HEAT TRANSFER IN ELECTRONIC PACKAGING

M. M. Yovanovich

Microelectronics Heat Transfer Laboratory
Department of Mechanical and Electrical Engineering
University of Waterloo, Waterloo, Ontario, Canada

ABSTRACT

The important contact and convection thermal resistances which occur at the numerous interfaces and at the air cooled interfaces in association with microelectronic packaging are considered in this paper. The contact, gap and joint conductance models based on plastic deformation of the contacting asperities are reviewed, and the most recent modifications to the relative contact pressure are presented. The contact conductance model is compared againt data obtained for four different metals, and a wide range of effective surface roughness and surface slope. With the exception of some light load data, the model predictions and the data are in very good agreement.

A recently proposed simple cuboid model is applied to the problem of predicting the heat transfer rates from rectangular heat sinks. The agreement between the model and some data is shown to be in excellent agreement over a range of Rayleigh number corresponding to boundary layer flow over the active heat sink surface.

1. INTRODUCTION

Heat transfer in microelectronics and in particular telecommunication systems of today occurs at several levels which are characterized by length scales which range from fractions of a micron at the device level up to one-to-two meters at the cabinet level as shown in Figure 1. Between these two extremes the nominal length scale ranges for the subsystems are: 1) contacting interfaces ($0.1 - 3 \times 10^{-6}\ m$), 2) solder balls, vias and layers ($.01 - 0.03\ mm$), 3) packages ($10 - 30\ mm$), 4) heat sinks ($10 - 100\ mm$), 5) printed circuit boards (PCBs) ($1 \times 250 \times 300\ mm$), 6) racks of PCBs forming many parallel channels whose spacing may vary from ($5 - 50\ mm$).

The heat produced by the many devices located within the numerous packages which are attached in different ways to the PCBs must be removed by the coolant fluid which (excluding exceptional applications) is ambient air under forced, buoyancy-induced, or mixed flow conditions.

The heat transfer from the device level to the PCB level is primarily by conduction through complex structures consisting of a wide range of materials of very different thermophysical properties and thickesses. The material, spreading and constriction resistances can be predicted quite accurately by means of the numerous thermal resistance models and solution methods developed over the past five decades and many of which are presented in the summary chapters of Yovanovich and Antonetti (1988) and Yovanovich (1991). Other models and methods are described in books devoted to cooling of microelectronic equipment, e.g. Aung (1988, 1991), Bergles (1990), Dean (1985), Ellison (1984), Kraus (1965), Kraus and Bar-Cohen (1983), Scott (1974), Seely and Chu (1972), and Steinberg (1982). Many papers have been presented in a series of ASME Heat Transfer volumes dealing with many aspects of microelectronic cooling. They are edited by Kelleher and Yovanovich (1981), Oktay and Bar-Cohen (1983), Witte, and Saxena, (1984), Oktay and Moffatt, (1985), Bar-Cohen (1986), Wirtz, (1988), Ortega, Agonafer, and Webb (1991), and Imber and Yovanovich (1991).

2. CONTACT CONDUCTANCE MODELS

Although great progress has been made in the understanding and the ability to predict and thereby control thermal contact resistance, there are however, many interfaces within the microelectronic subsytems which due their complexity are not tractable with the methods and models developed todate. The difficulties lie in several areas which can broadly be classified as geometric and mechanical. The true or appropriate surface characteristics (surface roughness and slope, etc) of machined surfaces are still very difficult

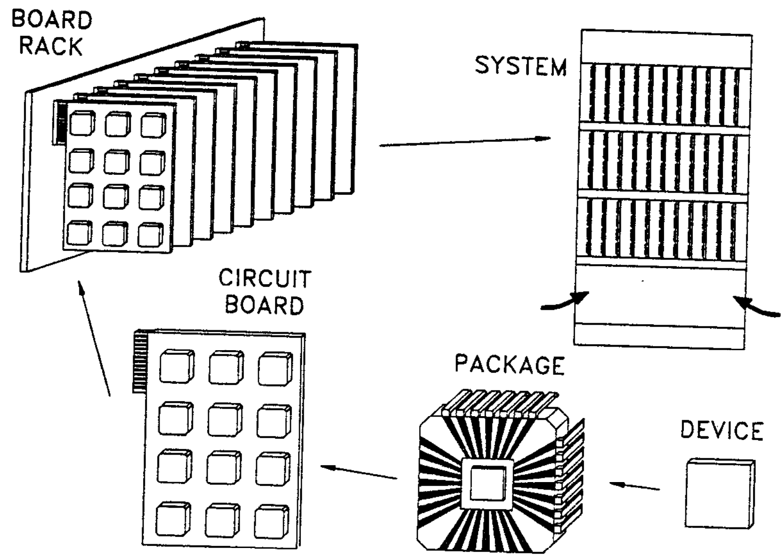

Fig. 1 Relative Scales of Subsystems from Device through System

to measure accurately for quantification for inclusion in a mechanical model.

Currently there are two mechanical models available for predicting the interaction of two rough metallic surfaces which are free of oxides, or any other coatings/contaminants. One mechanical model assumes plastic deformation of the contacting asperities, and the other mechanical model assumes elastic deformation of the contacting asperities. The two mechanical models also differ in their description of the geometry of the contacting asperities (see Sridhar and Yovanovich (1993))

The fully plastic model with significant modifications to the Vickers microhardness model as reported by Yovanovich et al. (1982a, 1983) has been used for three-to-four decades to predict quite accurately contact conductances of interfaces formed by identical metals having nominally flat, rough surfaces which are gaussian as recently reported by Sridhar and Yovanovich (1993). They also showed that the plastic model predictions and the experimental data for relatively smooth surfaces under very light relative loads are in significant disagreement.

The several elastic deformation models were reviewed by Sridhar and Yovanovich (1993) and compared against the identical set of data used in the plastic model comparison. In general they found that certain elastic models were in close agreement with each other, but they were in poor agreement with the data of Hegazy (1985). The elastic contact conductance model will not be considered in this paper.

3. PLASTIC CONDUCTANCE MODEL

3.1 CONTACT CONDUCTANCE MODELS

The contact conductance model and its correlation equation proposed by Cooper, Mikic and Yovanovich (1969) assumed a constant value of microhardness which at that time was taken to be approximately three times the yield stress. This model will be called the CMY plastic contact conductance model. Yovanovich (1982a) proposed a more comprehensive and accurate correlation equation for the contact conductance (based on the CMY Model) over a wider range of the relative contact pressure P/H_c. He proposed the simple relationship:

$$C_c \equiv \frac{\sigma}{m}\frac{h_c}{k_s} = 1.25\left(\frac{P}{H_c}\right)^{0.95} \qquad (1)$$

which agrees with the theoretical values to within ± 1.5 % in the relative contact pressure range $1 \times 10^{-6} \leq P/H_c \leq 2.2 \times 10^{-2}$. The geometric parameters are: i) the effective surface roughness $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$, ii) the effective surface slope $m = \sqrt{m_1^2 + m_2^2}$. The subscripts 1 and 2 denote the surfaces or metals on either side of the interface. The effective interface thermal conductivity is $k_s = 2k_1k_2/(k_1 + k_2)$. The relative contact pressure consists of the nominal pressure $P = F/A_a$ where F is the total static load on the interface and A_a is the apparent or nominal contact area. The contact hardness H_c is a complex geometric-mechanical parameter.

Soon after Yovanovich et al. (1982b, 1983) discovered the importance of the surface microhardness layers (produced by work-hardening) as measured by the Vickers microhardness test. These Vickers measurements where correlated as a power-law in the form: $H_V = c_1 d_V^{c_2}$ where d_V is the mean value of the Vickers diagonal for a single indentation, and c_1 and c_2 are correlation coefficients.

An iterative mechanical model was proposed for determined the relative contact hardness P/H_c for a given surface roughness σ/m where σ and m are effective surface RMS roughness and surface slope. Very good agreement between the modified CMY model and some data for a small set of identical metals was reported by Yovanovich et al. (1982b, 1983). Song and Yovanovich (1988) proposed an explicit method for calculating the relative contact pressure given the surface roughness parameter σ/m and the Vickers correlation coefficients c_1 and c_2:

$$\frac{P}{H_c} = \left[\frac{P}{c_1\left(1.62 \times 10^{-6}\sigma/m\right)^{c_2}}\right]^{(1+0.07c_2)^{-1}} \quad (2)$$

The modified CMY plastic model with the above expression for the relative contact pressure requires Vickers hardness measurements for each new metal. Hegazy (1985) showed that the coefficient c_1 was related to the bulk hardness as measured by the Brinell hardness test. He also observed that the coefficient c_2 was nearly constant and he set it to the value -0.26 for the four metals which he tested.

The Vickers microhardness can be predicted by means of the recently developed general correlation equations (Sridhar and Yovanovich, (1994)):

$$H_V = c_1 d_V^{c_2} \quad (3)$$

where the correlation coefficients are obtained from the Brinell hardness H_B according to

$$\frac{c_1}{3178} = \left[4.0 - 5.77\left(H_B^\star\right) + 4.0\left(H_B^\star\right)^2 - 0.61\left(H_B^\star\right)^3\right] \quad (4)$$

where $H_B^\star = H_B/3178$, and

$$c_2 = -0.370 + 0.442\left(\frac{H_B}{c_1}\right) \quad (5)$$

These correlations are valid for the Brinell hardness range of $1300 - 7600\ MPa$. They are based on measurements made on five different metals (SS304, Ni200, two Zirconium alloys, Titanium alloy, and untreated and heat treated tool steel) giving the Rockwell hardness range $19 \leq HRC \leq 66$. The agreement between the correlation equations and the hardness data is very good.

The CMY plastic contact conductance model with the relative contact pressure determined as described above is compared against similar metal vacuum data obtained for four sets of metals (SS304, Ni200, Zr-Nb, Zr-4). The comparisons are presented in Figures 2 through 5 where the dimensionless contact conductance C_c is plotted against the relative contact pressure P/H_c. The data were obtained for a very wide range of effective surface roughness and surface slope: $6.6 \times 10^{-6} \leq \sigma/m \leq 59.8 \times 10^{-6} m$, and a range of contact microhardness. In all cases the data collapse onto a very narrow band, and with the exception of the light load data there is very good agreement between the data and the CMY model predictions. The details of the comparisons between the data of Hegazy (1985) and the modified CMY plastic contact conductance model are presented by Sridhar and Yovanovich (1993). They concluded that the modified CMY model can be used to predict the contact conductances of conforming, flat surfaces provided the appropriate contact microhardness is used to determine the relative contact pressure as described above. Further research was recommended to ascertain the reasons for the observed discrepancies in the light load range for certain metals.

The CMY model was also shown by Antonetti and Yovanovich (1985) to be capable of predicting accurately the contact conductance at interfaces formed by two conforming surfaces one of which was bead blasted creating an isotropic rough surface, and the second surface was coated with a smooth, uniform layer of silver which was vapor deposited on a ground and lapped Ni200 substrate. Antonetti and Yovanovich (1985) introduced an effective conductivity and an effective contact hardness due to the softer and higher conductivity silver layer to be used in the CMY plastic contact conductance model in order to use it to predict the effect of the silver layers.

3.2 GAP CONDUCTANCE MODEL

The thermal resistance of gases trapped within the thin gap which is formed whenever two conforming rough surfaces are placed in mechanical contact can be predicted by the statistical model proposed by Yovanovich et al. (1982c). This model is in the form of an integral which relates the dimensionless gap conductance C_c to two dimensionless geometric parameters Y/σ and M/σ and the gas-to-solid thermal conductivity ratio:

$$C_g \equiv \frac{\sigma}{m}\frac{h_g}{k_s} = \frac{k_g}{k_s}\frac{1}{\sqrt{2\pi}}\int_0^\infty \frac{\exp\left[-\left(Y/\sigma - u\right)^2/2\right]}{M/\sigma + u}du \quad (6)$$

where u is a dummy variable and the relative mean plane separation Y/σ depends on the relative contact

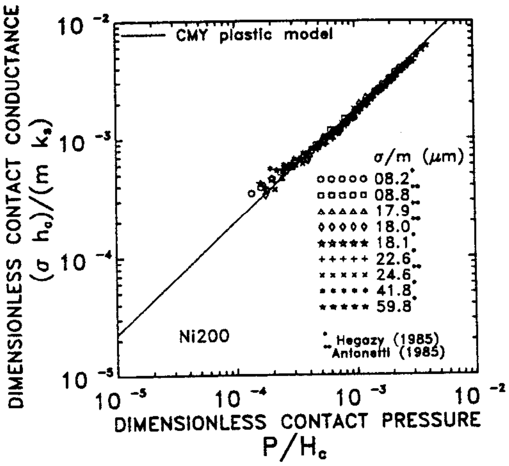

Fig. 2 Comparison of CMY plastic model with Ni200 data

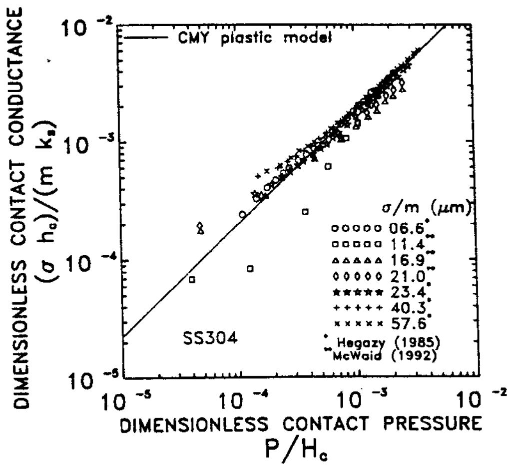

Fig. 3 Comparison of CMY plastic model with SS304 data

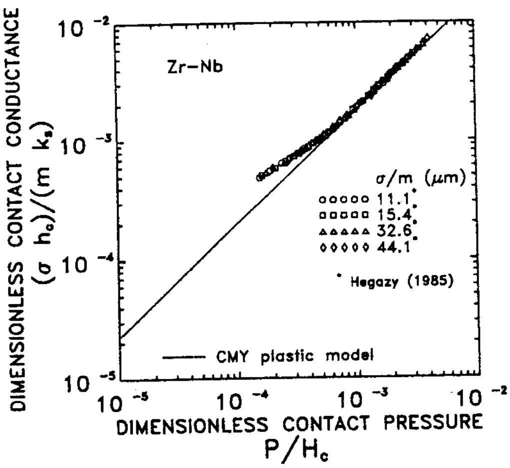

Fig. 4 Comparison of CMY plastic model with Zr-Nb data

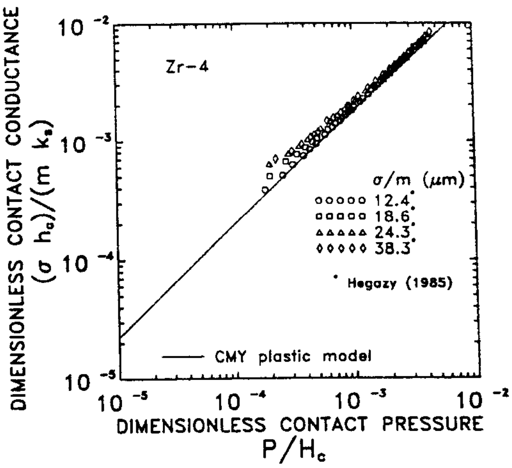

Fig. 5 Comparison of CMY plastic model with Zr-4 data

pressure through

$$\frac{Y}{\sigma} = \sqrt{2}\, erfc^{-1}\left(\frac{2P}{H_c}\right) \tag{7}$$

and $erfc^{-1}(x)$ is the inverse complementary error function. The gas parameter M depends on several other gas parameters such as the specific heat ratio, the Prandtl number, the accommodation coefficients and the molecular mean free path of the trapped gas at the mean temperature and gas pressure within the gap.

Wesley and Yovanovich (1985) showed that the gap model is in very good agreement with experimental measurements for interfaces which occur in Nuclear applications.

Negus and Yovanovich (1988) developed simple approximations for the integral (including the factor $1/\sqrt{2\pi}$) which appears in the gap model. They proposed

$$I_g = \frac{f_g}{Y/\sigma + M/\sigma} \tag{8}$$

with the following two correlation equations for the gas function f_g:

$$f_g = 1.063 + 0.0471\left(4 - \frac{Y}{\sigma}\right)^{1.68}\left(\ln\frac{\sigma}{M}\right)^{0.84} \tag{9}$$

for $2 \le Y/\sigma \le 4$ and $0.01 \le M/\sigma \le 1$, and

$$f_g = 1 + 0.06\left(\frac{\sigma}{M}\right)^{0.8} \tag{10}$$

for $2 \le Y/\sigma \le 4$ and $1 \le M/\sigma < \infty$. The maximum error between the correlations and the numerical values obtained from the gap integral is about 2 %.

Song et al. (1992) compared the gap conductance model against several other earlier models which were based on various approximations, and data obtained for two metals (SS304 and Ni200) having wide ranges of effective surface roughness and effective surface slope respectively: $0.07 \leq \sigma \leq 11.8 \mu m$, and $0.020 \leq m \leq 0.205$. Two gases were used in the tests: helium and nitrogen, over an important range of gas pressure: 10 – 700 Torr. They reported very good agreement between the gap conductance model and the data, and they found that certain other gap conductance models were in very poor agreement with each other and the data. Some differences between the gap model and light load, low pressure data were observed and discussed.

3.3 JOINT CONDUCTANCE MODEL

For interfaces in which gases are trapped in the gaps, it is convenient to deal with the contact and gap contributions separately. Since the contact and gap heat transfer rates are approximately independent for most practical applications, then assuming negligible radiation heat transfer across the gaps, the overall or joint conductance is modeled as

$$h_j = h_c + h_g \tag{11}$$

Using $\sigma/(mk_s)$ to nondimensionalize the three conductances in the joint conductance relation gives the dimensionless joint conductance

$$C_j = C_c + C_g \tag{12}$$

The joint conductance model has been verified by Hegazy (1985), Wesley and Yovanovich (1985) and more recently by Song et al. (1992).

4 HEAT SINKS

Thermally induced buoyancy effects are not always sufficient to cool the high density microelectronic packages attached to present day high density circuit boards. In many situations thermal enhancement techniques, such as heat sinks, must be used to increase the effective surface area for convection heat transfer to lower the thermal resistance between source and sink. The irregular geometries of heat sinks present great challenges to thermal analysts in determining the thermal-fluid boundary conditions at the fluid-solid interfaces.

A simple, accurate model for calculating the natural convection heat transfer from isothermal rectangular heat sinks using a flat plate boundary layer model has been proposed by Culham et al. (1994). Several heat sink geometries have been examined over a range of Rayleigh number between 10^3 and 10^{10}. The heat transfer performance of heat sinks, as given by the Nusselt number, is determined for each test based on the isothermal body temperature and the square root of the wetted surface area. Results obtained using a conjugate model, META (Culham et al. (1991)), are compared against an analytically based correlation and experimental data.

Microelectronics applications present a diverse mix of geometric configurations, thermophysical properties and flow conditions which must be factored into thermal modeling tools used for design or reliability assessment. The geometries encountered in heat sink assemblies are difficult to model using analytical techniques because of the complex fluid flow around and between the various components of the heat sink. Most heat transfer analyses of heat sinks and extended surfaces are based on correlated Nusselt number versus Rayleigh number plots derived from empirical studies. Van de Pol and Tierney (1973) presented a Nusselt versus Rayleigh relationship for natural convection cooling of vertical fins attached to a base plate, based on the experimental data of Welling and Wooldridge (1965), obtained over a range $0.6 < Ra^* < 100$. Jones and Smith (1970) performed a similar study for rectangular fin assemblies facing upward and downward in relation to the gravity vector. Nusselt versus Rayleigh plots were presented over a range $2 \times 10^2 < Ra < 6 \times 10^5$. In both studies, the correlation equations were restricted to a fixed range of geometric and flow conditions, limiting their use as general purpose design tools.

Some conjugate analytical solutions have been presented for rectangular heat sinks cooled by forced convection. Shvets and Didenko (1984) developed a conjugate fin model where the heat transfer coefficient was taken to be constant along the fin length. Garg and Velusamy (1986) used a boundary layer solution based on the Blasius equation to calculate a non-uniform boundary condition as the coupling condition in their iterative model. Similar models are not available for natural or mixed convection cooling.

Raithby and Hollands (1975, 1985) have presented models for predicting natural convection heat transfer from rectangular heat sinks. Since their models are general (capable of dealing with laminar and turbulent flow), they contain several semiempirical parameters which make them difficult to apply to heat sinks of interest in microelectronics.

Aihara (1991) presents heat sink models for vertical rectangular fin arrays. His models are restricted in their application and they are complex because i) they require the fin efficiency; ii) an empirical fac-

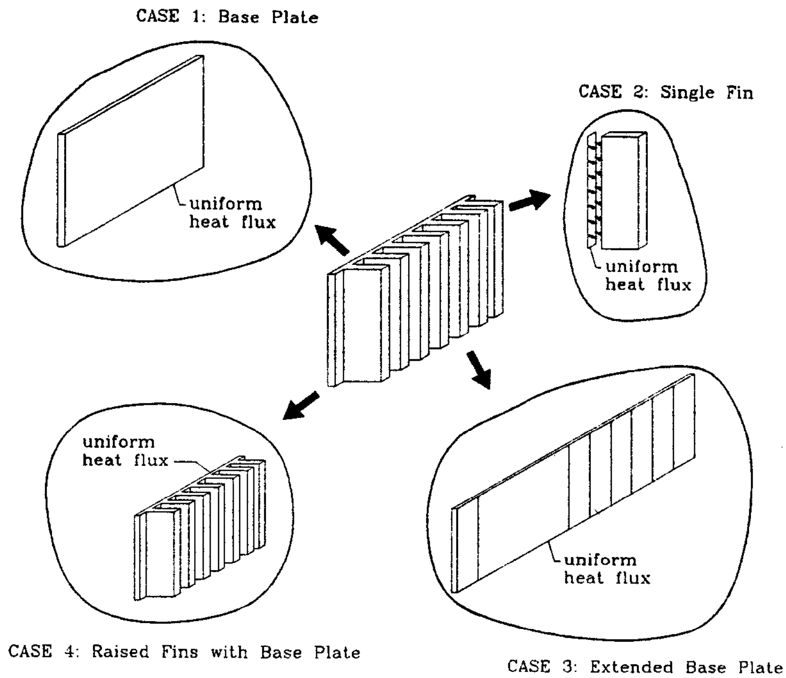

Fig. 6 Various Methods of Modeling Natural Convection Cooling of Rectangular Fins

tor which accounts for the effects of stagnant corners and fresh air inflow; iii) they are limited to particular ranges of certain geometric parameters; and iv) they are restricted to certain ranges of the Rayleigh number. These models are also difficult to apply to microlectronics applications.

A simple, yet accurate model, to predict laminar natural convection heat transfer from vertical rectangular heat sinks is required by the microelectronic and telecommunications industries. One such model will be presented in a subsequent section.

5 CUBOID MODEL

An analytically based correlation proposed by Yovanovich and Jafarpur (1993) for natural convection cooling of isothermal cuboids of arbitrary size. The Nusselt number is based on the linear superposition of the diffusive and convective limits for an isothermal body. The area-average Nusselt number, where the characteristic length is the square root of the total wetted surface area is (Yovanovich (1987a, 1987b)):

$$\mathrm{Nu}_{\sqrt{A}} = \mathrm{Nu}^{\infty}_{\sqrt{A}} + F(\mathrm{Pr}) G_{\sqrt{A}} \mathrm{Ra}^{1/4}_{\sqrt{A}} \qquad (13)$$

where the diffusive limit is calculated by

$$\mathrm{Nu}^{\infty}_{\sqrt{A}} = \frac{3.192 + 1.868(L/H)^{0.76}}{\sqrt{1 + 1.189(L/H)}} \qquad (14)$$

for a cuboid with dimensions $H \times W \times L$.

The "universal" Prandtl number function is

$$F(\mathrm{Pr}) = \frac{0.670}{[1 + (0.5/\mathrm{Pr})^{9/16}]^{4/9}} \qquad (15)$$

with the body-gravity function proposed by Lee et al. (1991), given as

$$G_{\sqrt{A}} = \left[\frac{1}{A} \iint_A \left(\frac{P \sin\theta}{\sqrt{A}} \right)^{1/3} dA \right]^{3/4} \qquad (16)$$

where P is the local perimeter of the body, perpendicular to the flow direction, θ is the angle subtended by the gravity vector and the normal to the surface, and A is the total wetted surface area of the body.

The body gravity function, $G_{\sqrt{A}}$, for an arbitrary rectangular fin with base plate dimensions, $H \times W \times t$, and N_f fins with dimensions, $L \times b \times H$, can be written as:

$$G_{\sqrt{A}} = 2^{1/8} \left[\frac{H(L \cdot N_f + t + W)^2}{(b \cdot L \cdot N_f + t \cdot W + H(L \cdot N_f + t + W))^{\frac{3}{2}}} \right]^{\frac{1}{4}} \qquad (17)$$

The base plate and fin height, H, are parallel to the gravity vector. The thickness of the base plate and the fin are t and b, respectively.

5.1 VERTICAL BASE PLATES AND FIN ARRAYS

Rectangular Fin

Many high powered electronic components cannot liberate sufficient quantities of heat given the high thermal impedance associated with air cooling. For situations where natural convection is a prerequisite, simple passive heat sinks can be attached to heat sources to enhance heat dissipation by increasing the available surface area exposed to the air flow.

A conventional rectangular fin, as shown in Figure 6, can be very effective as a means of lowering the thermal resistance between a heat source and the surrounding cooling fluid. The thermal resistance to the fluid sink can be characterized as $R_f = 1/(h \cdot A)$, where A is the total area wetted surface area. Given the narrow range of heat transfer coefficients available ($1 <$ h (W/m^2K) < 10) for buoyancy-induced air cooling, the most effective means of lowering the fluid-side thermal resistance is by increasing the area of the wetted surface.

The rectangular fin assembly shown in Figure 6, can be modeled as a collection of individual components consisting of a series of fins and a uniformly heated base plate or as a composite structure where all components simultaneously interact. Four unique examples of rectangular fin simulations are depicted in Figure 6. These examples consist of

- *Base Plate* - the base plate from the rectangular heat sink is modeled as a flat plate with a uniform heat flux boundary condition over the base surface.

- *Single Fin* - a single fin from the fin assembly is modeled as a flat plate with a uniform heat flux boundary condition applied over one end.

- *Extended Base Plate* - the entire heat sink assembly is modeled as a base plate with an equivalent wetted surface area. A uniform heat flux boundary condition is applied over a surface equivalent to the area of the original base plate.

- *Raised Fins with Base Plate* - the entire fin assembly is modeled as a base plate with attached extended surfaces. A uniform heat flux boundary condition is applied over the base plate. This option allows for the addition of a contact resistance between the fins and the base plate.

The wetted surfaces of a rectangular fin generally encounters two different types of flow which includes a conventional boundary layer flow over those surfaces where the formation of the boundary layer occurs unrestricted and a transitional region which leads to developing or fully-developed channel flow between individual fin sections. For a given surface area, the largest influence on the Rayleigh number ($\text{Ra}_{\sqrt{A}} = g\beta \text{Pr} \Delta T(\sqrt{A})^3/\nu^2$) is introduced through a change in the air pressure and in turn the kinematic viscosity, ν. Lowering the air pressure produces an increase in the kinematic viscosity and a decrease in the Rayleigh number. The rate of boundary layer growth for $Ra_{\sqrt{A}} < 10^6$ tends to preclude boundary layer flow between fin sections as the boundary layers interact and a transitional or fully-developed flow regime is attained. The actual demarcation between boundary layer flow and channel flow is a function of the spacing between fins, however, for fin spacings in the range of 15 mm to 35 mm, a Rayleigh number of 10^6 appears to be the appropriate lower bound for developing flow conditions.

Karagiozis (1991) performed a series of experiments to determine the heat transfer characteristics of rectangular heat sinks air cooled by natural convection. He presented test data for a variety of fin geometries, ranging from a simple base plate, 150 mm $\times$ 170 mm $\times$ 9.5 mm to a fin assembly consisting of a base plate, 150 mm $\times$ 220 mm $\times$ 9.5 mm with five rectangular fins, 150 mm $\times$ 50 mm $\times$ 9.5 mm, attached in such a manner that minimized the thermal contact resistance between the base plate and the base of the fins. Tests were conducted in a vacuum chamber that allowed data collection over a range of Rayleigh number between 10^{-3} to 10^8.

Example 1: Base Plate An aluminum base plate, 150 mm $\times$ 220 mm $\times$ 9.5 mm with a uniformly specified heat flux, is considered. The base plate is cooled by natural convection with the primary flow direction and the gravity vector parallel to the side measuring 150 mm. Karagiozis (1991) presented empirical data for a slightly smaller base plate (150 mm $\times$ 170 mm $\times$ 9.5 mm) over a range of Rayleigh numbers between 10^2 and 10^9, where the Rayleigh number is based on the plate length, L, as the characteristic length. The analytical cuboid model of Yovanovich and Jafarpur (1993) result in a 2% increase in the Nusselt number when the fin length is increased from 170 mm to 220 mm. For the purpose of this investigation a characteristic length based on the square root of the wetted surface area is used. Yovanovich (1987a, 1987b) showed that $\sqrt{A}$ is the appropriate choice of characteristic length when comparing different body shapes suspended in various orientations because of its ability to provide a common reference when comparing the heat transfer response presented as $\text{Nu}_{\sqrt{A}}$

vs. $Ra_{\sqrt{A}}$.

Figure 7 shows the comparison of the data of Karagiozis (1991) and the analytical cuboid model of Yovanovich and Jafarpur (1993) for the 150 mm × 220 mm × 9.5 mm base plate. The agreement between the predictions of the cuboid model and the data is within 8% over the full range of Rayleigh number examined, with the maximum discrepancy occurring at $Ra_{\sqrt{A}} = 10^9$. Although data begins at 10^3, the Yovanovich cuboid model extends to $Ra_{\sqrt{A}} = 10^{-4}$, where the diffusive limit dominates and both data and the cuboid model show good agreement.

Example 2: Single Fin In a manner similar to the base plate described above, a single fin can be modeled as a flat plate where the applied heat flux occurs over one end of the plate. A necessary condition for modeling a fin as an isolated plate is the assumption that the boundary layer formed around the fin remains in the developing region. Karagiozis (1991) did not present experimental data for a base plate the size of a single fin. Experimental data presented in Figure 8 are for the 170 mm wide base plate described above.

The very good comparison between the experimental data of Karagiozis (1991) and the analytical model of Yovanovich and Jafarpur (1993) is shown in Figure 8.

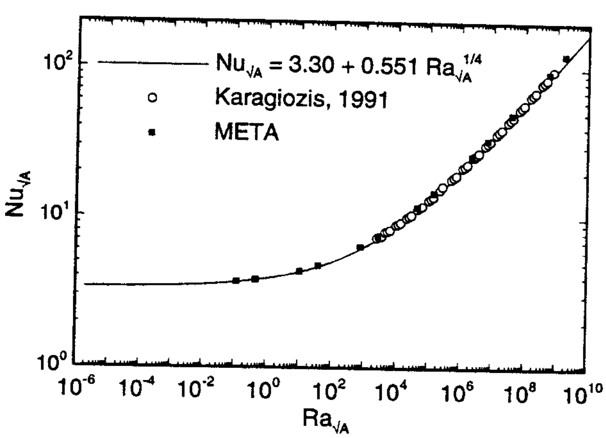

Fig. 7 Base Plate Natural Convection Cooling

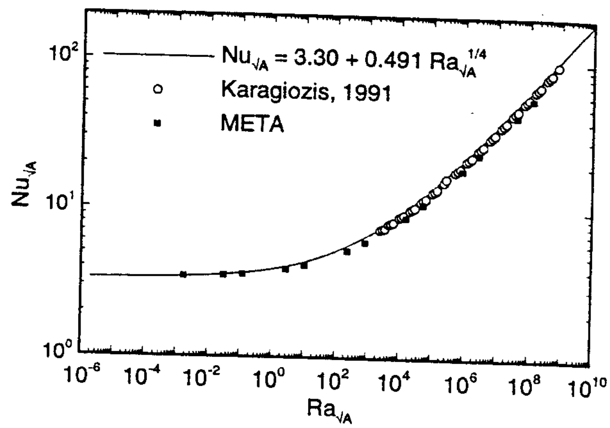

Fig. 8 Base Plate Natural Convection Cooling

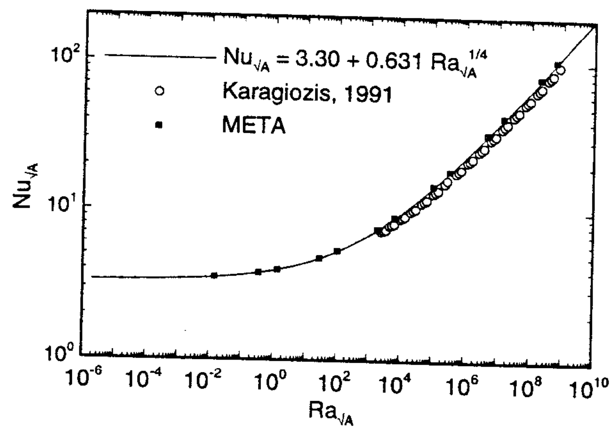

Fig. 9 Base Plate Natural Convection Cooling

Example 3: Extended Base Plate More practical applications of heat sinks involve the use of a base plate with several fins extending from the base plate at right angles. If the fins have a high thermal conductivity, the entire fin assembly can approach an isothermal condition while the increased surface area enhances heat transfer. One means of simulating a rectangular fin assembly using a flat plate model is to approximate the fin assembly as a flat plate of equivalent surface area. Again, this implies that the boundary layers formed between the fins remain in the developing regime over the full height of the fin.

Karagiozis (1991) also examined a rectangular fin with a base plate 150 mm × 220 mm × 9.5 mm with five rectangular fins 150 mm × 50 mm × 9.5 mm as shown in Figure 10 His tests were conducted over a range of Rayleigh between 10^2 and 10^9, where all tests below a Rayleigh number of 10^4 resulted in the bound-

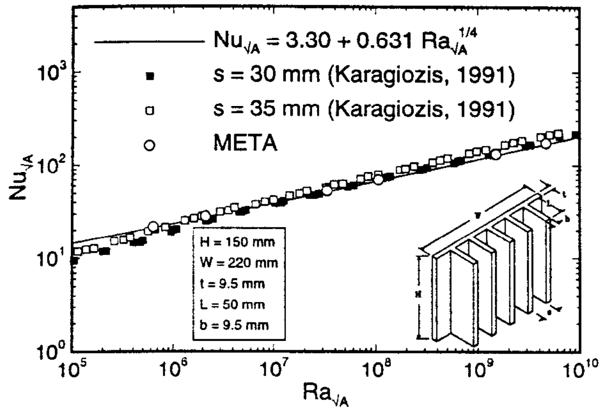

Fig. 10 Modeling Natural Convection from Rectangular Fins

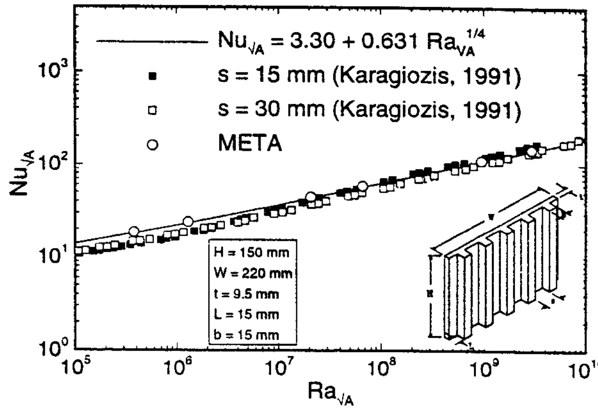

Fig. 11 Modeling Natural Convection from Rectangular Fins

ary layer becoming fully developed in the 30 mm spacing between adjacent fins.

Example 4: Raised Fin with Base Plate The heat sink fin assembly can be adequately modeled as a base plate of equivalent area. The development of boundary layers over the surface of the fins goes through three distinct regimes as the boundary layer grows over the fin surfaces. The first regime, associated with large Rayleigh numbers i.e., $Ra_{\sqrt{A}} > 10^6$, is the developing region where boundary layer growth occurs unrestricted. The total exposed surface area must be considered in the calculation of the characteristic length, namely $\sqrt{A}$. Low Rayleigh numbers, i.e., $Ra_{\sqrt{A}} < 1$ generally lead to situations where the boundary layers between adjacent fin sections interact almost immediately, resulting in a fully-developed flow over the entire height of the fin. In these instances, the exposed surface area is equivalent to the outside dimensions of the fin assembly. The transitional region, $1 \leq Ra_{\sqrt{A}} \leq 10^6$, results in a mixed condition where the effective surface area is neither the total exposed surface area nor the surface area associated with the outside dimensions of the fin assembly. The fully-developed and transitional regions are not be addressed in this paper.

The heat sink assembly presented in Figure 10 consists of a base plate 150 mm × 220 mm × 9.5 mm with five attached fins 150 mm × 50 mm × 9.5 mm. Karagiozis (1991) tested this heat sink for fin spacings of 30 mm and 3 5 mm over a range of Rayleigh number from $10^5 \leq Ra_{\sqrt{A}} \leq 10^{10}$. The data are in excellent agreement with the heat sink model of Yovanovich and Jafarpur (1993) over the range $10^5 \leq Ra_{\sqrt{A}} \leq 10^{10}$. Below $Ra_{\sqrt{A}} = 10^6$, the boundary layers forming over the fin sections begin to interact, marking the onset of fully-developed flow. Since the correlation of Yovanovich and Jafarpur (1993) only considers situations with developing boundary layer flow, a deviation between the model and the experimental data is expected below $Ra_{\sqrt{A}} = 10^6$.

Figure 11 presents a similar comparison of experimental data and the heat sink model. The fin assembly shown in Figure 11 consists of a base plate 150 mm × 220 mm × 9.5 mm with five attached fins 150 mm × 15 mm × 15 mm. As in the previous example, the agreement between the heat sink model the dat is very good, however, the experimental data of Karagiozis (1991) fall below the predictions at $Ra_{\sqrt{A}} \leq 10^7$. In addition the experimental data points for a fin spacing of 30 mm should result in a marginally higher Nusselt number than a fin spacing of 15 mm, but as shown in Figure 11 this trend is reversed. Since the experimental data have been obtained from the open literature, with no explanation for this trend, further insight into the differences between the experimental data and the predictions of the Yovanovich and Jafarpur (1993) heat sink model cannot be addressed at this time.

The excellent agreement obtained in comparisons between experimental data and the analytical cuboid and heat sink models indicate that a boundary layer model can be used for complex flow over three dimensional bodies if the appropriate characteristic length is used in determining the dimensionless groups, Nusselt and Rayleigh numbers. As demonstrated by Yovanovich (1987a, 1987b) and again in the comparisons presented here, the appropriate characteristic length for most three dimensional bodies is the square root of the total wetted surface area. It should be clearly noted that the boundary layer equations are only applicable if the boundary layer growth occurs unrestricted over the full length of the body, especially over the length of the U-shaped channels formed by rectangular fins in a heat sink assembly.

6. CONCLUDING REMARKS

The most recent modifications to the contact, gap and joint conductance models are presented. The relative contact pressure which depends on the contact microhardness (which is related to the Vickers microhardness) can now be predicted accurately given the bulk hardness (Brinell hardness) of the softer contacting metal. The correlation equations for the Vickers microhardness coefficients are valid for a wide range of metals. The dimensionless contact conductance model is shown to be in very good agreement with data obtained for four different metals.

The proposed simple heat sink model which is based on a simple cuboid model is shown to be in excellent agreement with data obtained for three single base plates over a wide range of the Rayleigh number. Subsequently it is shown that the simple heat sink model predicts quite accurately the data obtained for the base plates with five fins attached to them. Various fin spaces were tested. Agreement was observed for all tests.

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