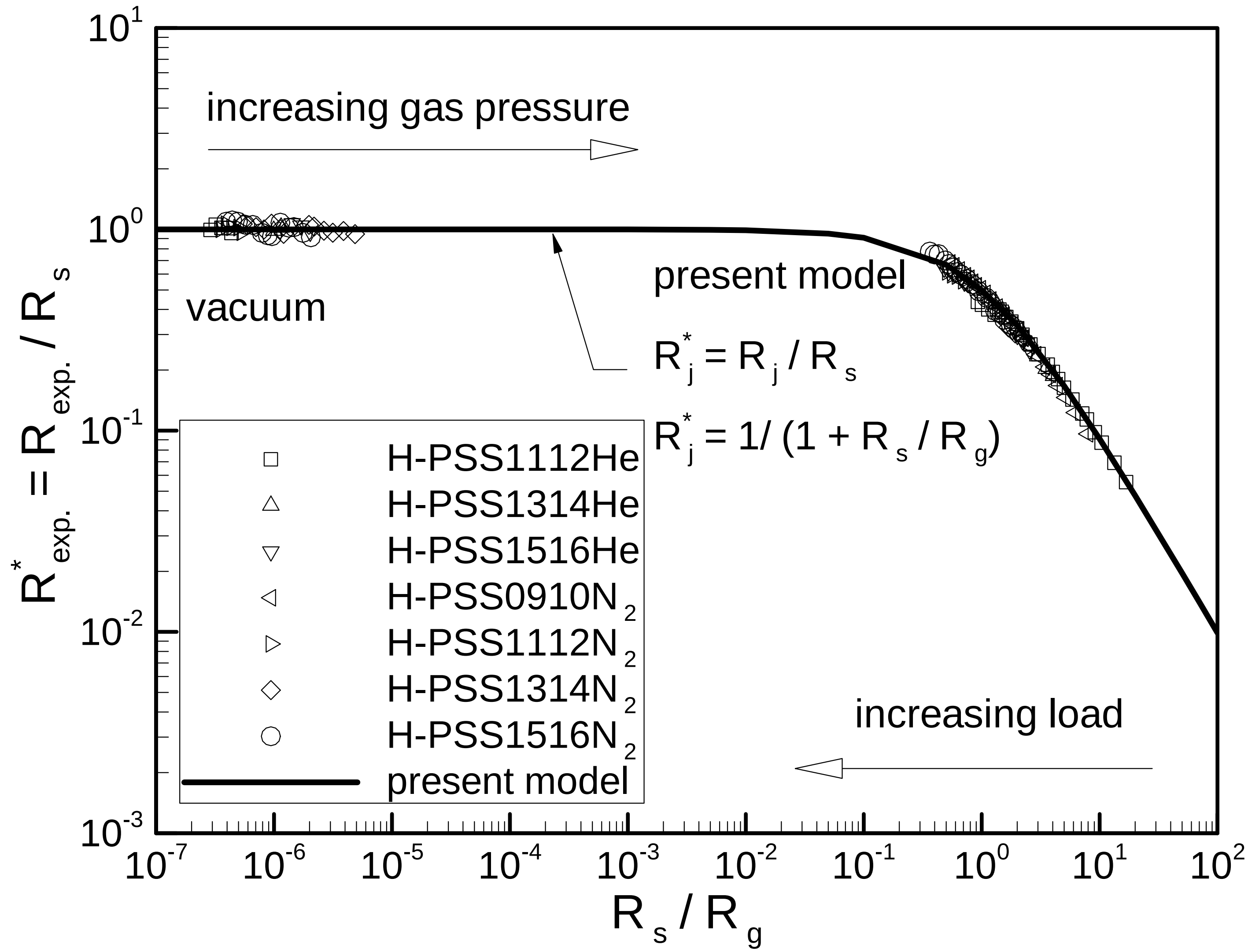
EXPERIMENTAL DATA

- Hegazy (1985)
  - 160 data point
  - four sets of SS 304 joints in N_2 and He
- Song (1988)
  - 350 data points
  - seven sets of SS 304 and Ni 200 in Ar, He, and N2

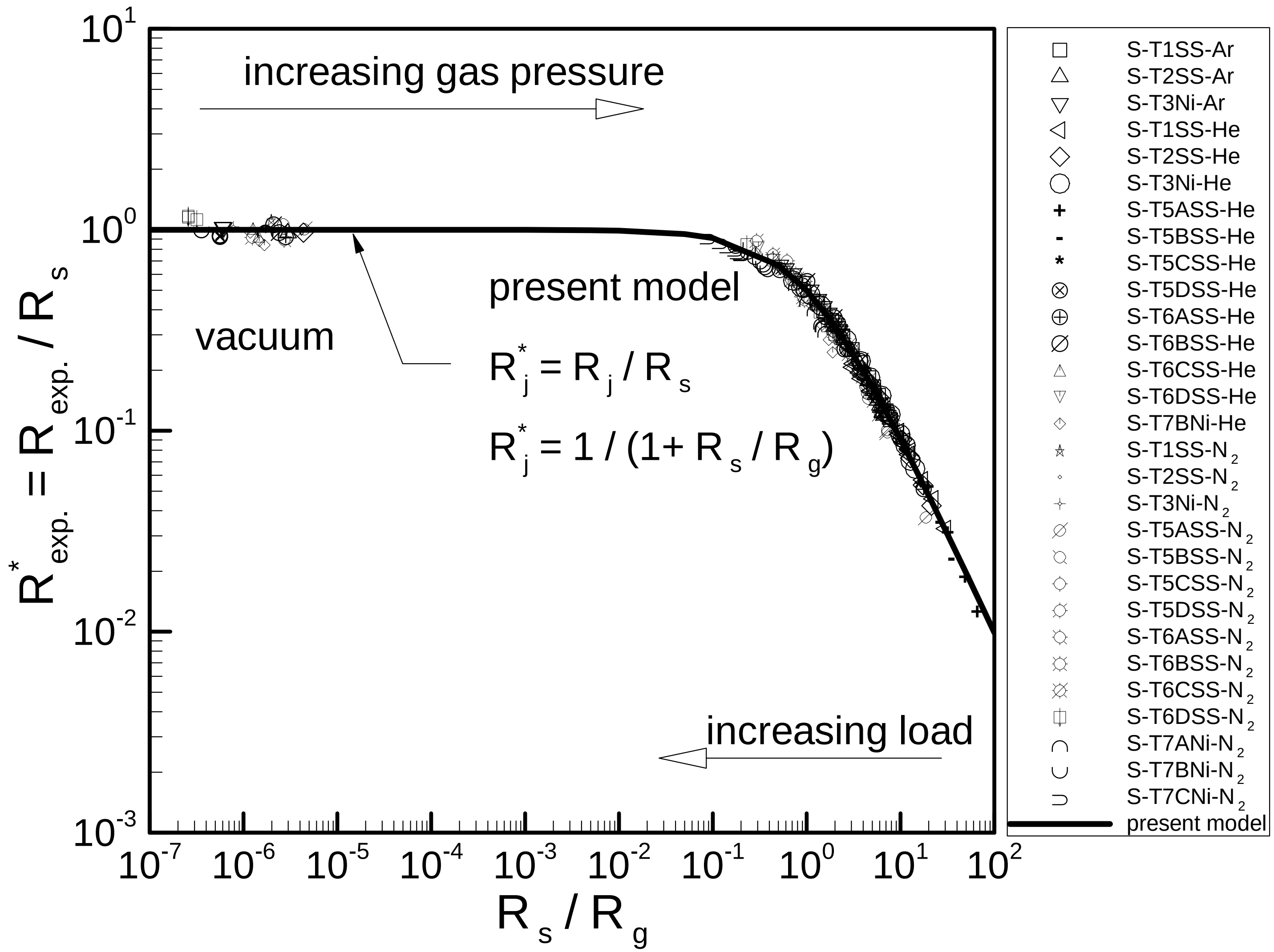
gas	k_g	Pr	TAC		0
	W/mK	–	–	–	nm
Ar	0.018 4.05E-5T	0.67	0.90	1.67	66.6
He	0.147 3.24E-4T	0.67	0.55	1.67	186
N_2	0.028 5.84E-5T	0.69	0.78	1.41	62.8

Parameter
$69.7 \le F \le 4357\ N$
$0.14 \le P \le 8.8\ MPa$
$19.2 \le k_s \le 72.5\ W/mK$
$0.08 \le m \le 0.205$
$10^{-5} \le P_g \le 760\ torr$
$0.55 \le TAC \le 0.9$
$1.52 \le \quad \le 11.8 \quad m$

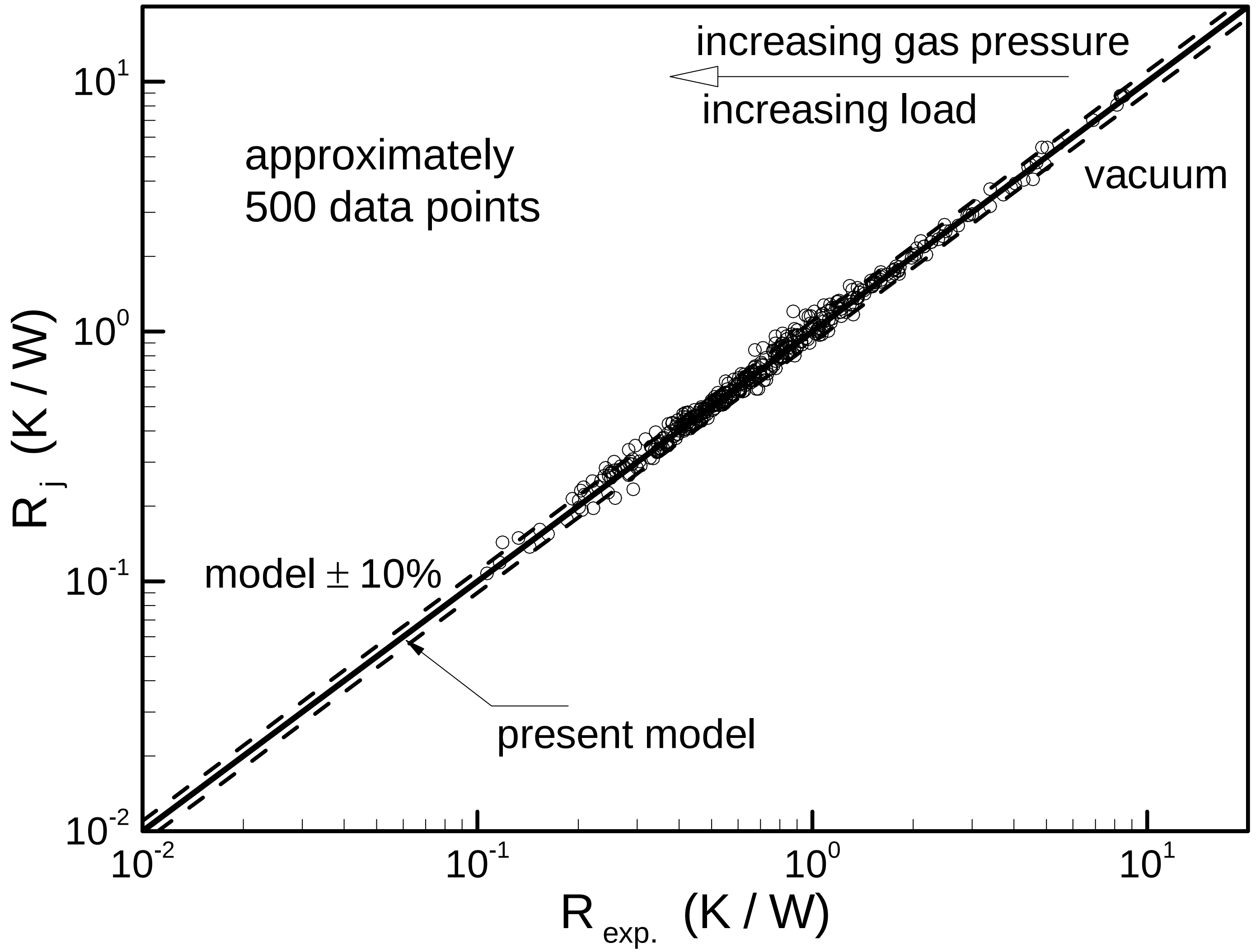
COMPARISON WITH HEGAZY DATA



COMPARISON WITH SONG DATA



COMPARISON WITH ALL DATA



SUMMARY AND CONCLUSIONS

- a compact model was developed for conforming rough joints in gaseous environment
- model covers four regimes of gas heat conduction, continuum, temperature-jump or slip, transition, and free molecular
- model accounts for gas and solid mechanical and thermal properties, gas pressure and temperature, surface roughness, and applied load
- for engineering applications, d = Y

SUMMARY AND CONCLUSIONS

- a correlation for inverse complementary error function was developed, with 2.8 percent max error over $10^{-9} < x < 1.9$.

- parametric studies showed
  - with constant gas pressure, at light loads $R_g << R_s$, thus most of the heat transfer occurs through gas. As load increases R_j decreases and $R_s << R_g$
  - with constant load, at very low gas pressures $R_g >> R_s$. As gas pressure increases R_g decreases and (at light loads) becomes controlling component of R_j.

SUMMARY AND CONCLUSION

- comparison with Yovanovich et al. (1982) model showed
  - small difference for slip and free molecular regimes
  - larger difference for continuum regime (atmospheric pressure) at relatively high loads
- model was compared with more than 510 experimental data points of Hegazy (1985) and Song (1988):
  - SS 304 and nickel 200, three gases: argon, helium and nitrogen
  - data covered a wide range of surface characteristics, load, thermal and mechanical properties and gas pressure
  - model showed good agreement with RMS relative difference approximately 7.3 percent.

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