.NALYSIS OF THREE-DIMENSIONAL CONJUGATE HEAT :RANSFER PROBLEMS IN MICROELECTRONICS

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UMMARY

This study is concerned with the development of an analytic-numerical ethod for modelling multi-layered rectangular printed circuit boards with bitrarily located, surface mounted finite heat sources. Heat conduction is ;sumed to be steady-state and fully three-dimensional in the solid, with ıch layer having a homogeneous thermal conductivity. The cooling of the)lid is achieved through a forced air flow boundary condition which in-udes the influence of an upstream heat flux from the solid into the fluid. 1 analytical Fourier series solution is used, and the boundary conditions e approximately satisfied using a continuous least-squares criterion. A ear set of symmetric equations is obtained for determining the unknown :ies coefficients. Unlike fully numerical, domain discretization schemes, :s procedure does not require discretization, making it particularly attrac-/e for three dimensional conduction problems. The model can simulate :sh-mounted heat sources, arbitrarily located on opposite surfaces of a rect-.gular domain with convective cooling. Results are presented illustrating e effects of multiple laminates and heat source spacing, with applications cooling microelectronic circuit boards.

INTRODUCTION

The thermal analysis of heat source modules arbitrarily located on the rface of rectangular multi-layered boards, is currently of practical interest the microelectronics industry. The typical system shown in Figure 1, is mprised of a solid, multi-layered conductive board, usually of alternating reglass and copper layers, to which the electronic chip components are attached. These chips can be treated as heat sources which dissipate electrical energy as heat, through the chip casing to the fluid, and via pins and surface mountings to the printed circuit board.

Of particular interest to electrical designers is the die temperature T_D, located within the chip module. This is unknown, but the heat dissipation rate Q can be specified for the problem. The thermal resistances, between the die and fluid R_{PF}, and between the die and board R_{PB} (see Figure 1 inset), will vary depending on package construction [1]. Experimental and numerical validations in [2] have shown that by assuming R_{PF} and R_{PB} to be negligible, hence $T_D \equiv T_B \equiv T_W$, reasonable temperature estimates can be obtained for heat source modules located in a strip-wise fashion along the board, in the direction of flow (x). Inherent also, is the assumption that the small module thickness does not significantly affect the flow, so that the fluid essentially sees a smooth heated wall surface.

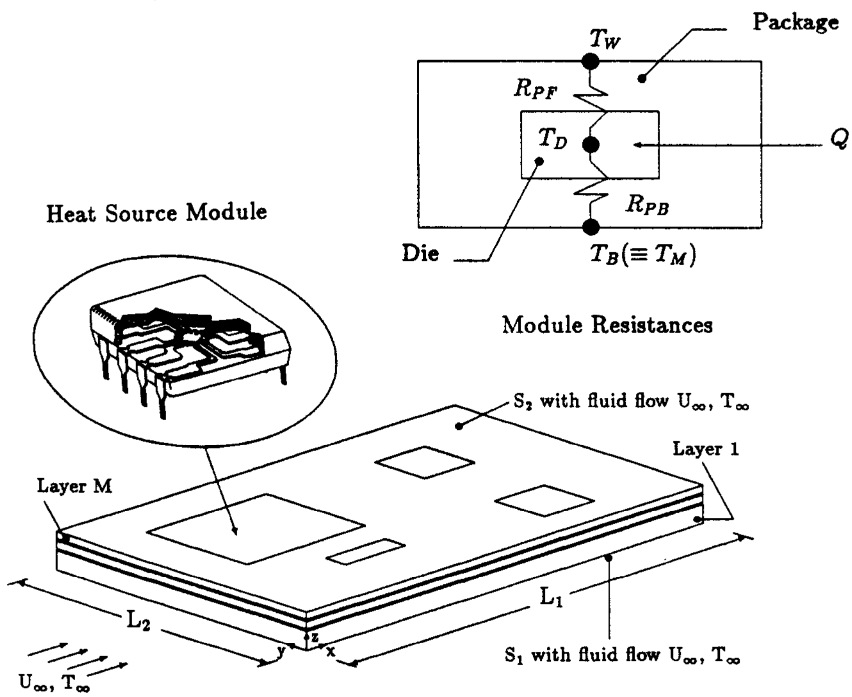

Figure 1: 3D Thermal Board with Heat Source Modules

If the heat source modules are small relative to the overall dimensions of the solid substrate, two-dimensional analyses will not adequately predict the temperatures as noted in [3]. Also, because the board will be non-isothermal, a conjugate fluid-solid analysis must be undertaken which accounts for up-stream heating of the air flow by the solid conducting medium. Theoretically this amounts to a solution of the energy-momentum equation on the fluid-side, for it's velocity u and temperature field T,

$$\frac{d}{dx}\int_0^{\delta_t} uT\,dz \;=\; -\alpha\frac{\partial T_W}{\partial z}\,, \tag{1}$$

which is obtained from boundary layer assumptions, and the solid-side Lapla-
:ian equation in each layer,

$$\frac{\partial^2 T_m}{\partial x^2} + \frac{\partial^2 T_m}{\partial y^2} + \frac{\partial^2 T_m}{\partial z^2} \;=\; 0\,, \tag{2}$$

vith boundary conditions. Equations (1) and (2) can be simultaneously solved numerically using finite element or finite difference methods. This vould involve discretization of both the fluid and solid domains requiring problem-specific mesh generation. For a smooth, non-isothermal wall, the wall-fluid temperature may be approximated [4,5] using boundary layer the- ory approximations [6] for forced convection flow, of the form

$$T_W - T_\infty \;=\; g_f U_\infty^{-1/2} x^{-1/2}\int_0^x \frac{q_W(\xi)d\xi}{\left(1-(\xi/x)^{1/c_1}\right)^{1/c_2}}\,. \tag{3}$$

T_∞ and U_∞ are free stream quantities, and g_f, c_1 and c_2 vary depending on vhether the flow is laminar or turbulent, wall shear conditions, and assumed elocity profiles [4,5]. Numerous researchers have studied similar problems vith different assumptions and methods [7-11]. A film coefficient can usually e extracted from (3) for use in the classical convection boundary condition eeded for the exposed surfaces;

$$-\,k\frac{\partial T_W}{\partial z} - h(x,T_W)(T_W - T_\infty) + q(x,y) \;=\; 0\,. \tag{4}$$

The film coefficient h and heat flux q in (4) may both vary positionally, but ince h will also be temperature dependent, it must be iteratively computed sing (3) and a solution to (2). Models using (4) automatically require eration to establish h, and are also automatically restricted to discretizing ie entire exposed surface. Although the method to be introduced here in easily incorporate the form (4) as shown in [12,13], this study directly ses the interface condition (3) thereby avoiding unnecessary intermediate erations and discretization.

. PRELIMINARY ANALYSIS

The origin of the local coordinate systems, in x, y, z of each layer, located at the bottom left corner as illustrated in Figure 1 for layer 1. etting $\theta_m \equiv T_m(x,y,z) - T_\infty$, with homogeneous thermal conductivity k_m, separable solution can be obtained in a straightforward manner,

$$\theta_m(x,y,z) \;=\; c_m z + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\cos(\lambda_i x)\cos(\beta_j y)[a_{m,i,j}\cosh(\gamma_{i,j}z) + {} + b_{m,i,j}\sinh(\gamma_{i,j}z)]\,, \tag{5}$$

which satisfies the insulated end conditions for each layer at $x = 0, L_1$, and $y = 0, L_2$, with constants

$$\lambda_i = \frac{i\pi}{L_1}\ ,\ \beta_j = \frac{j\pi}{L_2}\ ,\ \gamma_{i,j} = \sqrt{\lambda_i^2 + \beta_j^2}\ . \tag{6}$$

The board ends are treated as insulated since the aspect ratio of the exposed $x-y$ surface to the total z-thickness is generally $> 100:1$. Each of the layers are in perfect contact with each other, thus satisfying the following boundary conditions along the whole of each interface,

$$\theta_m(x, y, t_m) = \theta_{m+1}(x, y, 0)\ , \tag{7}$$

$$\kappa_m \frac{\partial \theta_m}{\partial z}(x, y, t_m) = \frac{\partial \theta_{m+1}}{\partial z}(x, y, 0)\ ;\ \kappa_m = \frac{k_m}{k_{m+1}}\ . \tag{8}$$

Using orthogonality, the constants in (5) may be related using (7) and (8), and it can be shown that in proceeding from layer 1 to layer M,

$$a_{m+1,i,j} = a_{m,i,j}\cosh(\gamma_{i,j}t_m) + b_{m,i,j}\sinh(\gamma_{i,j}t_m)\ ;\ i,j = 0,1,2,\ldots, \tag{9}$$

$$b_{m+1,i,j} = \kappa_m(a_{m,i,j}\sinh(\gamma_{i,j}t_m) + b_{m,i,j}\cosh(\gamma_{i,j}t_m))\ ;\ i,j = 0,1,2,\ldots, \tag{10}$$

$$a_{m+1,0,0} = a_{m,0,0} + c_m t_m\ ;\ c_{m+1} = \kappa_m c_m\ . \tag{11}$$

In (9) and (10), the relations hold whenever i and j are both not zero. Tables and relations are provided in [14] for three types of unmixed boundary conditions imposed on layer 1 at $z = 0$ (surface S_1), which initialize the first layer constants for use in (9)-(11). The special case where S_1 also has mixed boundary conditions with multiple heat flux and convective sections, is considered later in section 4.

Using (5)-(11), we can write the general series form on surface S_2 (layer M, $z = t_M$),

$$\theta_M(x, y, t_M) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(d_{i,j}\phi_{i,j}(t_M) + \sigma_{i,j}(t_M))\cos(\lambda_i x)\cos(\beta_j y)\ , \tag{12}$$

$$\frac{\partial \theta_M}{\partial z}(x, y, t_M) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}(d_{i,j}\phi'_{i,j}(t_M) + \sigma'_{i,j}(t_M))\cos(\lambda_i x)\cos(\beta_j y)\ , \tag{13}$$

with

$$\phi_{i,j}(z) = F_{1,i,j}\cosh(\gamma_{i,j}z) + F_{2,i,j}\sinh(\gamma_{i,j}z)\ ;\ i,j = 0,1,2,\ldots, \tag{14}$$

$$\sigma_{i,j}(z) = F_{5,i,j}\cosh(\gamma_{i,j}z) + F_{6,i,j}\sinh(\gamma_{i,j}z)\ ;\ i,j = 0,1,2,\ldots, \tag{15}$$

$$\phi_{0,0}(z) = F_3 z + F_4\ ;\ \sigma_{0,0}(z) = F_7 z + F_8\ . \tag{16}$$

The derivative functions ϕ' and σ' are obtained by differentiating the above with respect to z. The F_i along with the $d_{i,j}$ are outlined in [14]. For clarity, the $d_{i,j}$ are expressible in terms of $a_{i,j}, b_{i,j}$ and c, depending on the S_1 conditions.

From the fluid-side, the general wall condition to be satisfied is given by (3). We shall consider $R_{PF} \equiv R_{PB} \equiv 0$; thus the form (3) is continuous along the wall-fluid interface, i.e. on S_2, $T_W(x) - T_\infty \equiv \theta_M(x, y, t_M)$. The form (3) also allows for surfaces S_1 and S_2 to have independent fluid free-stream properties through the constant g_f.

3. SOLUTIONS FOR MIXED CONDITIONS ON S_2 ONLY

Since $R_{PF} = R_{PB} = 0$, the positive heat flux entering the fluid from surface S_2, for use in (3), may be written as

$$q_{w,2}(\xi) = -k_M \frac{\partial \theta_M(\xi, y, t_M)}{\partial z} + q_{S2} , \tag{17}$$

where q_{S2} may take on finite or zero values across the exposed surface. Substituting (12), (13) and (17) into (3), we obtain after a little manipulation,

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} d'_{i,j}\psi_{i,j}(x, y) - f(x, y) = 0 , \tag{18}$$

where

$$d'_{i,j} = d_{i,j}F_{1,i,j}\cosh(\gamma_{i,j}t_M) , i, j = 0, 1, 2, \dots \; ; \; d'_{0,0} = d_{0,0} \tag{19}$$

$$\psi_{i,j}(x, y) = \cos(\beta_j y)\left(\rho_{i,j}(t_M)\cos(\lambda_i x) + k_M\rho'_{i,j}(t_M)\int_0^x g_2(\xi, x)\cos(\lambda_i\xi)d\xi\right) \tag{20}$$

$$\rho_{i,j}(t_M) = 1 + \frac{F_{2,i,j}}{F_{1,i,j}}\tanh(\gamma_{i,j}t_M) \; ; \rho_{0,0}(t_M) = \phi_{0,0}(t_M) \tag{21}$$

$$\rho'_{i,j}(t_M) = \gamma_{i,j}\left(\frac{F_{2,i,j}}{F_{1,i,j}} + \tanh(\gamma_{i,j}t_M)\right) \; ; \rho'_{0,0}(t_M) = \phi'_{0,0}(t_M) \tag{22}$$

$$f(x, y) = \int_0^x g_2(\xi, x)q_{S2}d\xi - \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\cos(\beta_j y)\Big\{\sigma_{i,j}(t_M)cos(\lambda_i x) + \sigma'_{i,j}(t_M)k_M\int_0^x g_2(\xi, x)\cos(\lambda_i\xi)d\xi\Big\} . \tag{23}$$

We define the quadratic functional on S_2,

$$I_{S2} = \int_0^{L_1}\int_0^{L_2}\left[\sum_{i=0}^{N}\sum_{j=0}^{N} d'_{i,j}\psi_{i,j}(x, y) - f(x, y)\right]^2 dxdy . \tag{24}$$

By minimizing this variational form, a system of linear equations results which may be used for the solution of a finite number of unknown $d'_{i,j}$. Thus, taking $\partial I_{S2}/\partial d'_{p,n} = 0$, we obtain the $(N+1)^2$ linear equations

$$Rd' = G , \tag{25}$$

with

$$R_{p,n,i,j} = \sum_{s=1}^{J_s} \int\int_s \psi_{p,n}(x,y)\psi_{i,j}(x,y)dxdy \; ; p,n,i,j, = 0,1,2,\ldots N, \quad (26)$$

$$G_{p,n} = \sum_{s=1}^{J_s} \int\int_s \psi_{p,n}(x,y)f(x,y)dxdy \; ; p,n = 0,1,2,\ldots N \,. \quad (27)$$

where $\int\int_s$ denotes an area integration over every section s on S_2, where ψ or f may change value with x and y. Since (3) is positionally dependent on coordinate x, and not y, the total number of sections J_s necessary is only dependent on the number of heat source locations [14].

Solving (25) will yield an approximate set of $(N+1)^2$ coefficients $d'_{i,j}$, which can be post-processed to yield the necessary solution coefficients in (5) for the thermal fields. However because of the earlier assumptions that $T_W \equiv T_B \equiv T_D$, the integrations in (26) can be singly represented by a total area integration $\int_0^{L_1}\int_0^{L_2}$ over S_2, regardless of heat source location, since ψ is unaffected. The R matrix can be computed once for a given problem. Further details on particular integrations can be found in [14].

4. MIXED BOUNDARY CONDITIONS ON S_1 AND S_2

Here we assume that heat flux specified sections with forced convection cooling, are also arbitrarily located on both surfaces S_1 and S_2. This is representative of electronic circuit boards with chip modules mounted on sides of the board. This results in two unknown coefficients which have to be determined in (5), as opposed to one in (12). A procedure detailed in [12,13] can be used to relate coefficients in (5) in a similar manner that was shown for the solution of the coefficients in section 3. The location and intensity of heat sources, including fluid properties, may differ on S_2 from S_1. We will briefly outline the methodology in the following.

The general condition on surface S_1, with forced convection and arbitrarily placed heat sources, may be written as

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty} a_{1,i,j}\psi_{1,i,j}(x,y) + b_{1,i,j}\psi_{2,i,j}(x,y) - f_1(x,y) \;=\; 0, \quad (28)$$

where ψ_1 and ψ_2 can be established using a similar methodology as in section 3 to derive (18), and outlined in detail in [14]. We can then define on S_1 the form

$$I_{S1} = \int_0^{L_1}\int_0^{L_2} \left[\sum_{i=0}^{N}\sum_{j=0}^{N} a_{1,i,j}\psi_{1,i,j}(x,y) + b_{1,i,j}\psi_{2,i,j}(x,y) - f_1(x,y) \right]^2 dxdy, \quad (29)$$

and on S_2

$$I_{S2} = \int_0^{L_1}\int_0^{L_2}\left[\sum_{i=0}^{N}\sum_{j=0}^{N} a_{M,i,j}\psi_{3,i,j}(x,y) + b_{M,i,j}\psi_{4,i,j}(x,y) - f_M(x,y)\right]^2 dxdy, \tag{30}$$

or

$$I_{S2} = \int_0^{L_1}\int_0^{L_2}\left[\sum_{i=0}^{N}\sum_{j=0}^{N} a_{1,i,j}\psi_{5,i,j}(x,y) + b_{1,i,j}\psi_{6,i,j}(x,y) - f_2(x,y)\right]^2 dxdy, \tag{31}$$

where ψ_5, ψ_6 can be related recursively to ψ_3 and ψ_4 using a simple algorithm based on the forms (9)-(11). We now take two minimizations of forms (29) and (31), i.e. let $I = I_{S1} + I_{S2}$, thus we take

$$\frac{\partial I}{\partial a_{1,p,n}} = 0\ ,\ \frac{\partial I}{\partial b_{1,p,n}} = 0\ , \tag{32}$$

to obtain the $2(N+1)^2$ equations

$$\begin{bmatrix} R_1 & R_2 \\ R_3 & R_4 \end{bmatrix} \begin{Bmatrix} a_1 \\ b_1 \end{Bmatrix} = \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix}\ . \tag{33}$$

for the solution of $a_{1,i,j}$ and $b_{1,i,j}$. Particular entries of (33) are similar in form to those in (26), and are detailed in [14].

5. NUMERICAL RESULTS AND DISCUSSION

Numerical cases were studied examining the effect of heat source location on a laminated, 7-layer board. Figure 2 illustrates results for a single heat source centrally located (see inset) on the top board surface S_2, with forced convection cooling (3) on both sides, S_1 and S_2, of the board. Parameters are given in Table 1. The dimensionless heat source x-width/L_1, W_x, was fixed at 0.2, and the y-width/L_2, W_y, was varied between a small hot-spot size of 0.2 to a strip size of 1.0.

Similarly, Figures 3 and 4 show results for two heat sources of different size; a larger heat source trailing the first, and the larger preceeding the second, in the x-direction of coolant flow. All figures show the temperatures of the top surface S_2, from $x = 0$ to L_1, along a constant y-line located centrally on the source (denoted by - - - - in the insets). In all cases a two-dimensional $x - z$ strip solution would correspond to $W_y = 1$; a two-dimensional $x - y$ spatial solution, denoted by — — in the graphs, corresponds to a line of symmetry (insulated boundary) being drawn midway through the total z-thickness of the board (and $q \equiv q/2$). This $x - y$ spatial solution is also sometimes determined by assuming an effective thermal conductivity of the substrate, thereby treating the z-temperature drop as negligible.

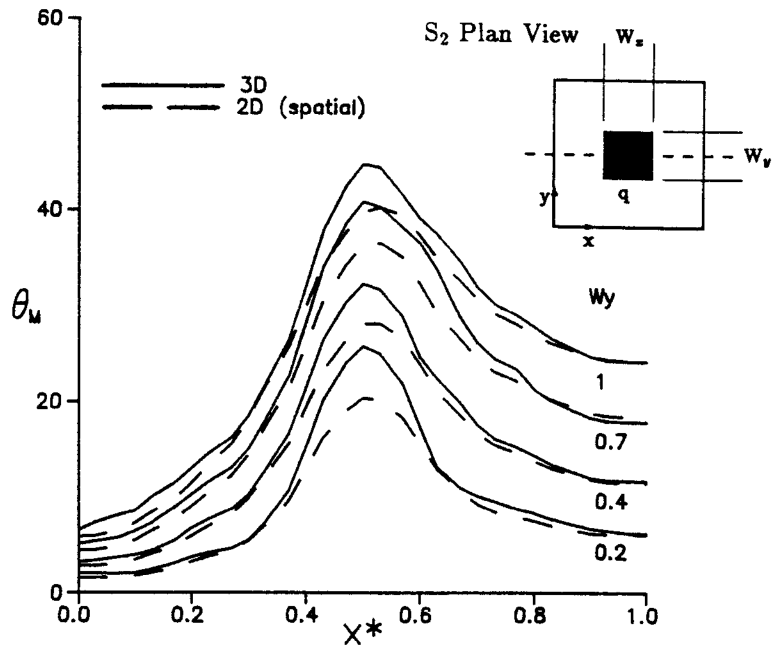

Figure 2: Temperatures For a Single Heat Source

Differences can be noted between the possible two-dimensional approximations to the three-dimensional thermal spreading problem, and these are always largest at the heat source. In all instances, the spatial solution will underestimate, and the strip solution will overestimate, the temperatures in the system. Further results are presented in [14].

The continuous least-squares Fourier solution to the heat conduction problem has been shown ([12,13]) to give stable estimates for the coefficients as the truncation value N is increased. As is typical with Fourier series type solutions, this value controls the level of error in convergence, as opposed to the degree of discretization required with numerical methods such as finite element, finite difference and even the boundary element method. For all cases considered herein, it was found that at most, a 5% temperature difference was observed between taking a truncation value as little as $N = 10$, compared with $N > 50$, to which the temperature solution could be converged to higher decimal degree of accuracy, but with significantly increased computation time.

Board	
(fibreglass)	(copper)
$t_1, t_3, t_5, t_7 = 3.8 \times 10^{-4}$ m	$t_4 = 3.6 \times 10^{-5}$ m
$k_1, k_3, k_5, k_7 = 0.4$ W/mK	$t_2, t_6 = 1.8 \times 10^{-5}$ m
$L_1 = L_2 = 0.1$ m	$k_2, k_4, k_6 = 386$ W/mK

Fluid (Air-laminar flow) $\quad g_f = 0.73 Pr^{-1/3} \nu^{1/2} k_f^{-1}\ m^2K/s^{1/2}W$
$U_\infty = 3$ m/s ; $T_\infty = 20^oC$
$k_{f,1} = k_{f,2} = 2.6 \times 10^{-2}$ W/mK ; $Pr = 0.71$
$c_1 = 1$; $c_2 = 3/2$; $\nu = 15 \times 10^{-6} m^2/s$

Heat Source : centroids at $x^* = x/L_1$, $y^* = y/L_2$	
Figure 2	$x^* = y^* = 0.5$; $W_x = 0.2$; $W_y = 0.2, 0.4, 0.7, 1.0$
	$q = 5000\ W/m^2$
Figure 3	$W_{x1} = 0.1$; $W_{y1} = 0.1, 1.0$; $x^* = 0.15$; $y^* = 0.85, 0.5$
	$W_{x2} = 0.4$; $W_{y2} = 0.4, 1.0$; $x^* = 0.60$; $y^* = 0.40, 0.5$
	$q_1 = 1000\ W/m^2$; $q_2 = 4000\ W/m^2$
Figure 4	$W_{x1} = 0.4$; $W_{y1} = 0.4, 1.0$; $x^* = 0.40$; $y^* = 0.60, 0.5$
	$W_{x2} = 0.1$; $W_{y2} = 0.1, 1.0$; $x^* = 0.85$; $y^* = 0.15, 0.5$
	$q_1 = 4000\ W/m^2$; $q_2 = 1000\ W/m^2$

Table 1: Parameters for Numerical Studies

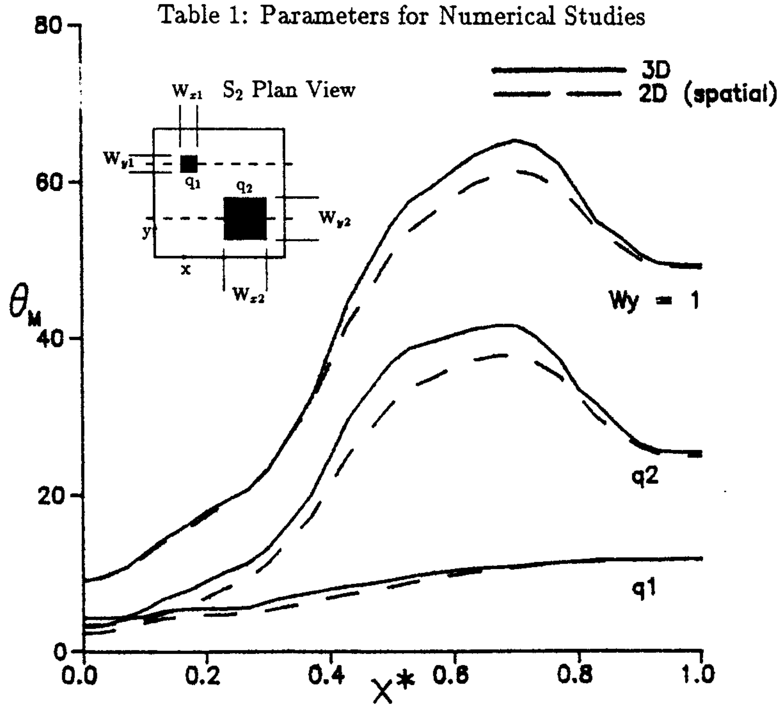

Figure 3: Effects of a Larger Downstream Heat Source

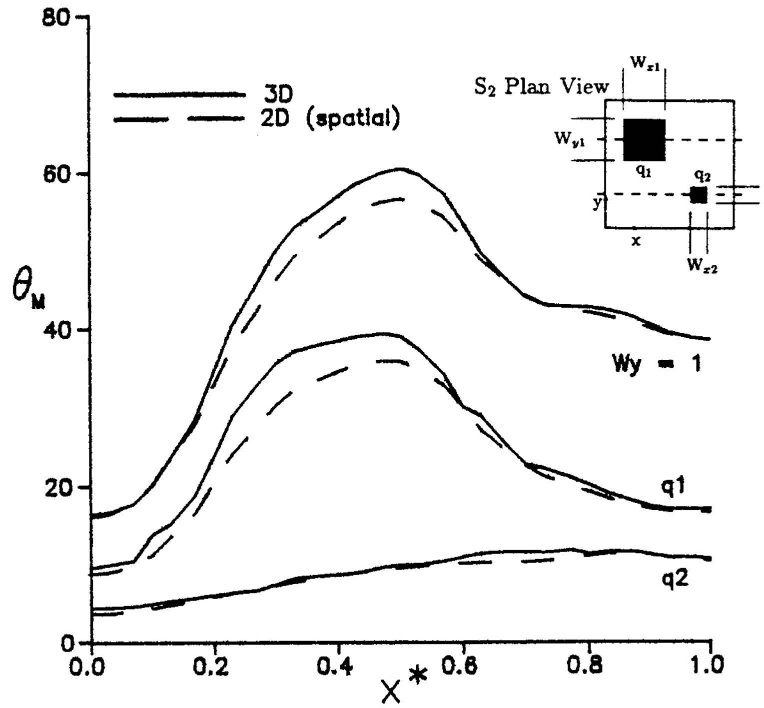

Figure 4: Effects of a Larger Upstream Heat Source

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