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THERMAL WAKE EFFECTS IN PRINTED CIRCUIT BOARDS

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A simple, one-dimensional analytical model is presented that characterizes thermal wake effects for a convection cooled flat plate with discrete, surface mounted isoflux heat sources. Using integral solution techniques, the model predicts the temperature rise induced at downstream locations due to heating of the boundary layer by upstream sources. This analytical solution can be part of a coupled solid-fluid model to quickly and accurately evaluate conjugate heat transfer for air-cooled electronics applications. The thermal wake model is validated using numerical simulations for a wide range of test cases, and the average difference between the numerical results and analytical predictions is less than 2% for all test cases.

1. Introduction

As packaging densities and power dissipation levels increase in printed circuit boards, it becomes imperative that designers have the ability to accurately predict temperature and heat flux profiles. Many models used to calculate local temperatures in PCBs populated with low-profile IC packages use iterative procedures that account for the conjugate behaviour of heat transfer between the fluid and solid domains. These models generally consist of separate procedures for solving the energy equation in the solid and the fluid domains. The domains are then coupled, typically by using a heat transfer coefficient, to locally match fluid and solid temperatures over the full extent of the fluid/solid interface.

Solid-side models range from conventional domain discretization schemes, such as finite volume[1] or finite element methods,[2] to analytical procedures, such as Fourier series methods.[3,4] These methods are well documented and will not be addressed in this paper. There are a limited number of approaches available for the selection of a fluid-side model to obtain a boundary condition along the solid interface. The simplest and computationally least intensive procedure is the use of a uniformly specified heat transfer coefficient based on correlation equations. Unfortunately, this approach can lead to significant inaccuracies in the calculation of temperature and heat flux, especially in applications with distributed heat sources, such as PCBs. The second approach and the most computationally intensive is the full solution of the Navier Stokes equations to obtain a detailed mapping over the entire fluid domain. This approach provides a wealth of information beyond what is required to solve the conjugate heat transfer problem at the fluid/solid interface. However, the extensive run time required for numerical computations makes this method a poor choice for a design tool.

An alternate approach presented in this study provides both the speed and accuracy required for an efficient design tool. Since conjugate modelling requires a matching of temperature and temperature gradient between the fluid- and solid-domains at the fluid/solid interface, a detailed temperature map of the boundary layer is not required. However, the temperature of the fluid, adjacent to the solid interface must be accurately predicted and for this boundary layer, integral methods provide a convenient and practical method for calculating fluid temperatures and heat transfer coefficients.

2. Model Development

Models used to simulate heat transfer in printed circuit boards must allow for the arbitrary placement of electronics packages over the surface of a PCB. Packages located near the leading edge of the PCB are cooled by the approaching fluid that is typically at ambient temperature. However, packages positioned downstream from the leading edge of the PCB are located in the thermal wake of upstream packages and see a cooling fluid that is above the temperature of the ambient fluid. Localized predictions of fluid temperature can be separated into two components; a temperature rise due to localized heating and a temperature rise due to the accumulation of heat in the boundary layer from upstream sources.

The calculation of temperature rise in a fluid as it passes over a flat surface with arbitrarily located heats sources, flush mounted with the surface of a substrate is well documented.[5–10] One approach that is commonly used to approximate the non-uniform, flux specified boundary conditions that result from discrete heat sources on a conductive substrate is to discretize the flux profile in the flow direction as shown in Fig. 1 to produce a piece-wise continuous profile of uniform step changes in heat flux. While the distributed flux specified boundary condition is a result of the conductive substrate, the fluid-side model itself is assumed to have no direct connection to the substrate other than through the flux-specified boundary condition imposed along the fluid-solid interface.

In this type of piecewise analysis of the problem, regions of zero flux may occur if the substrate is assumed to be non-conductive i.e., $k_s = 0$ W/(m · K), such as at the leading edge region of the board shown in Fig. 1. However, in all practical applications the conductivity of the printed circuit board is greater than zero and conjugate heat transfer occurs, resulting in a positive heat flux over all regions of the board. For the conjugate problem, the same

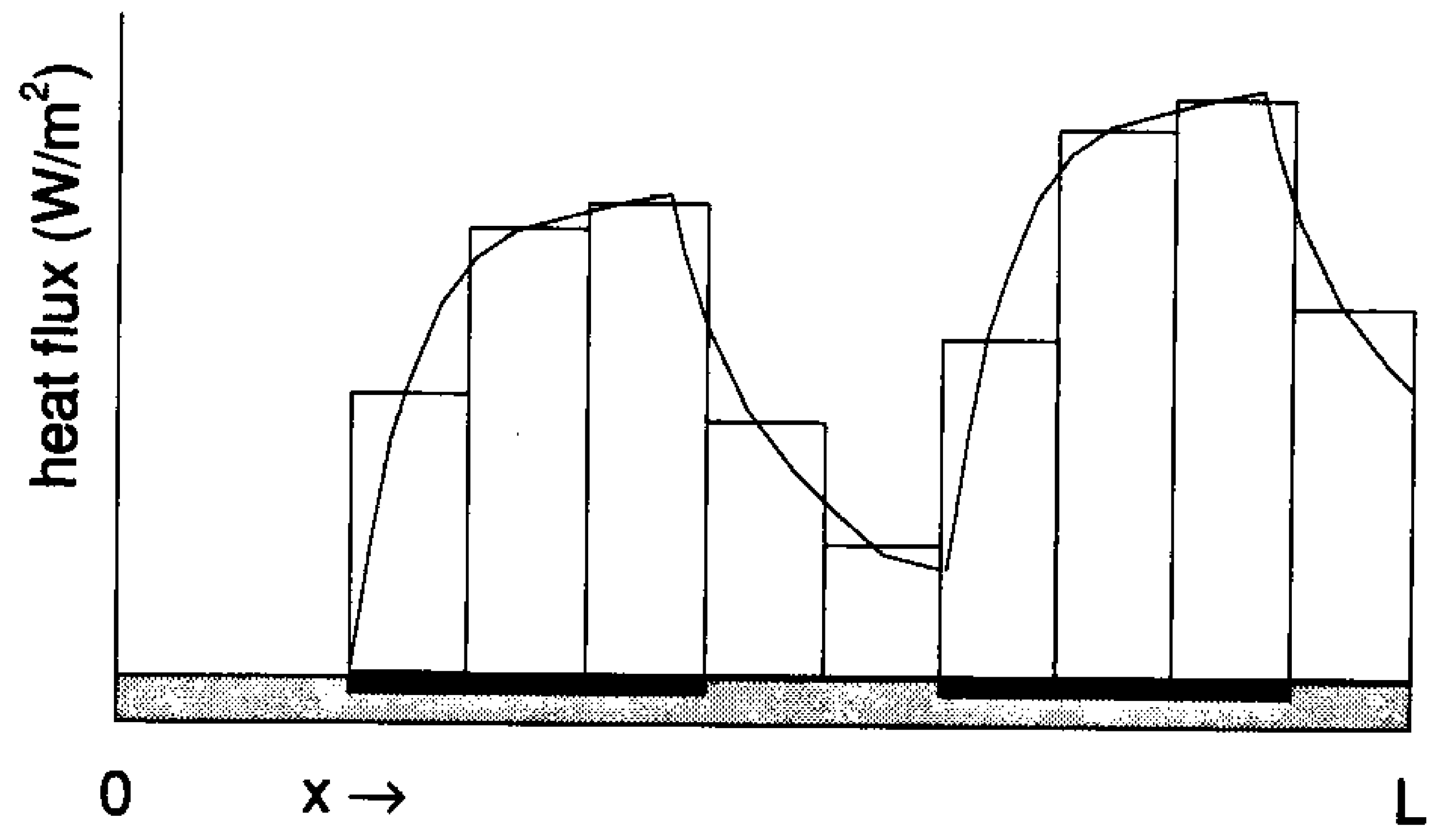

Fig. 1. Piece-wise continuous profile of heat flux.

procedure of discretizing the board into a series of small regions can be used with success, where the local heat flux can be approximated as uniform.

The following model for quantifying the thermal wake effect is based on forced convection flow (or natural convection using an equivalent flow velocity) where fluid temperatures are influenced only by upstream heat sources. The advection of heat is considered to be uni-directional in the primary flow direction, x. The model does not account for any heat transfer in the boundary layer that is not in the primary flow direction.

The positional coordinate in the x-direction varies between 0 and L, where L is the total length of the printed circuit board. When calculating the temperature or wake effect at some arbitrary point x in the range of $0 \to L$, it is not necessary to include any heat contribution in the range $x \to L$, because of the directional effects discussed previously. It is therefore convenient to temporarily fix the end of the plate at x to create a simulated plate with values that range between 0 and x. A new local variable ξ is introduced as the coordinate over the range of $0 \to x$. It is mathematically convenient to normalize the position variable ξ with respect to the total length of the simulated plate such that $0 \le \xi/x \le 1$, as shown in Fig. 2.

Wake model

The fluid temperature at the surface of an adiabatic flat plate with a discrete heat source having a uniformly specified heat flux boundary condition,[9,10] can be written as:

$$\Delta T_f(x) = \frac{\mathrm{Pr}^{-1/3}\mathrm{Re}_x^{-1/2} \cdot x \cdot q}{0.454 \cdot k_f} \cdot \{[(1-\xi_1/x)^{1/3} - (1-\xi_2/x)^{1/3}]\} \quad (1)$$

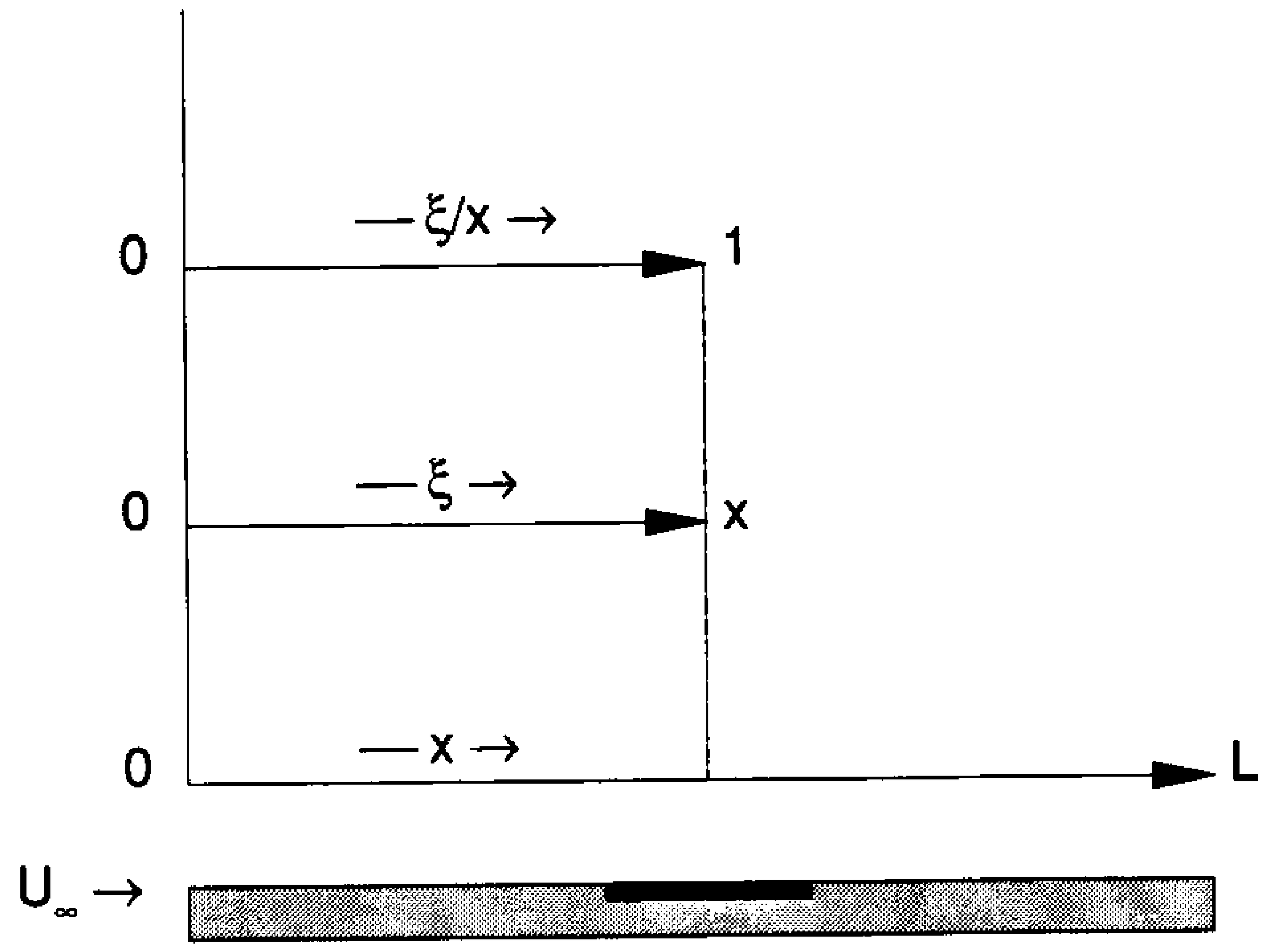

Fig. 2. Redefined coordinate system for thermal wake model.

where:

ξ_i/x = normalized position vector

ξ_1 = leading edge of the heat source

ξ_2 = trailing edge of the heat source

Since the fluid temperature at x is only a function of local heating and upstream effects, all sources downstream of x have no influence on the fluid temperature at x. The assumption that x is the artificial end of the plate guarantees that the normalized position vector (ξ/x), which is normalized with respect to x and not L, will range between 0 and 1.

As more sources are added to the surface of a circuit board, the fluid temperature at some arbitrary location x is no longer just a function of localized heating effects, since upstream wake effects must be considered. Using the form of the temperature solution given in Eq. (1) for a single source problem, a multiple heat source solution can be written as:

$$\Delta T_f(x) = \frac{\mathrm{Pr}^{-1/3}\mathrm{Re}_x^{-1/2} \cdot x \cdot q_N}{0.454 \cdot k_f} \cdot \{\text{wake effects} + \text{local effects}\} \quad (2)$$

The temperature solution for a multiple heat source problem or a problem where the board is discretized to approximate a non-uniform flux distribution as a piece-wise continuous distribution, can be written as

$$\Delta T_f(x) = \frac{\mathrm{Pr}^{-1/3}\mathrm{Re}_x^{-1/2} \cdot x}{0.454 \cdot k_f} \cdot \{q_1[(1-\xi_1/x)^{1/3} - (1-\xi_2/x)^{1/3}] + q_2[(1-\xi_3/x)^{1/3} - (1-\xi_4/x)^{1/3}] + \cdots + q_N[(1-\xi_{2N-1}/x)^{1/3} - (1-\xi_{2N}/x)^{1/3}]\} \quad (3)$$

where in a conjugate problem the q's will always be greater than zero.

The right hand side of Eq. (3) can be rearranged so that the bracketed term consists of the product of a dimensionless heat flux and a dimensionless geometric term which represents the distance between upstream sources and the local heat source. Each value of heat flux is normalized with respect to the local value of heat flux at x.

$$\Delta T_f(x) = \frac{\mathrm{Pr}^{-1/3}\mathrm{Re}_x^{-1/2} \cdot x \cdot q_N}{0.454 k_f} \cdot \left\{\frac{q_1}{q_N}[(1-\xi_1/x)^{1/3} - (1-\xi_2/x)^{1/3}] + \frac{q_2}{q_N}[(1-\xi_3/x)^{1/3} - (1-\xi_4/x)^{1/3}] + \cdots + [(1-\xi_{2N-1}/x)^{1/3} - (1-\xi_{2N}/x)^{1/3}]\right\} \quad (4)$$

By examining Eq. (4), it can be clearly seen that the series solution consists of a term which accounts for the local heating which is almost identical to the bracketed term in Eq. (1) plus a second group of terms that account for heating due to the wake effect. The wake terms act as dimensionless weighting functions that account for both upstream heat fluxes and location in relation to the local source of interest.

Local effects:

$$\Delta T_f(x) = \frac{\mathrm{Pr}^{-1/3}\mathrm{Re}_x^{-1/2} x\, q_N}{0.454 \cdot k_f} \{[(1-\xi_{2N-1}/x)^{1/3} - (1-\xi_{2N}/x)^{1/3}]\} \quad (5)$$

Wake effects:

$$\Delta T_f(x) = \frac{\mathrm{Pr}^{-1/3}\mathrm{Re}_x^{-1/2} x\, q_N}{0.454 \cdot k_f} \left\{\sum_{i=1}^{N-1} \frac{q_i}{q_N}[(1-\xi_{2i-1}/x)^{1/3} - (1-\xi_{2i}/x)^{1/3}]\right\} \quad (6)$$

3. Discussion

A flat plate consisting of two heat sources attached to an adiabatic wall, as shown in Fig. 3, is used to demonstrate the importance of incorporating a wake model into the calculation of local temperature. The influence of the wake is demonstrated through its dependence on two design parameters; relative source location and relative source strength. The relative source location, d^* is defined as the distance between the leading edges of the two packages, non-dimensionalized using the package length ℓ:

$$d^* = \frac{d}{\ell} \tag{7}$$

Based on this definition, two heat sources positioned with their edges in contact would result in a value of $d^* = 1$, while $d^* = 0$ represents the special case where the heat sources are superimposed into a single source, with $Q = Q_1 + Q_2$. The second parameter used in this study is the relative source strength Q_1/Q_2, defined as the ratio of the total heat flows for the upstream and downstream packages.

In order to validate the thermal wake model presented in this study, a series of numerical simulations are performed using FLOTHERM,[11] a commercial, finite-volume based software package. Numerical results are determined for each of five different relative source locations and three relative source strength ratios, as presented in Table 1.

The FLOTHERM model uses a two-dimensional domain with an infinitely thin board and large fluid region with a uniform velocity profile at the leading edge of the board to simulate the free stream. The analytical model for fluid temperature, given in Eq. (3), is a function of boundary layer heat transfer only and does not include a diffusive heat transfer component that is sometimes considered at very low Reynolds number flow. A free stream velocity of $U = 5$ m/s is used to ensure that diffusion effects are minimal. The package length is set to $\ell = 0.05$ m to ensure that for all cases, the flow over the entire board remains well within the laminar region, $\mathrm{Re}_L < 500\,000$. Heat sources are modelled as isoflux sources, with heat flux values as shown in Table 2.

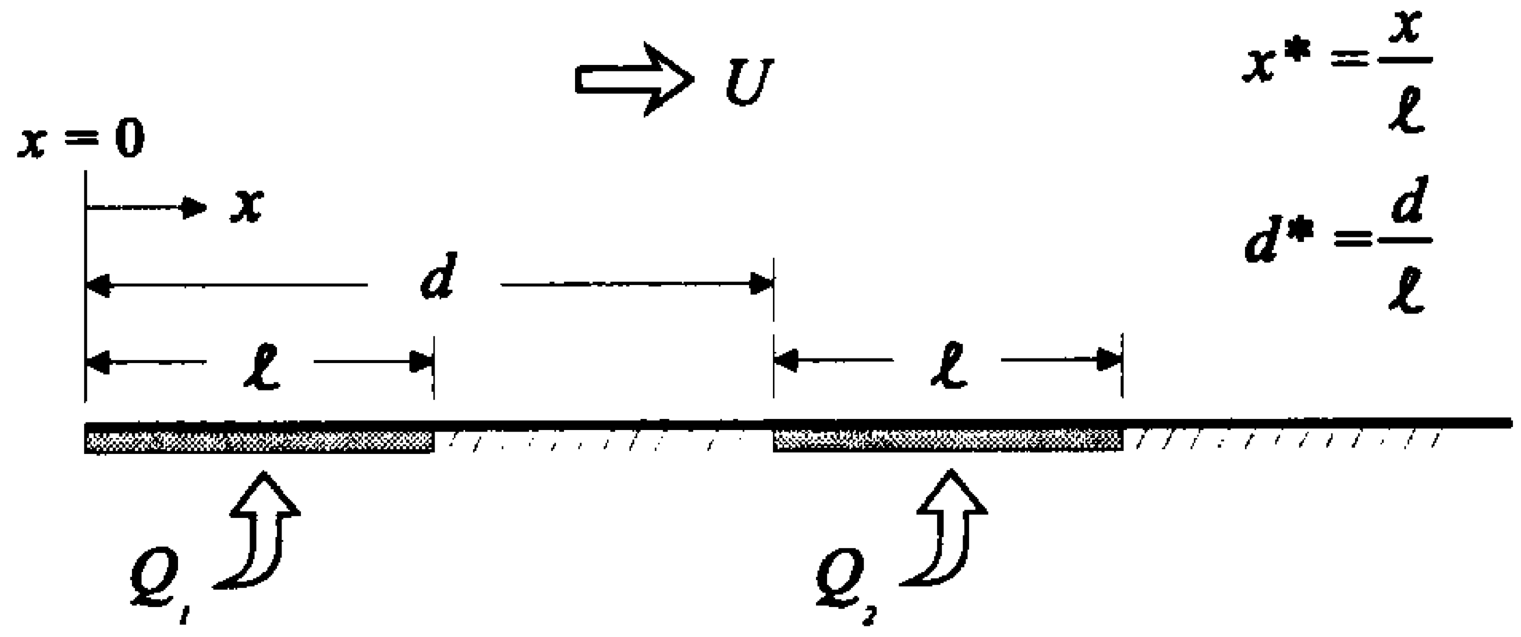

Fig. 3. Configuration of test case.

Table 1. Relative source strength and spacing for wake model simulations.

Test	Q_1/Q_2	d^*
1	1	0
2	1	1
3	1	2
4	1	3
5	1	4
6	0.5	0
7	0.5	1
8	0.5	2
9	0.5	3
10	0.5	4
11	2	0
12	2	1
13	2	2
14	2	3
15	2	4

Table 2. Isoflux heat source values.

Q_1/Q_2	q_1 (W/m^2)	q_2 (W/m^2)
0.5	500	1000
1.0	1000	1000
2.0	1000	500

Because the temperature results of the FLOTHERM simulations are for the layer of control volumes in the fluid adjacent to the wall, an extrapolation is necessary in order to directly compare the data with the board temperature predictions of the analytical model. The Fourier conduction equation has been modified to provide a correction factor:

$$T_s = T_f + \frac{q\Delta}{k_f} \tag{8}$$

where q is the uniform heat flux boundary condition, k_f is the thermal conductivity of the fluid, and Δ is the distance between the center of the control volume and the board, as shown in Fig. 4. Values for the correction factor used for the present cases, as a function of heat flux, are:

$$q = 1000\ \mathrm{W/m^2}, \quad \frac{q\Delta}{k_f} = 1.55$$

$$q = 500\ \mathrm{W/m^2}, \quad \frac{q\Delta}{k_f} = 0.77$$

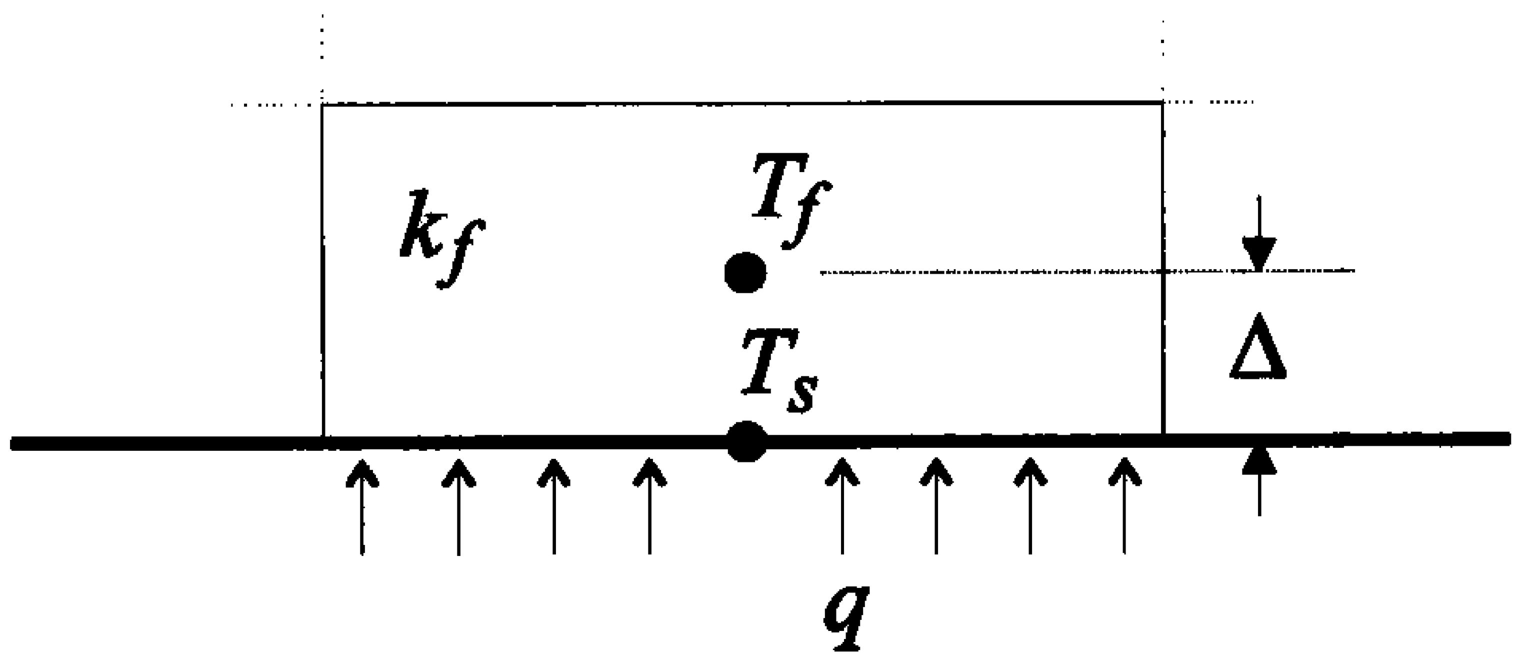

Fig. 4. Schematic for extrapolation of numerical results.

The corrected FLOTHERM results for local board surface temperature for each of the cases shown in Table 1 are presented in Figs. 5–7. In these plots, the dependent value is the dimensionless flow length x^*, defined as:

$$x^* = \frac{x}{\ell} \tag{9}$$

The local board temperature $T(x)$ has been non-dimensionalized using the maximum temperature of the upstream package, $T_{\max,1}$ as follows:

$$T^* = \frac{T(x) - T_\infty}{T_{\max,1} - T_\infty} \tag{10}$$

In each of the plots shown in Figs. 5–7, three different curves are presented: the dashed line represents the local temperature profile on the board when only the upstream package is heated; the dash-dot line is the temperature profile when only the downstream package is heated; and the solid line is the temperature profile resulting from both packages being active. The magnitude of the thermal wake effect on the downstream package can be evaluated by examining the difference between the solid and dash-dot lines starting at the leading edge of the downstream package.

Figure 5 presents a series of five plots for $d^* = 0$, 1, 2, 3 and 4 when $Q_1 = Q_2$. Figures 6 and 7 present the same set of plots for $Q_1/Q_2 = 1/2$ and 2, respectively. From these plots it can be clearly demonstrated that the thermal wake effect, characterized by the magnitude of the difference between the solid and dash-dot lines, decreases with an increase in the relative package spacing d^* or a decrease in the relative source strength Q_1/Q_2.

The analytical model presented in this study is validated using the results of the FLOTHERM simulations for the test cases shown in Table 1.

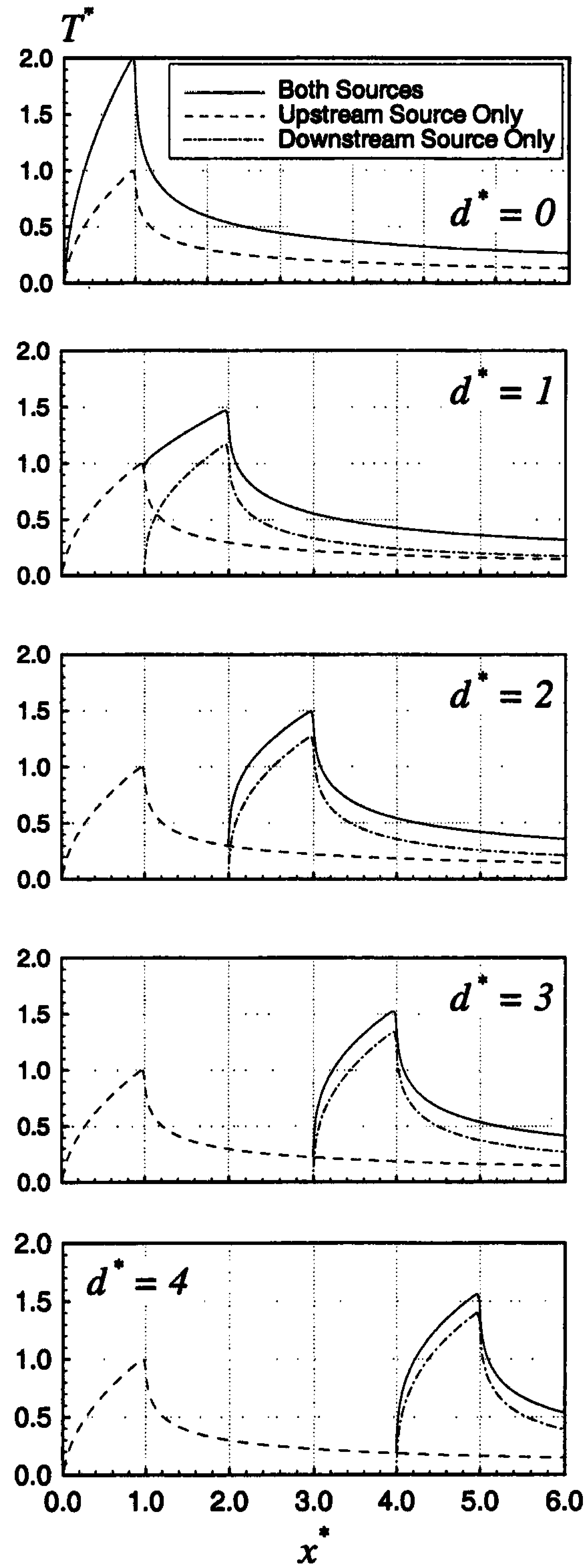

Fig. 5. Local board temperature profile for $Q_1 = Q_2$.

Figure 8 compares the FLOTHERM results with the predictions of the analytical model for each of the relative source strength values $Q_1/Q_2 = 0.5$, 1 and 2. The independent variable used in Fig. 8 is the inverse of the relative package spacing, while the dependent

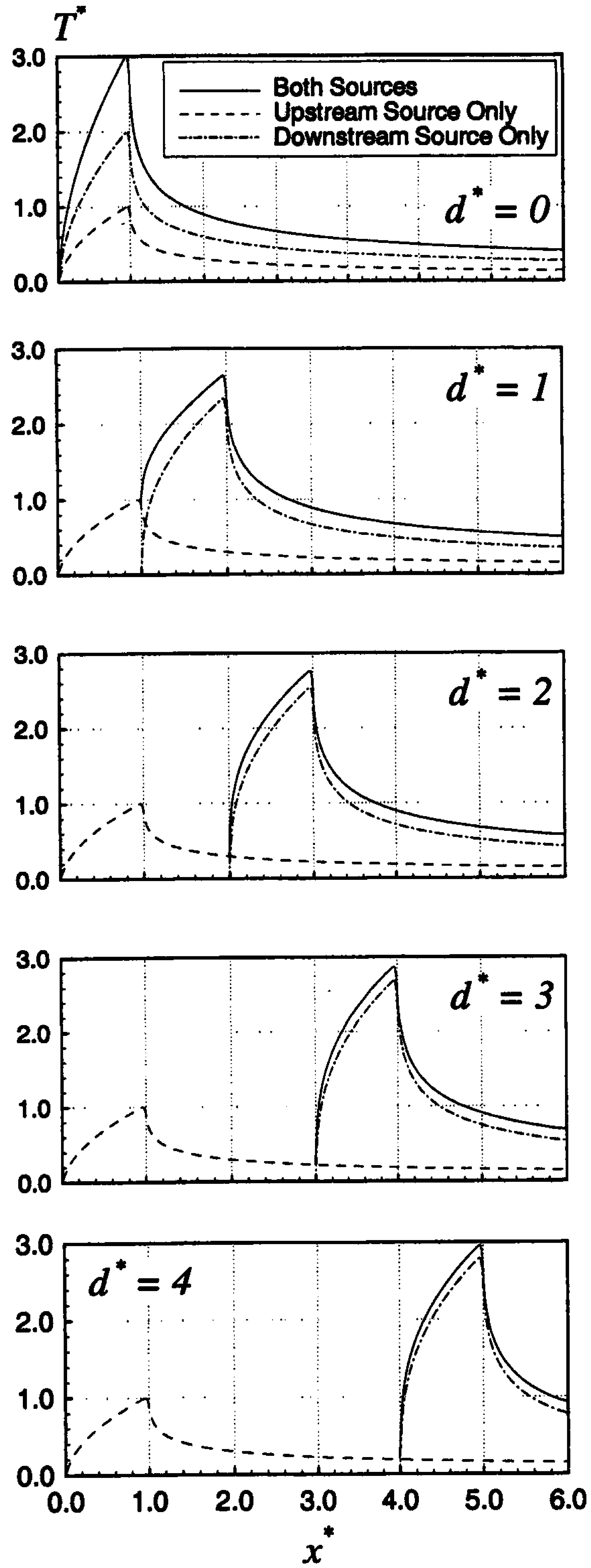

Fig. 6. Local board temperature profile for $Q_1 = Q_2/2$.

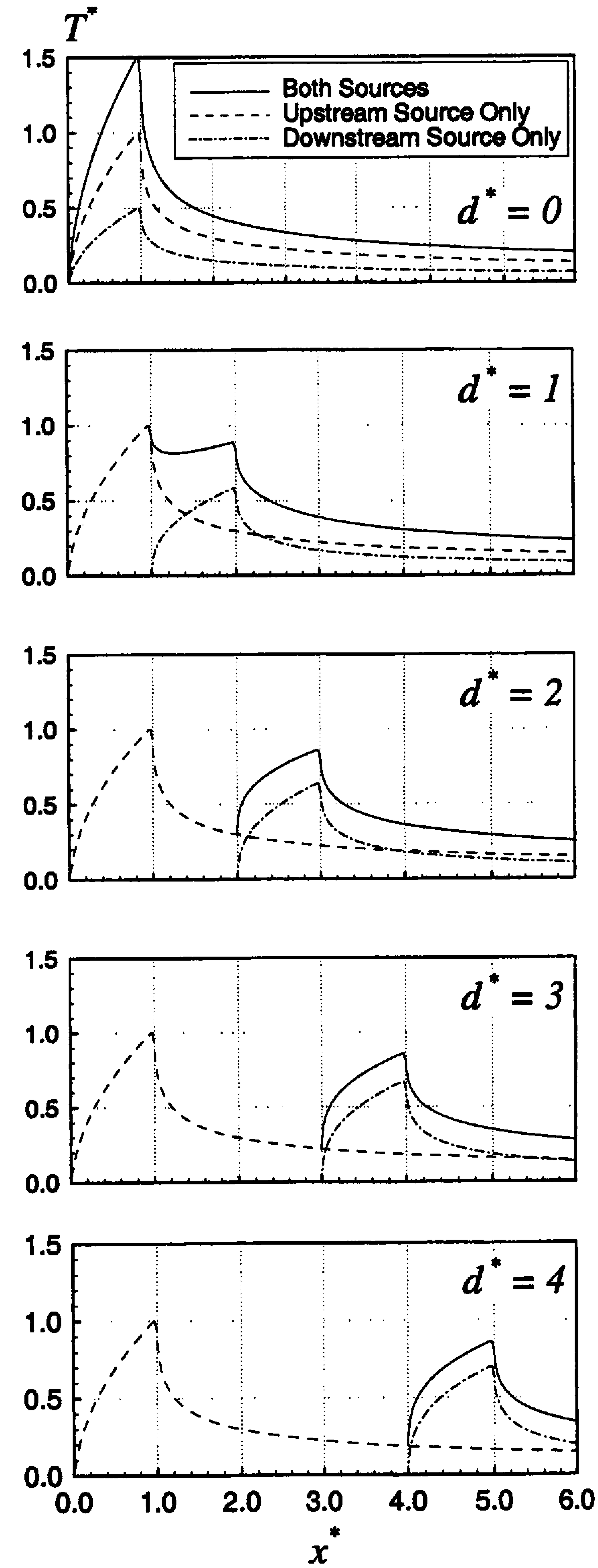

Fig. 7. Local board temperature profile for $Q_1 = 2Q_2$.

variable is the relative wake effect parameter θ^*, defined by the following dimensionless temperature ratio:

$$\theta^* = \frac{T_{\mathrm{mid},2}(\mathrm{wake} + \mathrm{local}) - T_\infty}{T_{\mathrm{mid},2}(\mathrm{local}) - T_\infty} \tag{11}$$

where $T_{\mathrm{mid},2}$(local) refers to the midpoint temperature of the downstream package when the upstream package is not heated i.e., $Q_1 = 0$, and $T_{\mathrm{mid},2}$(wake+local) is the downstream package midpoint temperature when both heat sources are active. By this

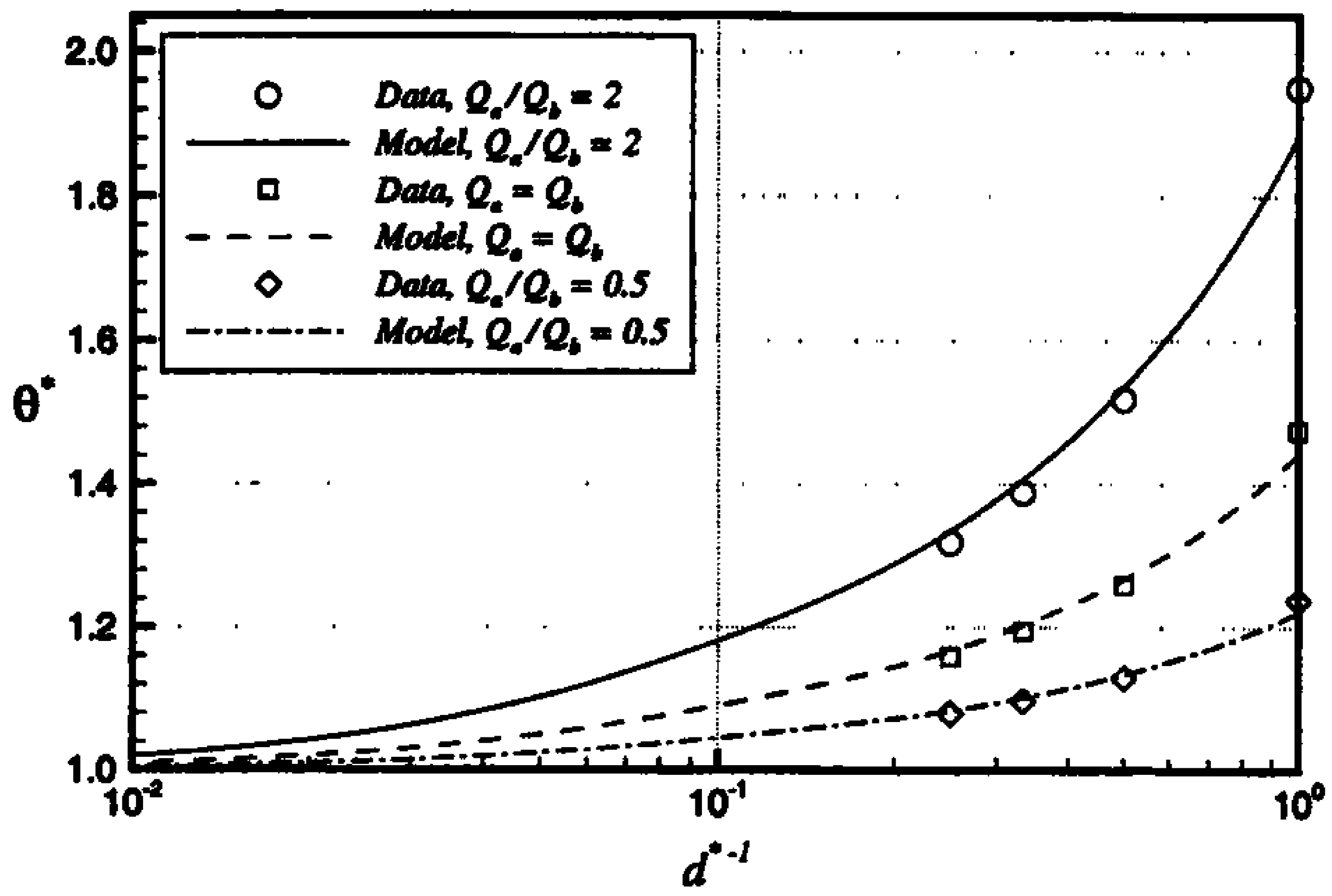

Fig. 8. Comparison of thermal wake model with numerical results.

definition it can be shown that as the effect of the wake increases, θ^* increases, and at the limiting case when the thermal wake from the upstream package has no effect, $\theta^* \to 1$.

Figure 8 clearly demonstrates the excellent agreement between the model and the numerical results, with differences of within $\pm 2\%$ for most cases. The poorest agreement occurs at $d^* = 1$ when that heat sources are next to each other, where diffusion effects within the FLOTHERM simulations may have contributed to the difference. The results shown in Fig. 8 also demonstrate the behaviour of the solution noted earlier; that the thermal wake effect can be reduced either by increasing the distance between packages or decreasing the relative source strength of the upstream package.

4. Conclusions and Recommendations

An analytical model is presented that predicts the one-dimensional thermal wake effects for a convection cooled flat plate with discrete, isoflux heat sources. This model can be easily adapted to the two-dimensional planar flow field of a printed circuit board by using the one-dimensional model for selected strips (Δy), perpendicular to the flow direction. The number of strips and the placement of the strips is arbitrary but should be based on the location of the discrete heat sources. The prudent placement of elements in the flow direction, Δx, and strips across the flow direction, Δy, will ensure the accurate calculation of $T(x, y)$, $q(x, y)$ and $h(x, y)$ over the two-dimensional planar surface of the plate. This model is proposed as a design tool for conjugate heat transfer modelling for air-cooled printed circuit boards for microelectronics applications. Validation of the model using numerical results generated by a commercial CFD code has shown the model to be accurate to within $\pm 2\%$ for most cases.

Acknowledgments

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Nomenclature

$d =$ source spacing; m
$d^* =$ relative source spacing; $\equiv d/\ell$
$h =$ heat transfer coefficient; $\mathrm{W/(m^2 \cdot K)}$
$k =$ thermal conductivity; $\mathrm{W/(m \cdot K)}$
$L =$ board length; m
$\ell =$ package length; m
$N =$ number of heat sources
$q =$ heat flux; $\mathrm{W/m^2}$
$Q =$ total heat flow rate; W
$\mathrm{Pr} =$ Prandtl number
$\mathrm{Re}_x =$ local Reynolds number
$T =$ temperature; °C
$\Delta T_f =$ temperature excess; $\equiv T_f - T_\infty$; °C
$T^* =$ dimensionless temperature; $\equiv \dfrac{T(x) - T_\infty}{T_{\max,1} - T_\infty}$
$U =$ free stream velocity; m/s
$x =$ flow length variable; $0 \le x \le L$
$x^* =$ dimensionless flow length; $\equiv x/\ell$

Greek Symbols

$\Delta =$ control volume half thickness; m
$\theta^* =$ relative wake effect;

$$\equiv \frac{T_{\mathrm{mid},2}(\mathrm{wake} + \mathrm{local}) - T_\infty}{T_{\mathrm{mid},2}(\mathrm{local}) - T_\infty}$$

$\xi =$ dummy variable; $0 \le \xi \le x$

Subscripts

$f =$ fluid
max = maximum source temperature
mid = source mid-point temperature
$s =$ solid
1 = upstream source
2 = downstream source
$\infty =$ free stream condition

Superscript

$* =$ dimensionless values

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