Thermal Spreading Resistance in Multilayered Contacts: Applications in Thermal Contact Resistance

Y. S. Muzychka,* M. R. Sridhar,* and M. M. Yovanovich†
University of Waterloo, Waterloo, Ontario N2L 3G1, Canada
and
V. W. Antonetti‡
Manhattan College, Riverdale, New York 10471

Application of highly conductive coatings to contacting surfaces is a commonly employed method to enhance thermal contact conductance. In many applications it is often necessary to apply an intermediate coating such that the conductive coating may be applied to a nonadhering substrate. In these instances, it is desirable to predict the effect that the intermediate and final coatings have on the spreading resistance. A solution for computing the thermal spreading resistance of a planar circular contact on a doubly coated substrate is presented. Also, a model is developed to compute the contact conductance between a bare substrate and a coated substrate. Comparisons are made with data obtained in the literature for which no analytical model was available. Solution of the governing equations and numerical computation of the spreading resistance were obtained using computer algebra systems.

Nomenclature

A_c, A_t, A_a	= area, m^2
A_{i_n}, B_{i_n}	= Fourier–Bessel coefficients
a, b	= two radii with $a < b$, m
C_L	= spreading correction factor
e	= natural log base
H_c	= contact microhardness, MPa
h_c	= contact conductance, W/m^{2}K
$J_0(x)$	= Bessel function of the first kind of order zero
$J_1(x)$	= Bessel function of the first kind of order one
K_{21}	= k_2/k_1 thermal conductivity ratio
K_{32}	= k_3/k_2 thermal conductivity ratio
k_0, k_1, k_2, k_3	= thermal conductivities, W/mK
$\mathcal{L}$	= characteristic length
N	= number of contacts
P	= contact pressure, MPa
Q	= total heat flow rate, W
R_s	= spreading resistance, K/W
r, z	= cylindrical coordinates, m
T	= temperature, K
$\bar{T}$	= area mean temperature, K
$t_1, t_2, \ldots, t_n$	= coating thicknesses, m
$z_1, z_2, \ldots, z_n$	= interface locations, m
β	= equation parameter
γ_n	= equation parameter
δ_n	= nth eigenvalue
ϵ	= contact spot aspect ratio a/b, or
	$\quad \sqrt{(A_c/A_t)} = \sqrt{(P/H_c)}$
λ	= separation constant
ρ_n	= boundary condition modification factor
σ/m	= rms roughness/mean asperity slope
τ_i	= relative coating thickness, t_i/a

ϕ	= equation parameter
ψ	= spreading parameter

Subscripts

a	= apparent contact area
c	= contact spot
i	= ith layer
n	= nth term in a series
t	= flux tube
0	= bare surface

Introduction

THERMAL spreading resistance has applications in predicting the contact conductance across semiconductor junctions and in thermal contact resistance models. Solution for the thermal spreading resistance of a planar heat source in perfect contact with a semi-infinite region has been examined by numerous researchers. Yovanovich and Antonetti[1] and Yovanovich[2] present a comprehensive review of the theory and application of spreading resistance for bare and singly coated surfaces.

Of particular interest is the solution for the spreading resistance of an array of contacts. As the spacing between contacts approaches the characteristic dimension of the contact, it becomes necessary to model the contact as a heat source in perfect contact with an insulated semi-infinite cylinder or flux tube. The theory of flux tubes is presented by Yovanovich[2] for bare surfaces and by Negus et al.[3] for bare surfaces having arbitrarily shaped contacts. Finally, Antonetti[4] presents the complete solution for a circular flux tube with a single and double coating.

This paper presents the general theory of multilayered flux tubes and discusses a particular case of a flux tube having two applied coatings. An application of the results in thermal contact resistance models is also presented through the development of a new model. Comparisons are then made with experimental data presented by Marotta et al.[5]

Problem Statement and Solution

The contact between two conforming rough surfaces in a vacuum may be modeled as an array of circular contact spots. The total heat transfer is then determined by combining all of the elemental flux tubes in parallel. The governing equation for each elemental flux tube is Laplace's equation in circular cylinder coordinates. If the flux tube is composed of N layers in the axial direction, as
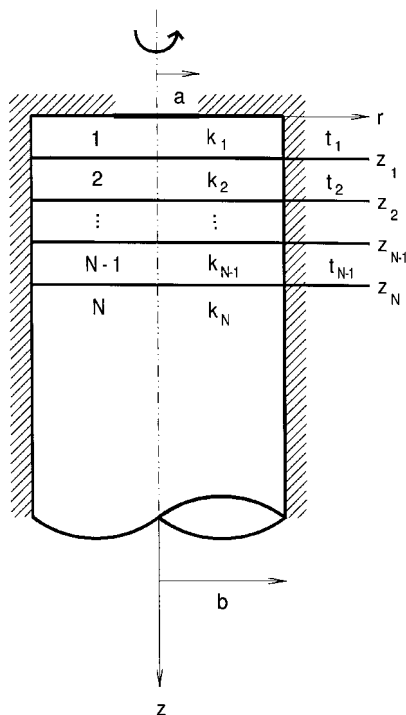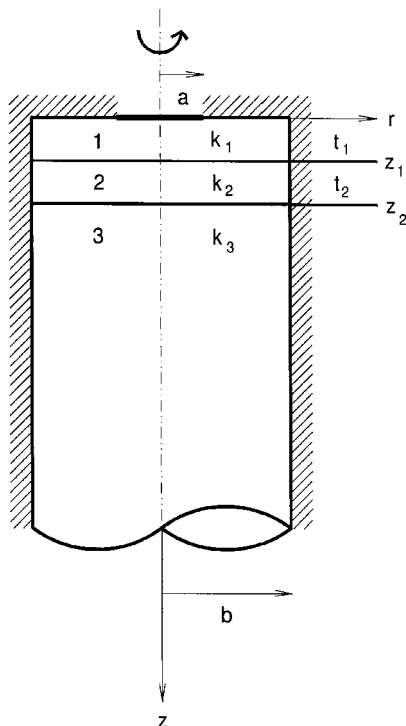
Fig. 1   Multilayered flux tube.

Fig. 2   Flux tube with two applied coatings.

shown in Fig. 1, then Laplace's equation must be written for each layer, resulting in a system of N equations and $2(N + 1)$ boundary conditions.

A system consisting of a cylindrical substrate and two base coatings is presented in Fig. 2. The governing equation in each coating and substrate is

$$\frac{\partial^2 T_i}{\partial r^2} + \frac{1}{r}\frac{\partial T_i}{\partial r} + \frac{\partial^2 T_i}{\partial z^2} = 0 \qquad (1)$$

for $i = 1, 2, 3$. The appropriate boundary conditions are summarized as follows.

Radial thermal boundary conditions, $z \geq 0$, $i = 1, 2, 3$:

$$\left.\frac{\partial T_i}{\partial r}\right|_{r=0} = 0, \qquad r = 0 \qquad (2)$$

$$\left.\frac{\partial T_i}{\partial r}\right|_{r=b} = 0, \qquad r = b \qquad (3)$$

Axial thermal boundary conditions, $0 \leq r \leq b$:

$$T_3(r, z \to \infty) = 0, \qquad z \to \infty \qquad (4)$$

$$T_2(r, z_2) = T_3(r, z_2), \qquad \left. k_2\frac{\partial T_2}{\partial z}\right|_{z=z_2} = \left. k_3\frac{\partial T_3}{\partial z}\right|_{z=z_2}$$
$$z = z_2 = t_1 + t_2 \quad (5)$$

$$T_1(r, z_1) = T_2(r, z_1), \qquad \left. k_1\frac{\partial T_1}{\partial z}\right|_{z=z_1} = \left. k_2\frac{\partial T_2}{\partial z}\right|_{z=z_1}$$
$$z = z_1 = t_1 \quad (6)$$

$$\left.\frac{\partial T_1}{\partial z}\right|_{z=0} = -\frac{Q}{\pi a^2 k_1}, \qquad 0 \leq r < a, \qquad z = 0$$

$$\left.\frac{\partial T_1}{\partial z}\right|_{z=0} = 0, \qquad a < r \leq b, \qquad z = 0 \qquad (7)$$

Solution for Spreading Resistance

The resulting system of three equations and eight boundary conditions is easily solved by analytical methods. Solutions to heat conduction problems in composite systems using integral transforms and separation of variables are discussed by Ozisik.[6] The problem as stated in the preceding section may be solved by separation of variables. The solution for the temperature field consists of two components: a uniform one-dimensional flow portion and a two-dimensional flow portion. The total thermal resistance of the system will then have the form

$$R_t = R_{1D} + R_s \qquad (8)$$

where R_{1D} is the total one-dimensional bulk resistance of each layer in the system and R_s is the spreading resistance.

The solution for the spreading resistance component may be obtained by considering the two-dimensional eigenvalue problem. Applying the method of separation of variables results in

$$T_i(r, z) = J_0(\lambda r)\left[A_i e^{-\lambda z} + B_i e^{\lambda z}\right] \qquad (9)$$

where the Bessel function $Y_0(\lambda r)$ has been eliminated due to the singularity at $r = 0$.

Application of the boundary condition along $r = b$ yields the characteristic equation

$$J_1(\delta_n) = 0 \qquad (10)$$

where $\delta_n = \lambda_n b$. The eigenvalues for this equation are well tabulated by Abramowitz and Stegun,[7] or they may be approximated to single precision using Stokes' approximation (see Gray and Mathews[8]). A modified Stokes approximation developed by Yovanovich[9] that provides greater accuracy is

$$\delta_n = \frac{\beta_n}{4}\left[1 - \frac{6}{\beta_n^2} + \frac{6}{\beta_n^4} - \frac{4716}{5\beta_n^6} + \frac{3,902,418}{70\beta_n^8}\right] \qquad (11)$$

where $\beta_n = \pi(4n + 1)$ and $n \geq 1$.

In this paper, exact values are used for the first five eigenvalues, and the remainder are computed using Eq. (11). The solution for each coating becomes

$$T_1(r, z) = \sum_{n=1}^{\infty} J_0(\lambda_n r)\left[A_{1_n} e^{-\lambda_n z} + B_{1_n} e^{\lambda_n z}\right] \qquad (12)$$

$$T_2(r, z) = \sum_{n=1}^{\infty} J_0(\lambda_n r)\left[A_{2_n} e^{-\lambda_n z} + B_{2_n} e^{\lambda_n z}\right] \quad (13)$$

Eliminating the singularity at $z \to \infty$ requires that $B_{3_n} = 0$, and the solution for the temperature in the substrate becomes

$$T_3(r, z) = \sum_{n=1}^{\infty} A_{3_n} J_0(\lambda_n r) e^{-\lambda_n z} \quad (14)$$

The five constants may be determined by applying the remaining boundary conditions at each interface and at the contact plane. Application of Eqs. (4–6) results in a system of four equations that may be solved for the constants A_{1_n}, B_{1_n}, A_{2_n}, and B_{2_n}. The solution to this system of equations was easily obtained using the computer algebra systems Maple[10] and Mathematica.[11] The four constants in terms of the unknown constant A_{3_n} are

$$A_{1_n} = \left(A_{3_n}/4\right)\left[(1 + K_{21})(1 + K_{32}) + (1 - K_{21})(1 - K_{32})e^{-2\delta_n \epsilon \tau_2}\right] \quad (15)$$

$$B_{1_n} = \left(A_{3_n}/4\right)\left[(1 - K_{21})(1 + K_{32})e^{-2\delta_n \epsilon \tau_1}\right.$$
$$\left. + (1 + K_{21})(1 - K_{32})e^{-2\delta_n \epsilon (\tau_1 + \tau_2)}\right] \quad (16)$$

$$A_{2_n} = \left(A_{3_n}/2\right)(1 + K_{32}) \quad (17)$$

$$B_{2_n} = \left(A_{3_n}/2\right)(1 - K_{32})e^{-2\delta_n \epsilon (\tau_1 + \tau_2)} \quad (18)$$

where $K_{21} = k_2/k_1$ and $K_{32} = k_3/k_2$ are the relative conductivities of adjacent layers, $\tau_1 = t_1/a$ and $\tau_2 = t_2/a$ are the relative thicknesses of each coating, $\delta_n = \lambda_n b$ are the eigenvalues of Eq. (10), and $\epsilon = a/b$ is the contact spot aspect ratio.

The final constant A_{3_n} is obtained by taking a Fourier–Bessel series expansion of the contact plane boundary condition Eq. (7). This results in the following relation for the isoflux condition:

$$A_{3_n} = \frac{8}{\pi} \frac{Q}{k_1 a} \frac{J_1(\delta_n \epsilon)}{\delta_n^2 J_0^2(\delta_n) \gamma_n} \quad (19)$$

where the dimensionless parameter γ_n that accounts for the effects of conductivity and thickness of each layer is defined as

$$\gamma_n = (1 + K_{21})(1 + K_{32}) - (1 - K_{21})(1 + K_{32})e^{-2\delta_n \epsilon \tau_1}$$
$$+ (1 - K_{21})(1 - K_{32})e^{-2\delta_n \epsilon \tau_2}$$
$$- (1 + K_{21})(1 - K_{32})e^{-2\delta_n \epsilon (\tau_1 + \tau_2)} \quad (20)$$

In contact conductance problems, the contact spot is assumed to be isothermal rather than isoflux. However, solution to the problem for an isothermal contact constitutes a mixed potential boundary value problem. To facilitate a solution, the equivalent isothermal boundary condition is assumed by imposing a heat flux distribution over the contact spot.

In the case of an isoflux contact, the temperature profile that results is parabolic, with the maximum temperature occurring at the centroid of the contact (Fig. 3). Alternatively, if one prescribes a parabolic heat flux distribution with the minimum at the centroid of the contact spot, a uniform temperature distribution will result. Yovanovich[12] and Negus et al.[13] discuss the solution for spreading resistance in semi-infinite domains for uncoated and coated surfaces for arbitrary heat flux distributions. Using the results of Yovanovich,[12] it can be shown that the isothermal contact may be modeled using the isoflux case by simply multiplying each term in the series by the factor

$$\rho_n = \begin{cases} \dfrac{\sin(\delta_n \epsilon)}{2 J_1(\delta_n \epsilon)}, & \text{isothermal} \\ 1, & \text{isoflux} \end{cases} \quad (21)$$

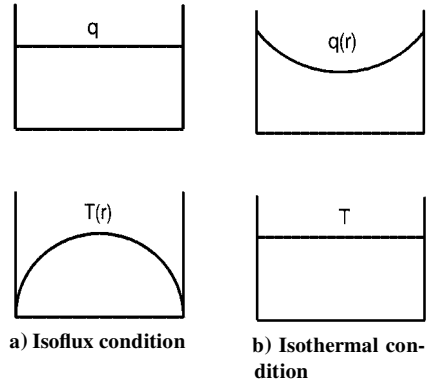**a) Isoflux condition**   **b) Isothermal condition**

Fig. 3   Isoflux and equivalent isothermal boundary conditions.

The results presented in a later section of this paper are based on the isothermal contact condition, rather than the isoflux contact condition. The difference between these two cases is approximately 8% as $\epsilon \to 0$ and $\tau_1 \to \infty$. In this limit, the solution approaches that of an isolated contact on a semi-infinite region.

Thermal Spreading Resistance Parameter

To use the results of the preceding section in contact resistance models, we must define the spreading resistance and the dimensionless spreading resistance parameter. The spreading resistance is defined as

$$R_s = (\bar{T}_{\text{contact}} - \bar{T}_{\text{cp}}/Q) \quad (22)$$

where

$$\bar{T}_{\text{contact}} = \frac{1}{\pi a^2} \int_0^a T_1(r, 0) 2\pi r \, \mathrm{d}r \quad (23)$$

is the mean temperature of the contact spot and

$$\bar{T}_{\text{cp}} = \frac{1}{\pi b^2} \int_0^b T_1(r, 0) 2\pi r \, \mathrm{d}r \quad (24)$$

is the mean temperature of the contact plane.

The spreading parameter ψ is defined with respect to the substrate thermal conductivity k_3,

$$\psi = 4k_3 \mathcal{L} R_s \quad (25)$$

where $\mathcal{L}$ is some characteristic length of the contact spot geometry. For the case of a circular contact $\mathcal{L} = a$, the radius of the contact. Extension of this solution for noncircular contacts is discussed later, and an alternative length $\mathcal{L}$ is proposed.

To examine the effect that each coating has on the spreading resistance, the solution for the spreading resistance parameter in the singly and doubly coated contacts will be presented in terms of the bare surface spreading resistance parameter. The spreading resistance parameters for the singly coated and bare surfaces may then be obtained as special cases from the spreading resistance parameter for the doubly coated surface.

Bare Surface

By setting $K_{21} = 1$ and $K_{32} = 1$, the bare contact spreading resistance parameter[12] becomes

$$\psi_{\text{bare}} = \frac{16}{\pi \epsilon} \sum_{n=1}^{\infty} \rho_n \frac{J_1^2(\delta_n \epsilon)}{\delta_n^3 J_0^2(\delta_n)} \quad (26)$$

Simple correlations that may be used in place of Eq. (26) have been developed by Negus and Yovanovich[14] that cover the range $0 \le \epsilon \le 0.9$ with a maximum error of 0.02%. These are

$$\psi_{\text{bare}}^T(\epsilon) = 1 - 1.40978\epsilon + 0.34406\epsilon^3 + 0.04305\epsilon^5 + 0.02271\epsilon^7 \tag{27}$$

$$\psi_{\text{bare}}^q(\epsilon) = 1.08076 - 1.41042\epsilon + 0.26604\epsilon^3$$
$$- 0.00016\epsilon^5 + 0.058266\epsilon^7 \tag{28}$$

where the superscripts T and q are the isothermal and isoflux boundary conditions, respectively.

Single Coated Surface

By setting $K_{32} = 1$, the single layer contact spreading resistance parameter becomes

$$\psi_{\text{single}} = \frac{16}{\pi\epsilon} \sum_{n=1}^{\infty} \rho_n \frac{J_1^2(\delta_n\epsilon)}{\delta_n^3 J_0^2(\delta_n)}$$
$$\times \left[K_{21} \frac{(1 + K_{21}) + (1 - K_{21})e^{-2\delta_n\epsilon\tau_1}}{(1 + K_{21}) - (1 - K_{21})e^{-2\delta_n\epsilon\tau_1}} \right] \tag{29}$$

Double Coated Surface

The two layer contact spreading resistance parameter is

$$\psi_{\text{double}} = \frac{16}{\pi\epsilon} \sum_{n=1}^{\infty} \rho_n \frac{J_1^2(\delta_n\epsilon)}{\delta_n^3 J_0^2(\delta_n)} \frac{\phi^+}{\phi^-} K_{21} K_{32} \tag{30}$$

where

$$\phi^\pm = (1 + K_{21})(1 + K_{32}) \pm (1 - K_{21})(1 + K_{32})e^{-2\delta_n\epsilon\tau_1}$$
$$+ (1 - K_{21})(1 - K_{32})e^{-2\delta_n\epsilon\tau_2}$$
$$\pm (1 + K_{21})(1 - K_{32})e^{-2\delta_n\epsilon(\tau_1 + \tau_2)} \tag{31}$$

The effect that each coating has on the spreading resistance parameter is easily seen in Eqs. (26), (29), and (30). Antonetti[4] computed results for the bare surface spreading resistance parameter and also tabulated values of the spreading resistance correction parameter for various $(\epsilon, \tau_1, \tau_2, K_{21}, K_{32})$ for both the isoflux and equivalent isothermal boundary conditions. The correction parameter for the spreading resistance in a layered system is defined as

$$C_L = \frac{\psi_{\text{single, double}}}{\psi_{\text{bare}}} \tag{32}$$

Results for the single layer spreading correction parameter have been reported in graphical form by Antonetti[4] and by Antonetti and Yovanovich.[15] Tabulation of the double layer spreading parameter would be too involved due to the large number of parameters involved, that is, $(\epsilon, \tau_1, \tau_2, K_{21}, K_{32})$. In a later section, a parametric analysis is conducted for comparison of experimental data with a new contact conductance model.

Isolated Contact $\epsilon \rightarrow 0$

As $\epsilon \rightarrow 0$, the contact becomes isolated, and the solution for a single contact on a half-space is obtained. Computing this special case requires several thousand terms; thus, the half-space solution should be used instead if computing resources are limited. However, with most computer algebra systems such as Maple[10] and Mathematica,[11] the computation time is quite reasonable, and there is no need to resort to the half-space solutions. The interested reader should refer to Negus et al.[13] for the procedure to obtain the half-space contact solutions.

Effect of Contact Spot Geometry

In many applications of spreading resistance, the contact spot may not be circular. Other shapes include square, triangular, or elliptic contacts. The model presented earlier is easily modified to account for a contact spot of arbitrary shape. The effect of contact spot geometry on spreading resistance was studied by Yovanovich

et al.[16] It was shown that the bare surface spreading parameter for an isolated contact on a semi-infinite region is a weak function of geometry when the spreading resistance is nondimensionalized using $\mathcal{L} = \sqrt{A_c}$ as a characteristic length, where A_c is the area of the contact spot. Negus et al.[3] also showed that the spreading parameters for semi-infinite flux tubes having various shapes are also weak functions of geometry if nondimensionalized using the square root of the contact area.

It can be shown that the spreading parameter for the singly and doubly coated contacts are also weak functions of the contact spot geometry. Thus, the solution given earlier may also be used for contact spots of arbitrary shape if the spreading parameter is defined as

$$\psi = 4k_3\sqrt{A_c}R_s \tag{33}$$

and the relative contact spot size is defined as

$$\epsilon = \sqrt{A_c/A_t} \tag{34}$$

where A_t is the cross-sectional area of the flux tube. The equivalent contact spot radius a is chosen such that $a = \sqrt{(A_c/\pi)}$.

## Application in Thermal Contact Conductance Models

An important application of thermal spreading resistance arises in the prediction of the thermal contact resistance between two contacting, nominally flat, rough surfaces. In many applications, the contact conductance is enhanced if one of the surfaces is coated with a high conductivity material such as a metallic coating.[4] In certain instances it is necessary to apply an intermediate coating to promote the adherence of the metallic coating. It is, therefore, desirable to assess the overall effect that each coating has on the enhancement (or reduction) of the thermal contact conductance. The authors have derived a general expression for determining the contact conductance of a doubly coated substrate in contact with a bare surface. Comparisons are then made with experimental data for diamondlike coatings (DLCs), which are presented by Marotta et al.[5]

Contact Conductance of Coated Interfaces

Contact resistance in a vacuum environment for the configuration shown in Fig. 4 is given by Antonnetti[4]:

$$R_c = \frac{1}{h_c A_a} = \frac{1}{N}\left[\frac{\psi_{\text{bare}}}{4k_0 a} + \frac{\psi_{\text{coated}}}{4k_3 a} \right] \tag{35}$$

where a is the mean contact spot radius, k_0 and k_3 are the conductivities of the upper bare surface and the substrate of the lower surface (Fig. 4), h_c is the contact conductance, A_a is the apparent contact area, N is the total number of contact spots, and ψ_{bare} and ψ_{coated} are
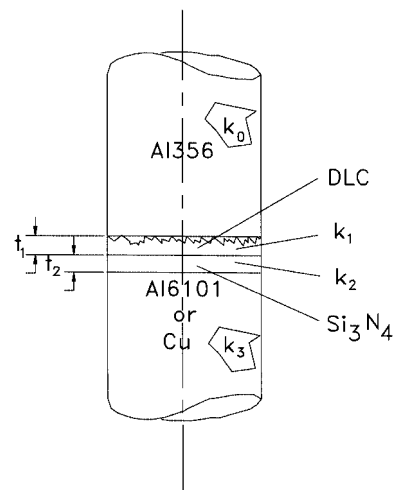
Fig. 4   Configurations considered for parametric analysis.

the thermal spreading parameters for the bare and coated surfaces, respectively.

Therefore, we have

$$h_c = \frac{2}{A_a}\left[\frac{2k_0 k_3}{(k_3 + k_0 \psi_{\text{coated}}/\psi_{\text{bare}})}\right]\frac{Na}{\psi_{\text{bare}}} \quad (36)$$

From Yovanovich[17] for a bare interface, the contact conductance is given by

$$h_{c,\text{bare}} = \frac{2}{A_a}\frac{2k_0 k_3}{k_0 + k_3}\frac{Na}{\psi_{\text{bare}}} \quad (37)$$

where k_0 and k_3 are the conductivities of the two bare surfaces in contact. Comparing the expression for contact conductance for the coated configuration in Fig. 4 and the bare interface given by Yovanovich,[17] we can rewrite the expression for contact conductance for the coated interface as follows:

$$h_c = h_{c,\text{bare}}\frac{k_0 + k_3}{k_3 + C_L k_0} = h_{c,\text{bare}}\text{TEF} \quad (38)$$

where the thermal enhancement factor (TEF) $= (k_0 + k_3)/(k_3 + C_L k_0)$ and $C_L = \psi_{\text{coated}}/\psi_{\text{bare}}$ is the spreading resistance correction factor, defined earlier, for the equivalent isothermal boundary condition.

From Yovanovich,[17] $h_{c,\text{bare}}$ is correlated as

$$h_{c,\text{bare}} = 1.25(m/\sigma)k_s(P/H_c)^{0.95} \quad (39)$$

Therefore,

$$h_c = 1.25(m/\sigma)k_s(P/H_c)^{0.95}\text{TEF} \quad (40)$$

where $k_s = 2k_0 k_3/(k_0 + k_3)$ is the harmonic mean thermal conductivity of the bare interface. If the TEF > 1 there is an enhancement in the contact conductance over the bare interface due to the presence of the coatings.

Comparison with Experimental Data

In this section we examine the effect of using DLC to enhance contact conductance. In practice DLCs cannot be directly applied on a substrate. Once the substrate surfaces are prepared, each test surface must be coated with a layer of silicon nitride (see Marotta et al.[5]). This coating is necessary to ensure the stability of the surface for the deposition of a DLC.

The experimental study by Marotta et al.[5] included two types of interfaces, each with three to four different coating thicknesses. The first type consisted of two aluminum substrates with upper specimen bare and lower specimen coated with silicon nitride and the DLC on top of it. The silicon nitride coating thickness was fixed at 3 μm, whereas the thickness of the DLC was varied from 0 μm (i.e., no coating) to 5 μm. The second type consisted of the same bare aluminum alloy on top, but the substrate of the coated specimen was changed to copper.

In the present work a comparison is made of the experimental results from Marotta et al.[5] with the present model using the bulk values of thermal conductivities. Then an estimate of the actual thermal conductivities of the coatings will be made with the aid of the present model.

To compute the contact conductance h_c as a function of the dimensionless contact pressure P/H_c, the correction factor C_L for each value of applied pressure for the given surface, material and thermal properties must be computed. To calculate the correction factor C_L at particular values of the dimensionless contact pressure (P/H_c) one requires the thermal conductivities k_0, k_1, k_2 and k_3, the thicknesses of the two coatings t_1 and t_2, and the mean contact spot radius a. The mean contact spot radius is determined using the approximation developed by Sridhar[18]:

$$a = 0.645(\sigma/m)(P/H_c)^{0.071} \quad (41)$$

The upper aluminum specimen (Al356) is the softer one, and it is assumed to undergo full plastic deformation. It is known from past experience[19] that aluminum alloys do not generally possess a hard

Table 1 Thermal conductivity of bulk materials and thin films[21]

Material	k_{film}, W/mK	k_{bulk}, W/mK
SiO$_2$	0.4–1.1	1.2–10.7
TiO$_2$	0.5–0.6	7.4–10.4
ZrO$_2$	0.04	——
Al$_2$O$_3$	0.72	20–46
MgF$_2$	0.58	15–30
Air	——	0.025
Oxides/fluorides	——	1.0–10
Diamond I, II	——	1200–2300
Silicon	——	150
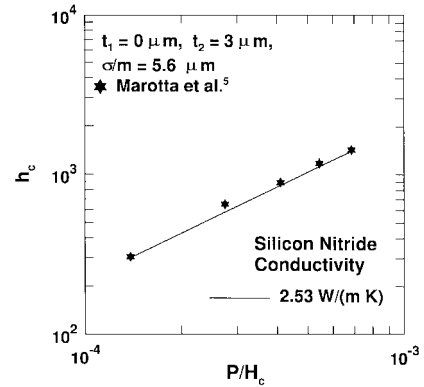
Fig. 5 Estimate of the conductivity of the silicon nitride layer for a typical interface.

surface layer, and the microhardness of the alloy is almost equal to the bulk hardness. Based on this assumption, the experimental data from Marotta et al.[5] were reduced using a single hardness value $H_c = 1256$ MPa. The correction factor C_L, TEF, and thus contact conductance h_c were computed by means of the computer algebra system Mathematica,[11] using about 2000 terms for each computation for the configurations tested by Marotta et al.[5]

Initial calculations revealed that the model overpredicted the data using the bulk values of thermal conductivities for thin films. It has been shown by Lambropoulos et al.[20,21] through measurements of the thermal conductivity of thin films that the value may be as much as two orders of magnitude lower than that of the corresponding bulk solid. A comparison of bulk and film conductivity is presented in Table 1. The measurements were made in air for a wide variety of thin films of oxides, flourides, nitrides, amorphous metals, and semiconductors.

With the aid of the experimental data of Marotta et al.,[5] an estimate of the thermal conductivity of the coatings is made by decreasing the thermal conductivity of the silicon nitride and the DLC in the model until the model and the data coincide. The first step was to estimate the conductivity of the silicon nitride coating. Figure 5 shows a comparison between the model (i.e., computed values) and data for the single layer of silicon nitride. The conductivity of the silicon nitride coating was decreased from its bulk value of 15 W/(mK) to an average value of 2.53 W/mK, where the computed values agree with the experimental data. The thermal conductivity of the silicon nitride layer varied between 2.43 and 2.71 W/mK. The mean value of 2.53 W/mK agrees well with the data of Volklein,[22] who obtained a value for a composite sandwich system of SiO$_2$–Si$_3$N$_4$ of 2.4 W/mK. The bulk values of SiO$_2$ and Si$_3$N$_4$ as reported by Volklein[22] are 12 and 17 W/mK, respectively.

Having estimated the conductivity of the silicon nitride coating, the conductivity of the DLC was then determined by decreasing the conductivity of the DLC until the model and data were in agreement. Figure 6 shows a comparison between the model and data for a system having a silicon nitride layer and a DLC layer. For one of the configurations examined by Marotta et al.[5] having a 1-μm DLC coating and a 3-μm silicon nitride layer, the conductivity of the DLC was found to vary between 3.37 and 3.98 W/(mK) with a
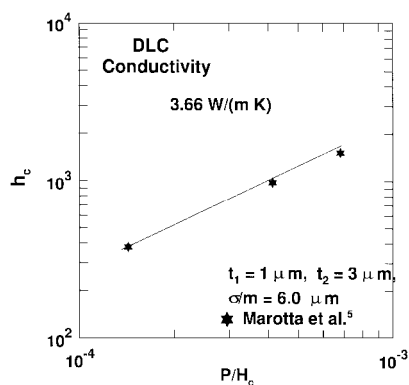
Fig. 6 Estimate of the conductivity of the DLC layer for a typical interface.

mean value of 3.66 W/mK. Morath et al.[23] reported values for thin films of amorphous diamond to vary between 5 and 10 W/mK and values for DLC to vary between 0.3 and 1 W/mK. The predicted value is in fair agreement with the published data for thin films of diamond and diamondlike carbon.

Comparison of the predicted values of thermal conductivity compare quite well with data of thin films reported by Lambropoulos et al.,[20,21] Volklein,[22] and Morath et al.,[23] where the authors reported measurements for the thermal conductivity of thin films of various materials. It should also be noted that the predictions of Lambropoulos et al.[20,21] were computed using the analysis of Dryden[24] for a point contact on a single coated half-space. The method outlined earlier for fitting the experimental data of Marotta et al.[5] to the analytic model is similar to the procedure used by Lambropoulos et al.[20,21] to determine the conductivity of thin films. Thus, an alternative method for determining the conductivity of thin films using contact conductance data has been developed.

Conclusion

The general theory for determining the spreading resistance for an isoflux or isothermal planar heat source in contact with a multilayered semi-infinite flux tube is presented. The solution is presented for several special cases that result for particular combinations of the physical parameters. In addition, extension of the solution for contacts of arbitrary shape was also discussed.

The solution to the governing equations and computation of numerical results were performed using the computer algebra systems of Maple[10] and Mathematica.[11] Both of these packages are capable of performing symbolic and numerical computations and provide an efficient means for computing the special functions that appear in the solutions.

Finally, a simple application of the theory of spreading resistance in multilayered contacts was discussed for the particular case of predicting the thermal contact resistance between two contacting planes. In this particular case, one of the substrates has been coated to enhance the thermal contact conductance between planes. It was found that the experimental data fell below the values computed using the model when the bulk values of the thermal properties were used. However, the correct trend in the data was predicted by the model. By using the model to match the experimental data, the thermal conductivity of each layer was predicted. The resulting values were much smaller than the reported bulk properties, but compared quite well with experimental results reported for thin films.

References

[1]Yovanovich, M. M., and Antonetti, V. W., "Application of Thermal Contact Resistance Theory to Electronic Packages," *Advances in Thermal Modeling of Electronic Components and Systems Vol. 1*, edited by A. Bar-Cohen and A. D. Kraus, Hemisphere, New York, 1984, pp. 79–128.
[2]Yovanovich, M. M., "Theory and Applications of Constriction and Spreading Resistance Concepts for Microelectronics Thermal Management," *Advances in Cooling Techniques for Computers*, edited by W. Aung, Hemisphere, New York, 1991, pp. 277–332.
[3]Negus, K. J., Yovanovich, M. M., and Beck, J. V., "On the Non-Dimensionalization of Constriction Resistance for Semi-Infinite Heat Flux Tubes," *Journal of Heat Transfer*, Vol. 111, No. 3, 1989, pp. 804-807.
[4]Antonetti, V. W., "On the Use of Metallic Coatings to Enhance Thermal Contact Conductance," Ph.D. Dissertation, Dept. of Mechanical Engineering, Univ. of Waterloo, Waterloo, ON, Canada, 1983.
[5]Marotta, E. E., Blanchard, D. G., and Fletcher, L. S., "The Thermal Contact Conductance of Diamond-Like Films," AIAA Paper 93-0845, Jan. 1993.
[6]Ozisik, N., *Boundary Value Problems of Heat Conduction*, International Textbook, Scranton, PA, 1968, Chap. 3.
[7]Abramowitz, M., and Stegun, I. A., *Handbook of Mathematical Functions*, Dover, New York, 1965, pp. 355–434.
[8]Gray, A., and Mathews, G. B., *A Treatise on Bessel Functions and Their Applications to Physics*, Macmillan, New York, 1952, pp. 79–90.
[9]Yovanovich, M. M., "ME 651 Advanced Heat Conduction," Course Notes, Univ. of Waterloo, Waterloo, ON, Canada, 1993.
[10]Maple, Ver. 3, Waterloo Maple Software, Waterloo, ON, Canada, 1984-1996.
[11]Mathematica, Ver. 2.2.3, Wolfram Research, Inc., Champaign, IL, 1987-1996.
[12]Yovanovich, M. M., "General Expression for Circular Constriction Resistances for Arbitrary Heat Flux Distribution," *Radiative Transfer and Thermal Control*, edited by A. M. Smith, Vol. 49, Progress in Astronautics and Aeronautics, AIAA, New York, 1976, pp. 301–308.
[13]Negus, K. J., Yovanovich, M. M., and Thompson, J. C., "Constriction Resistance of Circular Contacts on Coated Surfaces: Effect of Boundary Conditions," *Journal of Thermophysics and Heat Transfer*, Vol. 2, No. 2, 1988, pp. 158–164.
[14]Negus, K. J., and Yovanovich, M. M., "Application of the Method of Optimized Images to Steady Three-Dimensional Conduction Problems," American Society of Mechanical Engineers, Paper 84-WA/HY-110, Dec. 1984.
[15]Antonetti, V. W., and Yovanovich, M. M., "Enhancement of Thermal Contact Conductance by Metallic Coatings: Theory and Experiment," *Journal of Heat Transfer*, Vol. 107, No. 3, 1985, pp. 513–519.
[16]Yovanovich, M. M., Burde, S. S., and Thompson, J. C., "Thermal Constriction Resistance of Arbitrary Planar Contacts with Constant Flux," *Thermophysics of Spacecraft and Outer Planet Entry Probes*, edited by A. M. Smith, Vol. 56, Progress in Astronautics and Aeronautics, AIAA, New York, 1977, pp. 127–139.
[17]Yovanovich, M. M., "Thermal Contact Correlations," *Spacecraft Radiative Transfer and Temperature Control*, edited by T. E. Horton, Vol. 83, Progress in Astronautics and Aeronautics, AIAA, New York, 1982, pp. 82-95.
[18]Sridhar, M. R., "Elastoplastic Contact Models for Sphere-Flat and Conforming Rough Surface Applications," Ph.D. Dissertation, Dept. of Mechanical Engineering, Univ. of Waterloo, Waterloo, ON, Canada, 1994.
[19]Nho, K. M., "Experimental Investigation of Heat Flow Rate and Directional Effect on Contact Conductance of Anisotropic Ground/Lapped Interfaces," Ph.D. Dissertation, Dept. of Mechanical Engineering, Univ. of Waterloo, Waterloo, ON, Canada, 1990.
[20]Lambropoulous, J. C., Jolly, M. R., Amsden, C. A., Gilman, S. E., Sinicropi, M. J., Diakomihalis, D., and Jacobs, S. D., "Thermal Conductivity of Dielectric Films," *Journal of Applied Physics*, Vol. 66, No. 9, 1989, pp. 4230-4242.
[21]Lambropoulous, J. C., Jacobs, S. D., Burns, S. J., Shaw-Klien, L., and Hwang, S. S., "Thermal Conductivity of Thin Films: Measurement and Microstructural Effects," *Film Heat Transfer—Properties and Processing*, HTD-Vol. 184, No. H00671, American Society of Mechanical Engineers, New York, 1991, pp. 21–31.
[22]Volklein, F., "Thermal Conductivity and Diffusivity of a Thin Film SiO_2–Si_3N_4 Sandwich System," *Thin Solid Films*, Vol. 188, No. 1, 1990, pp. 27-33.
[23]Morath, C. J., Maris, H. J., Coumo, J. J., Pappas, D. L., Grill, A., Patel, V. V., Doyle, J. P., and Saenger, K. L., "Picosecond Optical Studies of Amorphous Diamond and Diamond Like Carbon: Thermal Conductivity and Longitudinal Sound Velocity," *Journal of Applied Physics*, Vol. 76, No. 5, 1994, pp. 2636-2640.
[24]Dryden, J. R., "The Effect of a Surface Coating on the Constriction Resistance of a Spot on an Infinite Half Plane," *Journal of Heat Transfer*, Vol. 105, No. 2, 1983, pp. 408–410.