AIAA-2003-0159

Conduction Shape Factor Models for 3-D Enclosures

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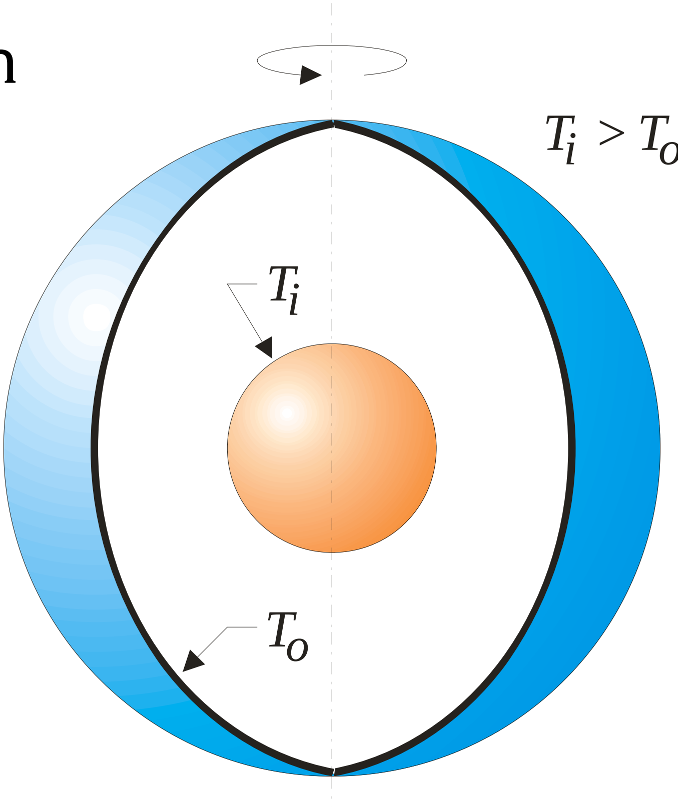
AIAA 41st Aerospace Sciences Meeting and Exhibit Conference
Reno, NV January 6, 2003

Outline

- Introduction
- Literature review
- Model development
- Application and validation of models
- Summary and Conclusions

Introduction

- Analytical models for conduction shape factors in enclosures
  - heated inner body
  - cooled surrounding enclosure
  - arbitrarily-shaped, concentric boundaries
  - isothermal boundary conditions
- Limiting case for natural convection models for enclosures



Literature Review

- Numerical data
  - Hassani[1] - concentric circular cylinders
    - concentric, base-attached double cones
  - Warrington et al.[2] - concentric cubes
    - sphere in cubical enclosure
    - cube in spherical enclosure
- Analytical model – Hassani and Hollands[3]

$$S = \left(S_0^m + S_\infty^m\right)^{1/m}, \quad S_0 = A_i / \delta, \quad S_\infty = 3.51\sqrt{A_i}$$

  - $m = 1$, concentric spheres – linear superposition
  - $m > 1$, other boundary shapes, dependent on geometry, inner area, aspect ratio
  - limited to geometrically similar boundary shapes

Problem Definition

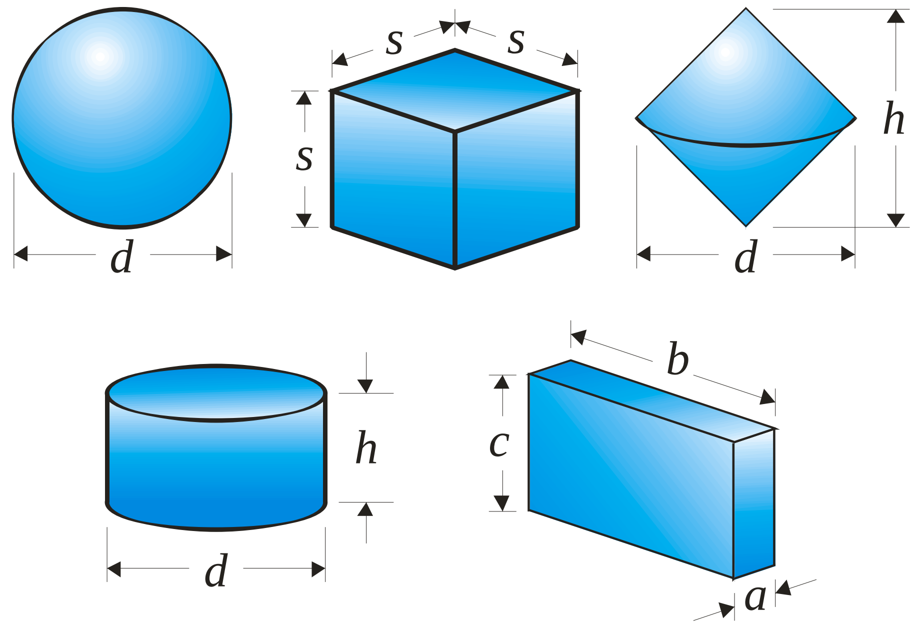
- Conduction shape factor

$$S = \iint_{A_i} - \frac{\partial \psi}{\partial n}\bigg|_{A_i} dA_i$$

$$\psi = \frac{T(\vec{r}) - T_o}{T_i - T_o}$$

- Dimensionless shape factor

$$S^*_{\sqrt{A_i}} = \frac{S}{\sqrt{A_i}} = \frac{Q}{k\sqrt{A_i}\,(T_i - T_o)}$$



Inner and Outer Boundary Shapes

Model Development

- Thermal resistance for spherical shells

$$R = \frac{1}{2\pi k}\left(\frac{1}{d_i} - \frac{1}{d_o}\right) \quad \Rightarrow \quad S^*_{\sqrt{A_i}} = \frac{2\sqrt{\pi}}{\left(1 - d_i / d_o\right)}$$

- Recast $R, S^*_{\sqrt{A_i}}$ based on gap thickness

$$\delta = \frac{d_o - d_i}{2} \quad \Rightarrow \quad R = \frac{\delta}{\pi k d_i (d_i + 2\delta)} \quad \Rightarrow \quad S^*_{\sqrt{A_i}} = \frac{\sqrt{\pi}\, d_i}{\delta} + 2\sqrt{\pi}$$

- General model based on concentric sphere solution

$$S^*_{\sqrt{A_i}} = \frac{\sqrt{A_i}}{\delta_e} + S^*_\infty$$

S^*_∞ = inner body conduction shape factor in full-space region

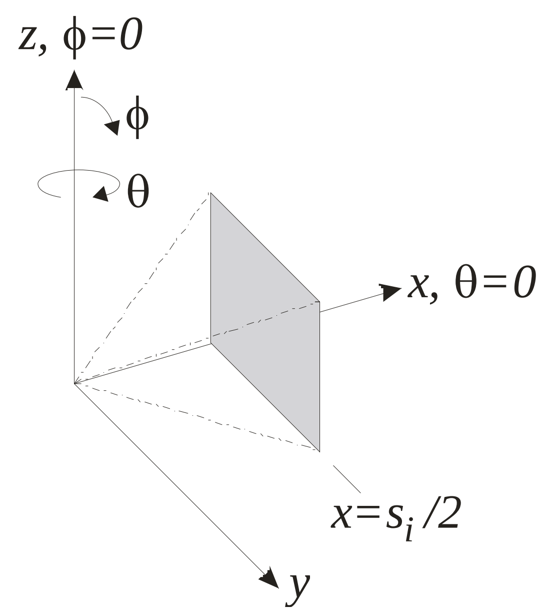
δ_e = effective gap spacing

Effective Gap Spacing – Integral Model

- Local gap thickness $\delta(\phi, \theta)$ can be calculated in spherical coordinates for certain geometries
- Example: cube in spherical enclosure

$$\delta(\phi,\theta) = \frac{d_o}{2} - \rho(\phi,\theta), \quad \rho(\phi,\theta) = \frac{s_i/2}{\sin\phi\cos\theta}$$

$$0 \le \theta \le \frac{\pi}{4} \qquad \tan^{-1}\sec\theta \le \phi \le \frac{\pi}{2}$$



- Effective gap spacing from area average

$$\delta_e = \frac{24}{\pi d_o^2} \int_0^{\pi/4} \int_{\tan^{-1}\sec\theta}^{\pi/2} \delta(\phi,\theta) \frac{d_o^2}{4} \sin\phi \, d\phi \, d\theta$$

Effective Gap Spacing – Two-rule Model

- Equivalent spherical enclosure preserves
  - inner body surface area, A_i
  - enclosed volume, V

$$d_i = \sqrt{A_i/\pi} \quad d_o = \left[6\left(V + V_i\right)/\pi\right]^{1/3} \quad V_i = \frac{A_i^{3/2}}{6\sqrt{\pi}} \quad \delta_e = \frac{d_o - d_i}{2}$$

- Conduction shape factor with two-rule model

$$S^*_{\sqrt{A_i}} = \frac{2\sqrt{\pi}}{\left[1 + 6\sqrt{\pi}\left(\frac{V^{1/3}}{\sqrt{A_i}}\right)^3\right]^{1/3} - 1} + S^*_\infty$$

Model Validation

- Models validated for seven enclosure configurations
  - geometrically similar boundary shapes
    - cubes, cylinders, base-attached double cones
  - different boundary shapes
    - cube in spherical enclosure
    - sphere in cubical enclosure
    - cuboid in cubical enclosure
    - circular cylinder in cubical enclosure
- Validation performed using numerical data
  - existing data from literature
  - FLOTHERM[7] CFD simulations

Geometrically Similar Boundary Shapes

- Concentric cubes

$$S^*_{\sqrt{A_i}} = \frac{2\sqrt{\pi}}{\left(1+\frac{\sqrt{\pi}}{6}\left[\left(\frac{s_o}{s_i}\right)^3 - 1\right]^{1/3}\right) - 1} + 3.391$$

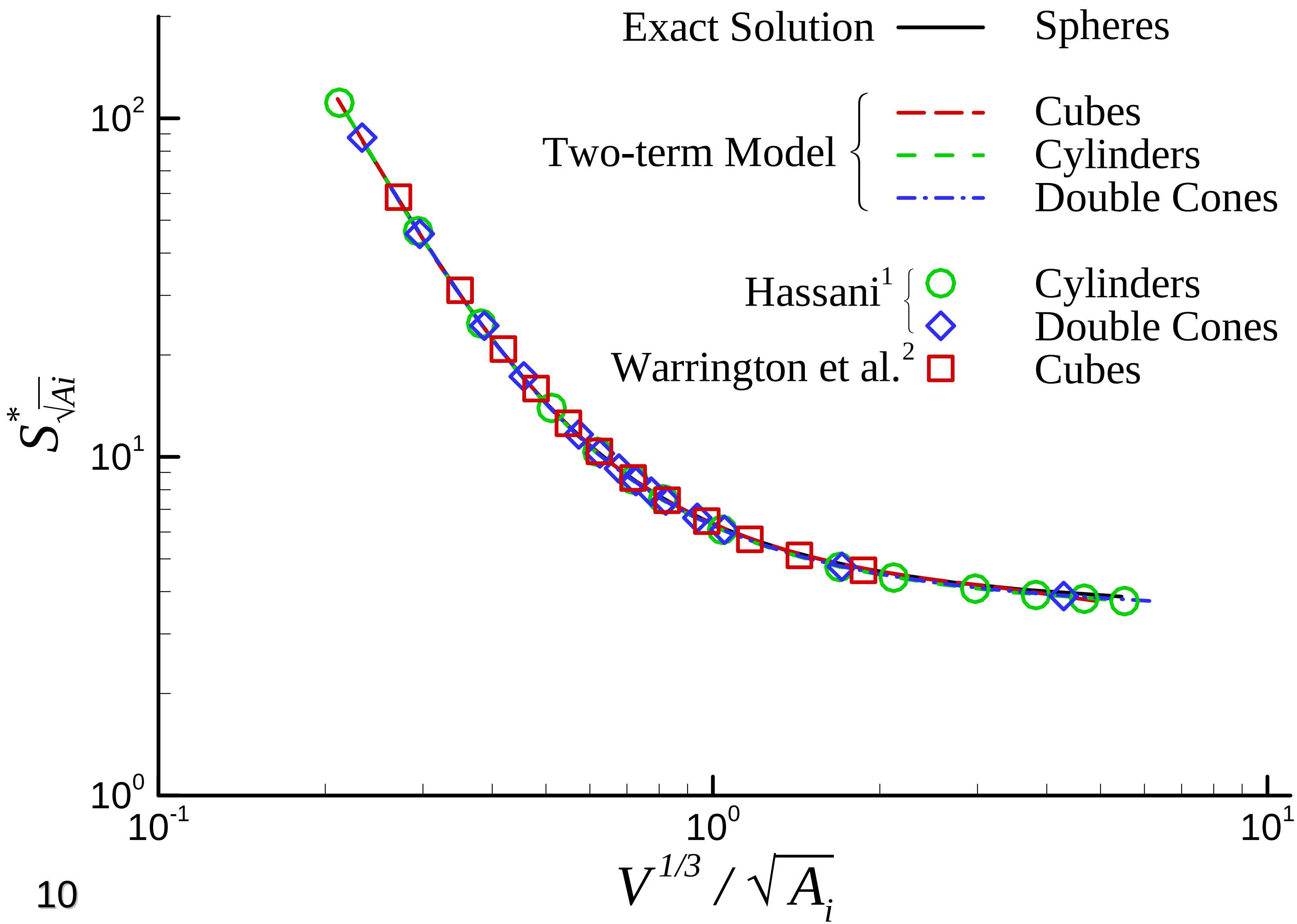
- Cylinders
  $h/d = 1$

$$S^*_{\sqrt{A_i}} = \frac{2\sqrt{\pi}}{\left(1+\frac{2}{\sqrt{6}}\left[\left(\frac{d_o}{d_i}\right)^3 - 1\right]^{1/3}\right) - 1} + 3.443$$

- Base-attached double cones
  $h/d = 1$

$$S^*_{\sqrt{A_i}} = \frac{2\sqrt{\pi}}{\left(1+\sqrt{2}\left[\left(\frac{d_o}{d_i}\right)^3 - 1\right]^{1/3}\right) - 1} + 3.471$$

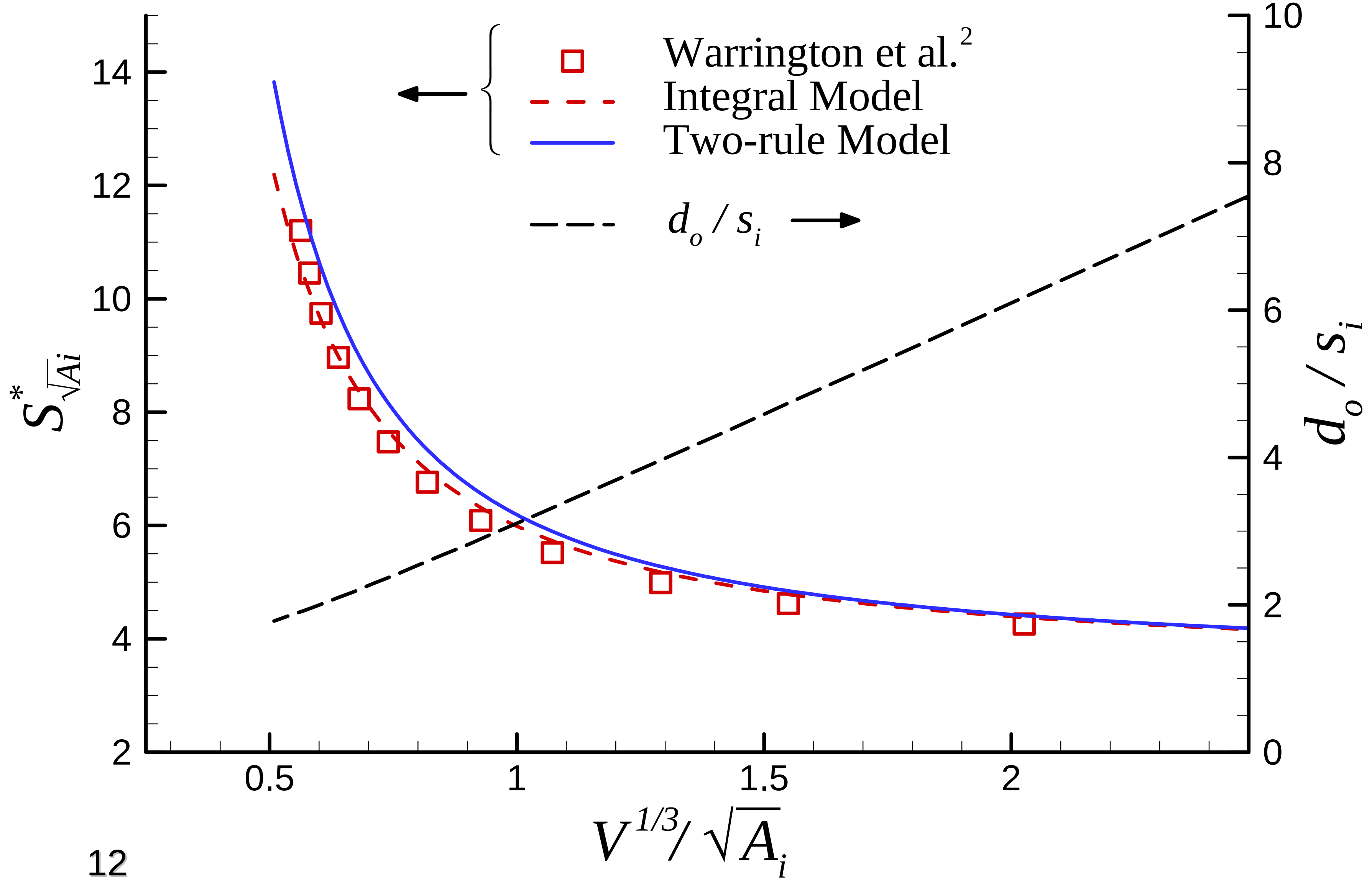
Geometrically Similar Boundary Shapes



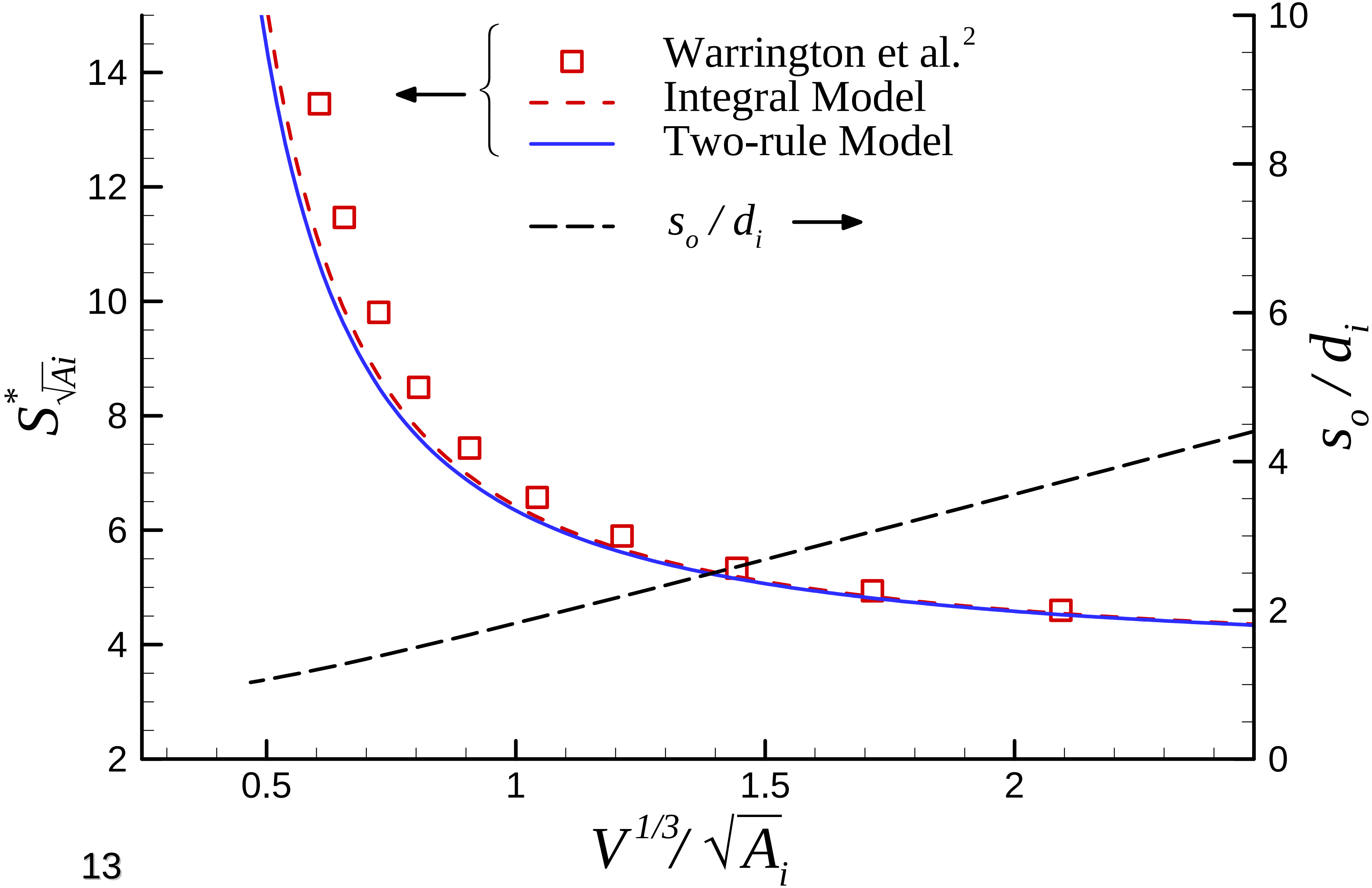
Enclosures with Different Boundary Shapes

- Cube in spherical enclosure
  - integral method ⇨ $\delta_e = s_i\left(\frac{1}{2}\frac{d_o}{s_i} - 0.6107\right)$
  - two-rule method
- Sphere in cubical enclosure
  - integral method ⇨ $\delta_e = d_i\left(0.6107\frac{s_o}{d_i} - \frac{1}{2}\right)$
  - two rule method
- Cuboid in cubical enclosure $(a\,,\, b = 3.785\,a\,,\, c = 2.175\,a)$
  - two-rule method
- Circular cylinder in cubical enclosure $(h/d = 0.5)$
  - two-rule method

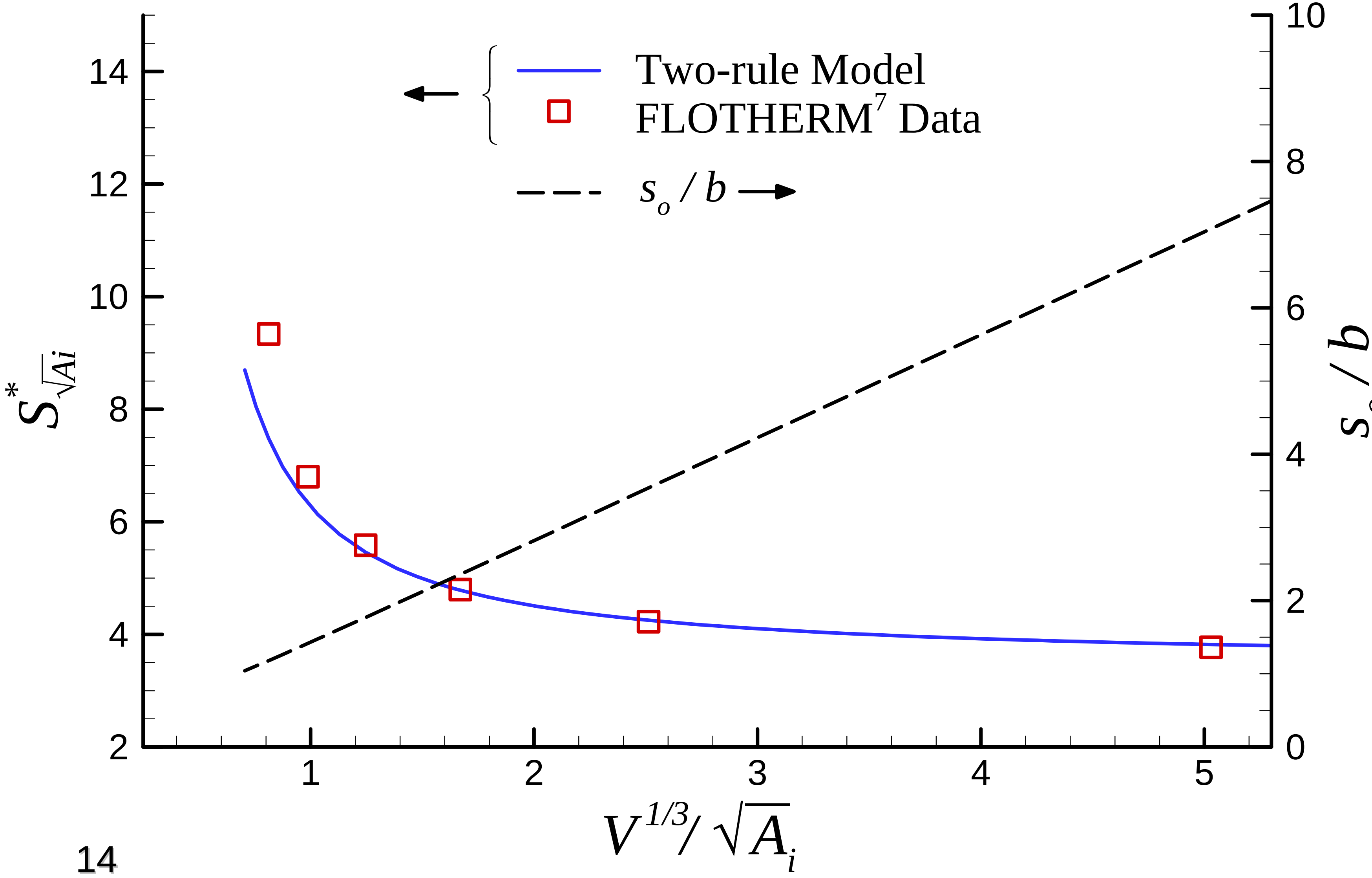
Cube in Spherical Enclosure



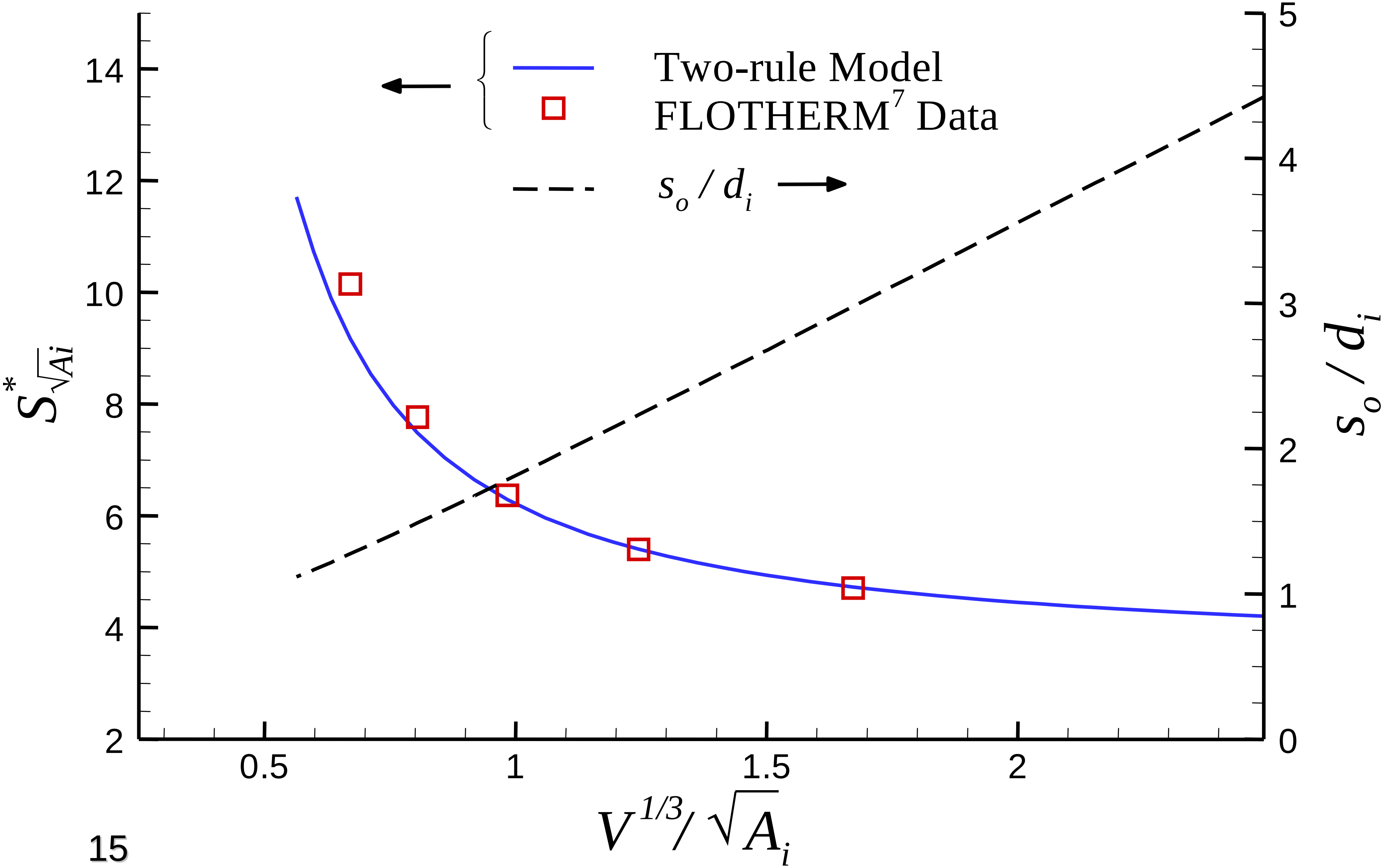
Sphere in Cubical Enclosure



Cuboid in Cubical Enclosure



Circular Cylinder in Cubical Enclosure



Summary and Conclusions

- General model for conduction shape factors for isothermal 3-D enclosures
- Two models for effective gap spacing
  - integral method – limited to specific geometries
  - two-rule method – applicable to all enclosures
- Excellent agreement (<3% RMS) with numerical data for geometrically similar boundary shapes
- Good agreement for enclosures with different boundary shapes
  - 3 – 5 % RMS when $V^{1/3} / \sqrt{A_i} > 1$

Acknowledgements

- MMO - Materials and Manufacturing Ontario
- CMAP - Center for Microelectronics Assembly and Packaging