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General Expression for Forced Convection Heat and Mass Transfer From Isopotential Spheroids

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GENERAL EXPRESSION FOR FORCED CONVECTION

HEAT AND MASS TRANSFER FROM ISOPOTENTIAL SPHEROIDS

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Abstract

A single, semi-empirical, correlation equation for laminar forced convection heat and mass transfer from isopotential spheroids is presented. It is based on blending two correlation equations of Yuge developed for isothermal spheres in air streams, and the diffusive body length, *square root of the total body surface area*, of Yovanovich recently proposed for laminar natural convection from complex bodies. The proposed correlation equation is in very good to excellent agreement over the full range of the Reynolds number $0 \le Re_{\mathcal{L}} \le 10^5$ with several other correlation equations developed for spheres and spheroids, but over limited ranges of the Reynolds number.

Nomenclature

A	surface area of the body $[\mathcal{L}^2 \mathcal{A}]$
$\sqrt{A}$	diffusive characteristic body length of Yovanovich
AR	aspect ratio of oblate and prolate spheroids
A/P	Pasternak-Gauvin characteristic body length
$\mathcal{A}$	dimensionless surface area $[A/\mathcal{L}^2]$
a	sphere radius
$2b$	spheroid equatorial diameter $[P/\pi]$
$C_D, C_{\mathcal{L}}, C_{\sqrt{A}}$	correlation coefficients
c	concentration in the extensive fluid
c_0	uniform body concentration
c_∞	concentration remote from body
D	sphere diameter
$\mathcal{D}$	molecular diffusivity
e	eccentricity of spheroids
h	heat transfer coefficient
h_m	mass transfer coefficient
$\vec{i}$	unit vector along flow direction
$J_1(\cdot)$	Bessel function of first kind of order one
k	thermal conductivity of the extensive fluid
$\mathcal{L}$	arbitrary characteristic body length
m	Reynolds number correlation parameter
$\dot{m}$	mass flow rate
$\dot{m}^*_{\mathcal{L}}$	dimensionless mass flow rate $[\dot{m}\mathcal{L}/A(c_0 - c_\infty)\mathcal{D}]$
$Nu_{\mathcal{L}}$	Nusselt number $[Nu_{\mathcal{L}} = h\mathcal{L}/k]$
n	Prandlt number correlation parameter; outward body normal
$\tilde{n}$	dimensionless outward body normal $[n/\mathcal{L}]$
P	maximum (equatorial) body perimeter
$Pe_{\mathcal{L}}$	Peclet number $[Re_{\mathcal{L}}\, Pr]$
Pr	Prandtl number $[\nu/\alpha]$
$\tilde{p}$	dimensionless pressure
Q	heat flow rate
$Q^*_{\mathcal{L}}$	dimensionless heat flow rate $[Q\mathcal{L}/A(T_0 - T_\infty)k]$
$Re_{\mathcal{L}}$	Reynolds number $[U_\infty \mathcal{L}/\nu]$
r	spherical coordinate
S	shape factor
Sc	Schmidt number $[\nu/\mathcal{D}]$
$Sh_{\mathcal{L}}$	Sherwood number $[Sh_{\mathcal{L}} = h_m \mathcal{L}/\mathcal{D}]$
T	temperature of extensive fluid
T_0	uniform body temperature
T_∞	fluid temperature remote from the body
x, y, z	cartesian coordinates
U_∞	uniform free stream velocity
$\vec{V}$	velocity vector
$\vec{\mathcal{V}}$	dimensionless velocity vector $[\vec{V}/U_\infty]$
$Y_1(\cdot)$	Bessel function of second kind of order one

Greek Letters

α	thermal diffusivity of the extensive fluid
α_1	Drake-Backer parameter $[\sqrt{2Re_D\, Pr}]$
β	dummy variable in Eq. (17)
θ	spherical coordinate
μ_s, μ_∞	fluid viscosity at surface and free stream temperatures
ν	kinematic viscosity of the extensive fluid
ρ	mass density of the extensive fluid
ρ_∞	density of fluid remote from the body
ϕ	dimensionless temperature or concentration potential $[(T - T_\infty)/(T_0 - T_\infty)]$ or $[(c - c_\infty)/(c_0 - c_\infty)]$
∇	del operator
$\tilde{\nabla}$	dimensionless del operator $[\mathcal{L}\, \nabla]$
∇^2	Laplacian operator
$\tilde{\nabla}^2$	dimensionless Laplacian operator $[\mathcal{L}^2 \nabla^2]$

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Introduction

Steady laminar forced convection heat (Nusselt number, Nu) and mass (Sherwood number, Sh) transfer from isopotential (isothermal or isoconcentration) oblate and prolate spheroids as well as spheres into an extensive fluid such as air or water have been investigated experimentally, theoretically and numerically by numerous researchers since the turn of the century.

The primary objective of these studies is to determine the relationships between the following dimensionless groups:

$$Q^*_{\mathcal{L}} = Nu_{\mathcal{L}} = \dot{m}^*_{\mathcal{L}} = Sh_{\mathcal{L}} = \frac{1}{\mathcal{A}} \iint_{\mathcal{A}} -\left(\frac{\partial \phi}{\partial \bar{n}}\right)_0 d\mathcal{A} \qquad (1)$$

and the independent physical and thermophysical parameters: Reynolds number, and Prandtl and Schmidt numbers for heat and mass transfer respectively, in order to calculate the overall heat transfer rate Q and the mass transfer rate $\dot{m}$. The relationship given above holds for any characteristic body length, $\mathcal{L}$, and all values of the dependent and independent parameters provided the analogy between heat and mass transfer is valid.

The heat (and mass transfer) problem is formidable because it requires the solutions of the dimensionless continuity, momentum and energy (mass) equations [1]:

$$\tilde{\nabla} \cdot \vec{\mathcal{V}} = 0 \qquad (2)$$

$$\vec{\mathcal{V}} \cdot \tilde{\nabla} \vec{\mathcal{V}} = -\tilde{\nabla} \tilde{p} + \frac{1}{Re_{\mathcal{L}}} \tilde{\nabla}^2 \vec{\mathcal{V}} \qquad (3)$$

$$\vec{\mathcal{V}} \cdot \tilde{\nabla} \phi = \frac{1}{Re_{\mathcal{L}}\, Pr} \tilde{\nabla}^2 \phi \qquad (4)$$

which are subject to the following dimensionless thermal and kinematic boundary conditions:

on the body: $\phi = 1$ and $\vec{\mathcal{V}} = 0$ (5)

remote from the body: $\phi \rightarrow 0$ and $\vec{\mathcal{V}} \rightarrow \vec{i}$ (6)

The steady bulk fluid flow is assumed to be parallel to the positive x−axis.

The general dimensionless heat and mass transfer solutions

$$Nu_{\mathcal{L}} = f(Re_{\mathcal{L}}, Pr, Pe_{\mathcal{L}}, \mathcal{L}, AR) \qquad (7)$$

and

$$Sh_{\mathcal{L}} = f(Re_{\mathcal{L}}, Sc, Pe_{\mathcal{L}}, \mathcal{L}, AR) \qquad (8)$$

are not available for all values of Reynolds, Péclet, Prandtl, and Schmidt numbers, characteristic body length, and aspect ratio which is defined to be the ratio of the maximum body length parallel to the flow divided by the maximum body length perpendicular to the flow. Thus oblate spheroids with minor axes parallel to the bulk flow have aspect ratios less than unity, and prolate spheroids with major axes parallel to the bulk flow have aspect ratios greater than unity. Spheres which can be considered to be isotropic bodies have aspect ratios of unity.

Table 1: Effect of characteristic body length on $Nu^\infty_{\mathcal{L}}$ and $C_{\mathcal{L}}$ for spheroids for $0 \leq Pe_{\mathcal{L}} \leq 1$ [9]

Body Shape	AR	$\mathcal{L}$	$Nu^\infty_{\mathcal{L}}$	$C_{\mathcal{L}}$	$\mathcal{L}$	$Nu^\infty_{\mathcal{L}}$
Prolate	5	P/π	1.069	0.572	A/P	4.274
Sphere	1	D	2	0.500	A/P	2
Oblate	0.2	P/π	2.617	0.468	A/P	1.431

Figures 1 and 2 illustrate the effect of the chosen characteristic body length on the numerical results for a prolate spheroid ($AR = 5$), a sphere ($AR = 1$) and an oblate spheroid ($AR = 0.2$). In Fig. 1 the area-mean Nusselt and the Péclet numbers are both based on the equatorial diameter, P/π, of the spheroids. The plotted results show the oblate spheroid to be more conductive than the sphere which is more conductive than the prolate spheroid. The differences between these spheroids at $Pe_{\mathcal{L}} = 0$ is very large. When the same data are plotted using another characteristic body length (surface area of body divided by the maximum perimeter) the data for the prolate spheroid lie above the sphere data which are above the oblate results. The difference between the bodies appears to be quite large and the data are shifted significantly upward and to the right. Masliyah and Epstein [9] developed the expression

$$Nu_{\mathcal{L}} = Nu^\infty_{\mathcal{L}} + C_{\mathcal{L}} Pe_{\mathcal{L}} \qquad (9)$$

for the spheroidal data shown in Fig. 1 provided $Pe_{\mathcal{L}}$ is less than unity. Their results are given in Table 1.

The difference between the $Nu^\infty_{\mathcal{L}}$ values for the prolate and oblate spheroids is 145% and the difference between the $C_{\mathcal{L}}$ values is 22% when $\mathcal{L} = P/\pi$. On the other hand, when $\mathcal{L} = A/P$ is used the difference between the $Nu^\infty_{\mathcal{L}}$ values are reversed and the difference between the oblate and prolate values becomes 199%. There is no change in the correlation coefficients $C_{\mathcal{L}}$.

Analytical solutions for the limiting condition of zero Reynolds number, called the diffusive regime, ($0 \leq Re_{\mathcal{L}} \leq 10^{-4}$), have been presented for oblate spheroids ($0 \leq AR < 1$), spheres ($AR = 1$), and prolate spheroids ($AR > 1$) [1]; and it was demonstrated that

$$Nu_{\mathcal{L}} = Sh_{\mathcal{L}} = f(\mathcal{L}, AR) \qquad (10)$$

Yovanovich [1] has proposed and shown that the square root of the total surface area is the body dimension which *best* characterizes the heat and mass transfer in the diffusive regime because it *minimizes* the effect of aspect ratio on the area-average Nusselt and Sherwood numbers.

The current state of knowledge about steady forced convection heat and mass transfer from isopotential, three-dimensional, bodies of arbitrary shape into an extensive flowing fluid is somewhat incomplete. Numerous theoretical expressions, graphical correlations and empirical equations have been developed to represent the coefficients for heat and mass transfer. However, the discrepancies between the expressions proposed for correlations and the

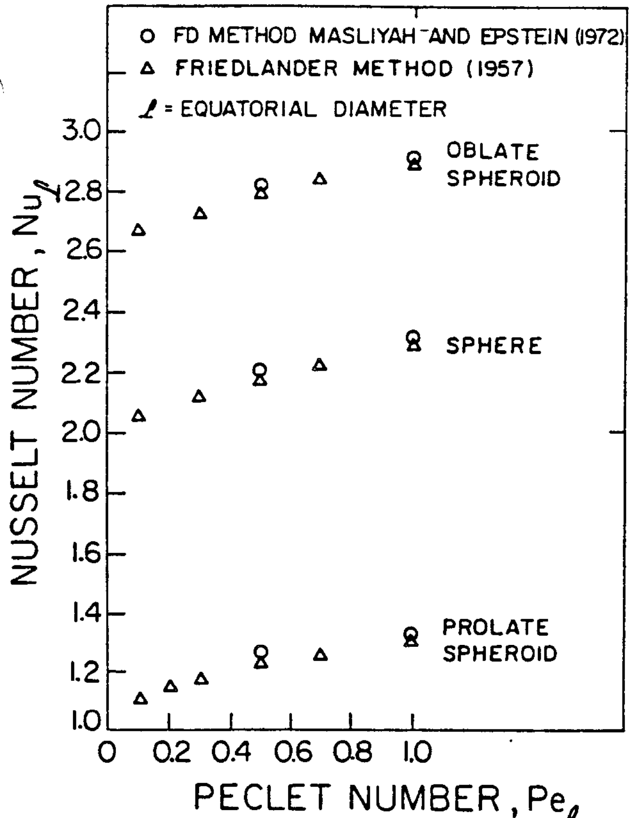

Fig. 1. Nusselt number for spheroids ($0.2 \leq AR \leq 5$) for Peclet number less than unity. Characteristic body length is the equatorial diameter.

different sets of experimental data have still not been completely resolved or explained. The theoretical results are mostly limited to the range of Reynolds number for which the postulates of laminar boundary-layer theory are applicable, i.e., ($10^3 \leq Re_{\mathcal{L}} \leq 10^5$). A completely satisfactory theory for the transition from the diffusive regime to the laminar regime ($10^{-4} \leq Re_{\mathcal{L}} \leq 10^3$), the laminar regime ($10^3 \leq Re_{\mathcal{L}} \leq 10^5$) or the turbulent regime ($Re_{\mathcal{L}} > 10^5$), is presently unavailable for bodies of arbitrary shape and aspect ratio.

Spheres for obvious reasons have received the greatest attention, followed by oblate spheroids. Prolate spheroids have been least studied.

The results of experimental, theoretical and numerical investigations have been reported as Nusselt or Sherwood numbers as functions of the independent hydrodynamic parameter (Reynolds number, $Re_{\mathcal{L}}$) and the independent property parameter (Prandtl number, Pr, for heat transfer and Schmidt number, Sc, for mass transfer). The results are frequently presented as correlation equations of the form:

$$Nu_{\mathcal{L}} = Nu_{\mathcal{L}}^{\infty} + C_{\mathcal{L}} Re_{\mathcal{L}}^{m} Pr^{n} \tag{11}$$

The first term on the right-hand side of Eq. (11) represents the contribution of the molecular limit to the Nusselt number as the Reynolds or Péclet numbers approach zero. This limiting value of the Nusselt number is a strong function of the characteristic length, $\mathcal{L}$, chosen to non-dimensionalize the results as demonstrated by Yovanovich [1-3], and verified by Hassani and Hollands [4] for laminar natural convection heat transfer from isothermal bodies of arbitrary shape into an extensive stagnant fluid.

The second term represents the effect of fluid motion to heat transfer; it consists of the product of the correlation coefficient, $C_{\mathcal{L}}$, and the Reynolds and Prandtl numbers. The correlation coefficient and the Reynolds number are also dependent on the choice of the characteristic body length. The coefficient implicitly depends on the Prandtl number and the value of the Reynolds number parameter, m. The Prandtl number ($Pr = \nu/\alpha$) is a dimensionless fluid property parameter and therefore should be independent of body shape and the characteristic length. The Prandtl number parameter, n, will dependent on the value of the Prandtl number.

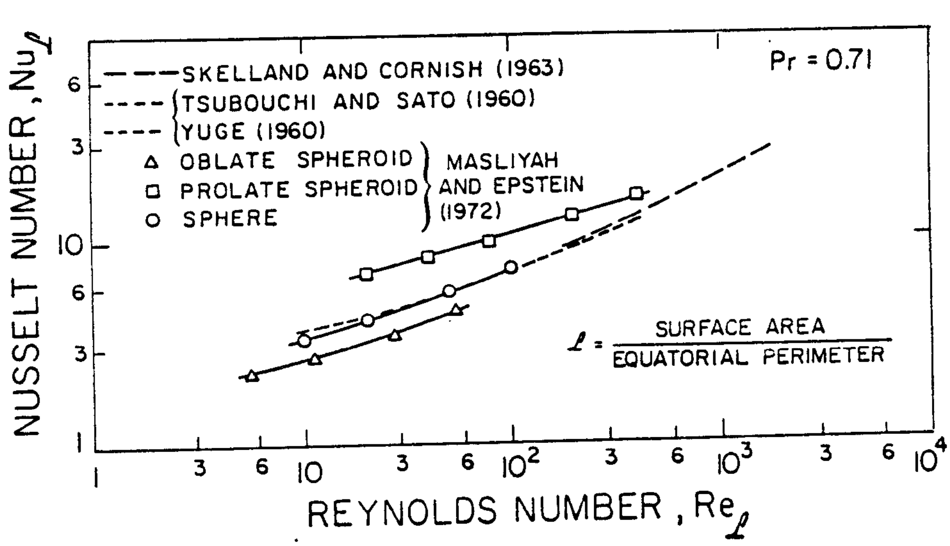

Fig. 2. Nusselt number for spheroids ($0.2 \leq AR \leq 5$) for large Reynolds number. Characteristic body length is the ratio of body area to equatorial diameter.

The primary shortcoming of the numerous empirical correlations is their failure to take into account the shape, aspect ratio of the bodies as well as using the most physically correct characteristic length in the Nusselt and Reynolds numbers.

There is a great need for a single, simple but accurate, correlation equation for steady, axisymmetric, laminar forced convection from isothermal bodies of arbitrary shape in the Reynolds number range: $0 \leq Re_{\mathcal{L}} \leq 10^5$.

The main objective of this paper is to review the pertinent theoretical, numerical and experimental works which deal with steady laminar forced convection heat and mass transfer from single, isopotential, spheroids into an extensive flowing fluid to establish a data base for the development of a single, simple, but accurate, correlation equation for area-mean Nusselt (or Sherwood) numbers for the diffusive to laminar flow regimes $0 \leq Re_{\mathcal{L}} \leq 10^5$. The numerous correlation equations which are based on different characteristic body lengths will be converted to new correlation equations based on the diffusive body length and compared to each other to determine which are most accurate over certain ranges of the Reynolds number.

A new correlation equation to be based on the characteristic body length first proposed by Yovanovich [1-3] for the diffusive regime and successfully employed in the development of simple, accurate correlation equations for natural convection from isothermal bodies of arbitrary shape, aspect ratio, and orientation will be developed and compared to the numerous correlations available for spheroids.

Heat and Mass Transfer Correlations for Spheres

Correlation equations developed for heat transfer from isothermal spheres into air streams from 1916 through 1963 are reviewed by McAdams [5] and Hsu [6] . Skelland [7] describes the correlation equations developed for heat and mass transfer from isopotential spheres and oblate spheroids; and Clift et al [8] review the theoretical, experimental and numerical results for heat and mass transfer from isopotential spheres, oblate and prolate spheroids through 1978.

According to Hsu [6] the earliest correlation equations were developed by Hughes (1916), Reiher (1925) and Lohrisch (1929) for heat transfer from isothermal spheres into air streams for which $Re_D > 10^3$. The correlation equations did not include the diffusive term and the Prandtl number, and were based on the sphere diameter, i.e., $\mathcal{L} = D$. It is interesting to note that the recent empirical data and correlation equation of Raithby and Eckert (1968) [10] given in Table 2 are in good agreement with the predictions of the correlation equations of the other authors given therein. For example, at $Re_D = 10^3$, the maximum difference of 12% occurs between the Reiher and Raithby-Eckert predicts; and at $Re_D = 10^4$ and 10^5 the maximum differences of 14 and 22% respectively occur between the Lohrisch and Hughes predicts. The other two

Table 2: Empirical correlation coefficients and parameters for spheres in air streams

Author	C_D	m	Re_D
Hughes (1916)	0.326	0.555	>1000
Reiher (1925)	0.35	0.56	>1000
Lohrisch (1929)	0.282	0.585	>1000
McAdams (1954)	0.33	0.60	20 – 150,000
Raithby-Eckert (1968)	0.257	0.588	3600 – 52,000

correlation equations predict values of Nu_D which agree to within ±5% and they lie close to the average of the values predicted by Lorisch and Hughes.

It can be seen in Table 2 that the Reynolds number correlation parameter, m, ranges from 0.555 to 0.60 and the corresponding correlation coefficient, C_D, ranges from 0.35 to 0.282. The fluid properties: thermal conductivity, k, and viscosity, μ, should be evaluated at the film temperature.

McAdams [5] correlated the air data of numerous investigators and recommended the correlation equation:

$$Nu_D = 0.33 Re_D^{0.6} \tag{12}$$

for $20 \leq Re_D \leq 1.5 \times 10^5$. Hsu [6] extended the correlation equation to include other gases by assuming $Pr = 0.74$ and the Prandtl number coefficient to be 1/3 giving:

$$Nu_D = 0.37 Re_D^{0.6} Pr^{1/3} \tag{13}$$

Beginning with the empirical and theoretical mass transfer research of Frössling [11] the heat and mass transfer correlation equations now included the diffusive term, $Nu_D^\infty = 2$, the Prandtl number with the exponent, $n = 1/3$, and the Reynolds number with the exponent, $m = 1/2$, therefore:

$$Nu_D = 2 + C_D Re_D^{1/2} Pr^{1/3} \tag{14}$$

This form of the correlation equation was derived from laminar boundary layer theory heat transfer by Kudryashev [6]. The correlation equation was studied by numerous investigators [7-9,12-22] for both heat and mass transfer from spheres. The excellent reviews of past work by Griffiths (1960) [17], Vliet and Leppert (1961) [14], Rowe et al (1965) [12], Maslyiah and Epstein (1971) [9] and Clift et al (1978) [8] should be consulted for the details of the analytical numerical and experimental work done from 1938 through 1978. Only those references which are based upon the form of Eq. (14) are considered here and are reported in Table 3 for convenience. It can be seen that the correlation coefficient, C_D, was reported to lie in the range $0.370 \leq C_D \leq 0.95$ for $0 \leq Re_D < 10^3$. For $Re_D > 10^3$ it was observed that the Reynolds number exponent should lie in the range $0.500 \leq m \leq 0.600$. For the subsequent sections of this paper which deal with corre-

Table 3: Correlation coefficients and parameters for heat and mass transfer from spheres

Author	Nu_D^∞	C_D	m	n	$Pr(Sc)$	Re_D
Frössling (1938)	2	0.55	1/2	1/3	0.6 – 2.7	2 – 1000
Kudryashev (1949)	2	0.33	1/2	0	0.71	
Drake (1952)	2	0.459	0.55	0.333	0.71	0.1 – 200,000
Ranz and Marshall (1952)	2	0.60	1/2	1/3	0.6 – 2.5	2 – 200
Tang, Duncan and Schweyer (1953)	2.1	0.42	1/2	1/3	0.71	50 – 1000
Hsu, Sato and Sage (1954)	2	0.544	1/2	1/3	1.0	50 – 350
Radusich (1956)	2.83	0.60	1/2	1/3	0.71	
Garner and Suckling (1958)	2	0.95	1/2	1/3	1200 – 1525	60 – 660
Griffiths (1960)	2	0.60	1/2	1/3	0.7	
	2	0.54	1/2	0.35	0.7	
Yuge (1960)	2	0.551	1/2	1/3	0.715	10 – 1800
	2	0.335	0.5664	1/3	0.715	1800 – 150,000
Vliet and Leppert (1961)	$1.2\ Pr^{0.3}$	0.53	0.54	0.3	2 – 380	1 – 300,000
Rowe, Claxton and Lewis (1965)	2	0.69	1/2	1/3	0.73	65 – 1750
	2	0.79	1/2	1/3	6.8	26 – 1150
Hughmark (1967)	2	0.60	1/2	1/3	< 250	1 – 450
	2	0.50	1/2	0.42	> 250	1 – 17
	2	0.40	1/2	0.42	> 250	17 – 450
	2	0.27	0.62	1/3	< 250	450 – 10,000
	2	0.175	0.62	0.42	> 250	450 – 10,000
Raithby and Eckert (1968)	2	0.235	0.606	1/3	0.71	3600 – 52,000
Masliyah and Epstein (1971)	2	0.500	1	1	0.71	0 – 1.4
Clift, Grace and Weber (1978)	1	0.757	0.47	1/3	0.70 – 0.73	100 – 4,000
	1	0.304	0.58	1/3	0.70 – 0.73	4,000 – 100,000
	1	0.724	0.48	1/3	$Pr > 188$ $Sc > 1100$	100 – 2,000
	1	0.425	0.55	1/3		2,000 – 100,000

lation equations based upon the diffusive body length, $\mathcal{L} = \sqrt{A}$, it should be noted that Eq. (11) which is based upon $\mathcal{L} = D$ can be converted to a new correlation equation in which $Nu_{\sqrt{A}}^\infty = 2\sqrt{\pi}$ and $C_{\sqrt{A}} = C_D(\sqrt{\pi})^{1-m}$ where $0.5 \le m \le 0.62$.

Sphere Correlation Equation Based on Diffusive Body Length

Before reporting the sphere correlation equations which are considered to be the most accurate over particular ranges of Reynolds number, the important analytical work of Drake and Backer (1952) [20] will be considered in its original development. They obtained an analytical solution for the energy equation which was approximated by

$$\rho c_p \frac{U_\infty}{r}\frac{\partial T}{\partial \theta} = k\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right] \qquad (15)$$

and subject to the following boundary conditions:

$$r = D/2, \quad T = T_0 \quad \text{and} \quad r \to \infty, \quad T \to T_\infty \qquad (16)$$

By means of the Laplace transform they obtained the area-average heat transfer coefficient and the Nusselt number as a function of the parameter $\alpha_1 = \sqrt{2Re_D\,Pr}$:

$$Nu_D = 2 + \frac{2}{\pi^2}\int_0^\infty \frac{(1+e^{-\pi\beta^2})(1+\beta^4)^{-1}d\beta}{[J_1^2(\alpha_1\beta) + Y_1^2(\alpha_1\beta)]\,\beta} \qquad (17)$$

where $J_1(\alpha_1\beta)$ and $Y_1(\alpha_1\beta)$ are Bessel functions of the first and second kinds of order one. Subsequently Drake [14] provided the simple correlation equation:

$$Nu_D = 2 + 0.459 Re_D^{0.55} Pr^{0.333} \qquad .1 \le Re_D \le 200{,}000 \qquad (18)$$

which agrees to within ±1% with the values predicted by the complex expression, Eq. (17). Equation (18) converts to

$$Nu_{\sqrt{A}} = 2\sqrt{\pi} + 0.594 Re_{\sqrt{A}}^{0.55} Pr^{0.333} \qquad (19)$$

for $0.2 < Re_{\sqrt{A}} < 3.5 \times 10^5$.

Yuge (1960) [22] reported two empirical equations which convert to the following expressions:

$$Nu_{\sqrt{A}} = 2\sqrt{\pi} + 0.734 Re_{\sqrt{A}}^{1/2} Pr^{1/3} \qquad 17.7 \le Re_{\sqrt{A}} \le 3{,}200 \qquad (20)$$

and

$$Nu_{\sqrt{A}} = 2\sqrt{\pi} + 0.431 Re_{\sqrt{A}}^{0.566} Pr^{1/3} \qquad 3{,}200 \le Re_{\sqrt{A}} \le 2.66\times10^5 \qquad (21)$$

Clift, Grace and Weber [8] gave two correlation equations based on data from numerous sources for heat transfer from isothermal spheres into air streams with 0.70 ≤

$Pr \leq 0.73$. Introducing the Prandtl number their correlation equations convert to the following expressions:

$$Nu_{\sqrt{A}} = \sqrt{\pi} + 1.025 Re_{\sqrt{A}}^{0.47} Pr^{1/3} \qquad 177 \leq Re_{\sqrt{A}} \leq 7,090 \tag{22}$$

and

$$Nu_{\sqrt{A}} = \sqrt{\pi} + 0.387 Re_{\sqrt{A}}^{0.58} Pr^{1/3} \qquad 7,090 \leq Re_{\sqrt{A}} \leq 1.77 \times 10^5 \tag{23}$$

Clift, Grace and Weber [8] also presented a mass transfer correlation equation which was derived from numerical data from several sources for $1 \leq Re_D \leq 400$ and $0.25 \leq Sc \leq 100$:

$$(Sh_D - 1)/Sc^{1/3} = [1 + (1/Re_D\, Sc)]^{1/3} Re_D^{0.41} \tag{24}$$

The above equation which correlates numerical data to within ±3% converts to the following heat transfer correlation equation:

$$Nu_{\sqrt{A}} = \sqrt{\pi} + 1.4 \left[1 + \frac{\sqrt{\pi}}{Re_{\sqrt{A}} Pr}\right]^{1/3} Re_{\sqrt{A}}^{0.41} Pr^{1/3} \tag{25}$$

for $1.77 \leq Re_{\sqrt{A}} \leq 709$ and $0.25 \leq Pr \leq 100$.

Whitaker (1972) [15] has proposed the following heat transfer correlation equation:

$$Nu_{\sqrt{A}} = 2\sqrt{\pi} + \left[0.533 Re_{\sqrt{A}}^{1/2} + 0.073 Re_{\sqrt{A}}^{2/3}\right] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4} \tag{26}$$

where

$$6.2 \leq Re_{\sqrt{A}} \leq 135,000$$
$$0.71 \leq Pr \leq 380$$
$$1.0 \leq \mu_\infty/\mu_s \leq 3.2$$

He claims the agreement is quite good when compared against the experimental data of Yuge [22], Kramers [16], and Vliet and Leppert [14]; the scatter of data around the correlation is ±30% at the very worst over the range of the parameters given above.

Heat Transfer Correlations for Spheroids

Skelland and Cornish (1963) [23] and Beg (1973, 1975) [24,25] measured sublimation rates of naphthalene oblate spheroids in air streams to obtain data on the effect of body shape or aspect ratio on overall mass transfer rates. The aspect ratio, AR, ranged from 1 (spheres) to 0.25 (oblate spheroids). Attempts were made to characterize the geometry of the spheroids in terms of several alternative body lengths. The correlation was observed to be best with the characteristic body length proposed by Pasternak and Gauvin (1960) [26] for all bodies, i.e., $\mathcal{L} = A/P$, where A is the total body surface and P is the maximum body perimeter normal to the flow. The diffusive body length of Yovanovich [1-3], $\mathcal{L} = \sqrt{A}$, and the Pasternak-Gauvin body length are simply related:

$$\mathcal{L}_{PG} = \frac{\sqrt{A}}{P} \mathcal{L}_Y \tag{27}$$

The dependent mass and heat transfer parameters, Sherwood and Nusselt numbers, and the independent flow parameter, Reynolds number, based on the Pasternak-Gauvin body length [26] or the diffusive body length are related in the following manner:

$$Sh_{PG} = \frac{\sqrt{A}}{P} Sh_{\sqrt{A}}, \quad Nu_{PG} = \frac{\sqrt{A}}{P} Nu_{\sqrt{A}} \tag{28}$$

and

$$Re_{PG} = \frac{\sqrt{A}}{P} Re_{\sqrt{A}} \tag{29}$$

For oblate spheroids the conversion factor is

$$\frac{\sqrt{A}}{P} = \frac{1}{\sqrt{2\pi}} \left[1 + \frac{1 - e^2}{2e} \ln \frac{1 + e}{1 - e}\right]^{1/2} \tag{30}$$

and for prolate spheroids it is

$$\frac{\sqrt{A}}{P} = \frac{1}{\sqrt{2\pi}} \left[1 + \frac{\sin^{-1} \dot{e}}{e\sqrt{1 - e^2}}\right]^{1/2} \tag{31}$$

where e is the spheroidal eccentricity.

For oblate spheroids ($AR < 1$), $e = \sqrt{1 - AR^2}$, and for prolate spheroids ($AR > 1$), $e = \sqrt{1 - AR^{-2}}$ When $AR = 1$, $\sqrt{A}/P = 1/\sqrt{\pi}$ and when $AR = 0$, $\sqrt{A}/P = 1/\sqrt{2\pi}$.

Skelland and Cornish (1963) [23] derived a single correlation equation for oblate spheroids for which $1/3 \leq AR \leq 1$. Their mass transfer equation converts to the following heat transfer equation:

$$Nu_{\sqrt{A}} = 0.985 Re_{\sqrt{A}}^{1/2} Pr^{1/3} \qquad 213 \leq Re_{\sqrt{A}} \leq 10,635 \tag{32}$$

Beg (1973, 1975) [24,25] developed two correlation equations for oblate spheroids for which $1/4 \leq AR \leq 1$. Beg did not distinquish between the sphere and oblate spheroid data, and therefore his correlation equation is valid for all aspect ratios. Beg's mass transfer correlation equations convert to the following heat transfer correlation equations when $AR = 1$ and $\sqrt{A}/P = 1/\sqrt{\pi}$:

$$Nu_{\sqrt{A}} = 0.825 Re_{\sqrt{A}}^{1/2} Pr^{1/3} \qquad 355 \leq Re_{\sqrt{A}} \leq 3545 \tag{33}$$

and

$$Nu_{\sqrt{A}} = 0.325 Re_{\sqrt{A}}^{0.61} Pr^{1/3} \qquad 3545 \leq Re_{\sqrt{A}} \leq 56,720 \tag{34}$$

Complex Mass Transfer Correlations for Spheroids

Clift et al [8] reported a complex correlation equation for mass transfer from spheroids with aspect ratios between 0.05 and 5, Reynolds numbers from 1 to 100 and Schmidt numbers between 0.7 and 2.4. They correlated the published numerical data to ±5% by means of the following equation:

$$\frac{Sh - Sh_0/2}{Sh_{sphere} - 1} = \frac{1.25}{1 + 0.25\, AR^{0.9}} \tag{35}$$

where

$$0.2 \le AR \le 5, \quad \text{and} \quad 1 \le Re_{2b} \le 100 \tag{36}$$

where both Sh and Sh_0 are based on the equatorial diameter, $2b$, of the spheroids. The reference Sherwood number, Sh_0, must be determined by means of the following relationship:

$$Sh_0 = \frac{S}{A}\mathcal{L} \tag{37}$$

where A is the total surface area, $\mathcal{L}$ is the characteristic body length, and S is the shape factor [8],

$$S = \frac{4\pi b\sqrt{1 - AR^2}}{\cos^{-1} AR} \qquad \text{oblates } (AR < 1) \tag{38}$$

and

$$S = \frac{4\pi b\sqrt{AR^2 - 1}}{\ln(AR + \sqrt{AR^2 - 1}} \qquad \text{prolates } (AR > 1) \tag{39}$$

In both shape factor expressions the equatorial diameter is $2b$ and it was chosen as the characteristic body length. The shape factor for a sphere of radius a is $4\pi a$.

For the same Reynolds number, the parameter Sh_{sphere} is obtained from Eq. (24).

A second correlation was reported by Clift et al [8] which is based on the numerical results obtained for spheroids; however, in this case the characteristic body length recommended by Pasternak and Gauvin [26], i.e., $\mathcal{L} = A/P$, is used. The alternate correlation equation has the form

$$\frac{Sh' - Sh_0'/2}{Sc^{1/3}} = \left[1 + \frac{(K')^3 - 1}{(Re')^{1/8}} + \frac{(Sh_0'/2)^3}{Re'Sc}\right]^{1/3} (Re')^{0.41} \tag{40}$$

and $1 \le Re' < 400$.

The parameter K' as shown by Sehlin [27] depends on the body shape, the aspect ratio as well as the chosen characteristic body length; it is plotted in Fig. 3 as a function of the aspect ratio, AR. The predictions of

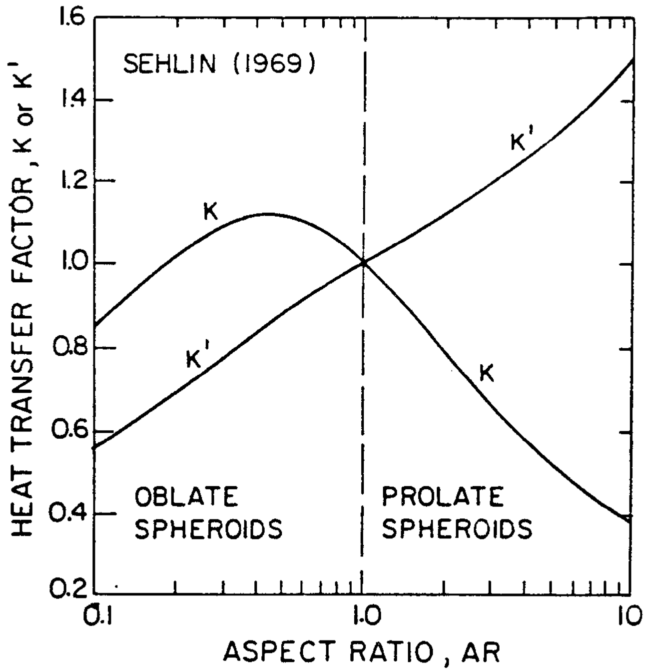

Fig. 3. Heat transfer factor of Sehlin [27] used in Eq. (40) for oblate and prolate spheroids.

Eq. (40) for dry air, $Pr = 0.71$, and spheroids having aspect ratios ranging from 0.05, a circular disk, to a slender prolate spheroid with an aspect ratio of 5 are presented in Fig. 4. It is seen that $\mathcal{L} = A/P$ gives values of $Nu_{\mathcal{L}}$ which appear to be strongly dependent on AR when the Reynolds number, also defined with the same characteristic body length, is small, i.e., $Re_{\mathcal{L}} = 1$. The dependence decreases with increasing Reynolds number as noted in Fig. 4. For details of the numerical data used in Fig. 4, the reader should consult Clift et al [8] for the pertinent references.

For convenience the various correlation coefficients developed for spheroids are reported in Table 4 for the three body lengths: equatorial perimeter (P/π), the Pasternak-Gauvin body length (A/P) and the Yovanovich diffusive body length ($\sqrt{A}$).

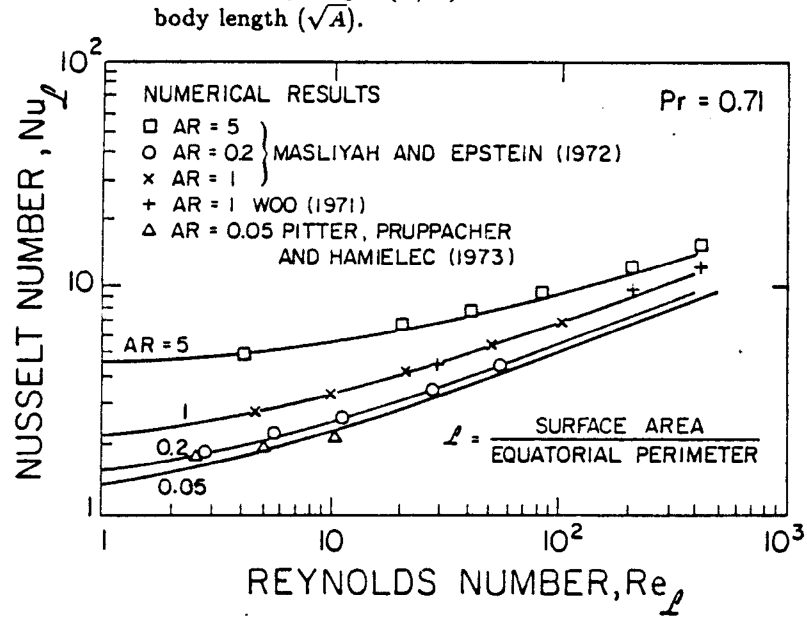

Fig. 4. Nusselt number for spheroids ($0.05 \le AR \le 5$) based on the Pasternak-Gauvin body length and $Pr = 0.71$ [8].

Table 4: Correlation coefficients and parameters for spheroids ($0.2 \leq AR \leq 5$)

Author	AR	$\mathcal{L}$	$Nu_{\mathcal{L}}^{\infty}$	$C_{\mathcal{L}}$	m	n	$Pr(Sc)$	$Re_{\mathcal{L}}$
Skelland and Cornish (1963)	1/3 – 1	A/P	0	0.74	1/2	1/3	2.4	120 – 6000
Masliyah and Epstein (1972)	0.2	A/P	1.431	0.468	1	1	0.7	0 – 1
	0.2	P/π	2.617	0.468	1	1	0.7	0 – 1
	1.0	A/P	2	0.500	1	1	0.7	0 – 1
	1.0	P/π	2	0.500	1	1	0.7	0 – 1
	5	A/P	4.274	0.571	1	1	0.7	0 – 1
	5	P/π	1.069	0.571	1	1	0.7	0 – 1
Beg (1973)	0	A/P	0	0.67	0.54	1/3	2.4	270 – 34,900
Beg (1975)	0.25 – 1	A/P	0	0.62	1/2	1/3	2.4	200 – 2,000
	0.25 – 1	A/P	0	0.26	0.6	1/3	2.4	2,000 – 32,000
Yovanovich (1987)	0.2	$\sqrt{A}$	3.430	0.468	1	1	0.7	0 – 1.41
	1.0	$\sqrt{A}$	3.545	0.500	1	1	0.7	0 – 1.41
	5.0	$\sqrt{A}$	3.791	0.572	1	1	0.7	0 – 1.41

General Correlation Equation for Spheroids

It is clear from the above review of the literature that numerous correlation equations have been proposed for steady laminar forced convection heat and mass transfer from isopotential spheroids into an extensive flowing fluid. The proposed correlation equations for spheres are based on the diameter and those proposed for oblate and prolate spheroids are based on either the equatorial diameter or the Pasternak-Gauvin characteristic body length. Skelland and Cornish [23] and Beg [24,25] have shown that the Pasternak-Gauvin body length is superior to all other body lengths, but this body length gives values of Nusselt (Sherwood) numbers for zero Reynolds number which are quite different for thin oblate spheroids and long prolate spheroids as seen in Tables 1 and 4 and Figure 4. The diffusive body length proposed by Yovanovich [1] gives values of $Nu_{\sqrt{A}}^{\infty}$ which differ by less than 10% as seen in Table 4. This body length when introduced into the two Yuge correlation equations developed for spheres in air streams yields Eqs. (20) and (21). These correlations can be *blended* into a single equation which should be accurate over the full range of Reynolds numbers. By simple trial and error analysis the following general expression for spheres which is accurate to within ±5% with Yuge's two equations is developed:

$$Nu_{\sqrt{A}} = 2\sqrt{\pi} + (0.200 Re_{\sqrt{A}}^{1/2} + 0.350 Re_{\sqrt{A}}^{0.566})\, Pr^{1/3} \quad (41)$$

which is valid for $0 \leq Re_{\sqrt{A}} \leq 2 \times 10^5$.

By simple geometric arguments the above equation can be modified to predict heat (mass) transfer from spheroids with $0 \leq AR \leq 5$:

$$Nu_{\sqrt{A}} = Nu_{\sqrt{A}}^{\infty} + (C_1 (P/\sqrt{A})^{1/2} Re_{\sqrt{A}}^{1/2} + C_2 Re_{\sqrt{A}}^{0.566})\, Pr^{1/3} \quad (42)$$

where P is the equatorial perimeter perpendicular to the bulk flow, $C_1 = 0.150$ and $C_2 = 0.350$. The diffusive limit $Nu_{\sqrt{A}}^{\infty}$ for spheroids has been presented by Yovanovich [1]. The general expression for spheres, Eq. (41), with $Pr = 0.71$ is compared with the correlation equations of Masliyah and Epstein, Hughmark, Clift et al, Raithby and Eckert, Yuge's two equations, Whitaker, and Drake in Figs. 5-7 over the full range of the Reynolds numbers. The agreement between the proposed general equation and the several correlation equations is seen to be excellent. The maximum difference between the proposed general equation and the Hughmark correlation equation occurs in the range $1 \leq Re_{\sqrt{A}} \leq 200$. Table 3 shows that Hughmark's correlation is identical to the correlation equations of Ranz and Marshall [18,19] and Griffiths [17]. The correlation equations of Frössling [11], Drake [20,14] and Clift et al [8], however, are in excellent agreement with the proposed general equation in this range of Reynolds numbers as seen in Figs 5 and 6.

Finally the general equation is compared with the correlation equations of Skelland and Cornish [23], Beg [24,25] and Pasternak and Gauvin [26] in Fig. 7. Because Skelland and Cornish, and Beg elected to correlate all spheroidal data, including sphere data, with a single equation, the effect of aspect ratios is not apparent.

The effect of aspect ratios is relatively small as seen in Table 5.

The difference between the values of the Nusselt number for spheres and prolate spheroids ($AR \leq 5$) is less than ±7% over the entire range of Reynolds number. The difference between the values of the Nusselt number for the oblate spheroids ($AR \geq 0.2$) is less than ±3% over the entire range of Reynolds number. The sphere can therefore be used to approximate heat and mass transfer from oblate and prolate spheroids for aspect ratios between 0.2 and 5 provided the diffusive body length is used in the Reynolds and Nusselt (Sherwood) numbers. The geometric parameter $(P/\sqrt{A})^{1/2}$ is also a relatively weak function of the geometry and aspect ratios. For example, this parameter takes the values 0.942, 1.331 and 1.548 for the prolate spheroid ($AR = 5$), sphere ($AR = 1$) and oblate spheroid ($AR = 0.2$) respectively. This parameter differs by approximately 40% for the aspect ratio range $0.2 \leq AR \leq 5$, which is quite large.

Table 5: Effect of aspect ratios on Nusselt numbers for spheroids ($0.2 \le AR \le 5$) and $Pr = 0.71$

AR	0.2	1.0	5
$Re_{\sqrt{A}}$	$Nu_{\sqrt{A}}$	$Nu_{\sqrt{A}}$	$Nu_{\sqrt{A}}$
0.01	3.46	3.58	3.83
0.1	3.58	3.68	3.91
1	3.95	4.03	4.23
10	5.24	5.25	5.34
10^2	9.74	9.56	9.29
10^3	25.60	24.78	23.40
10^4	81.70	78.91	73.95
10^5	280.8	272.0	255.7

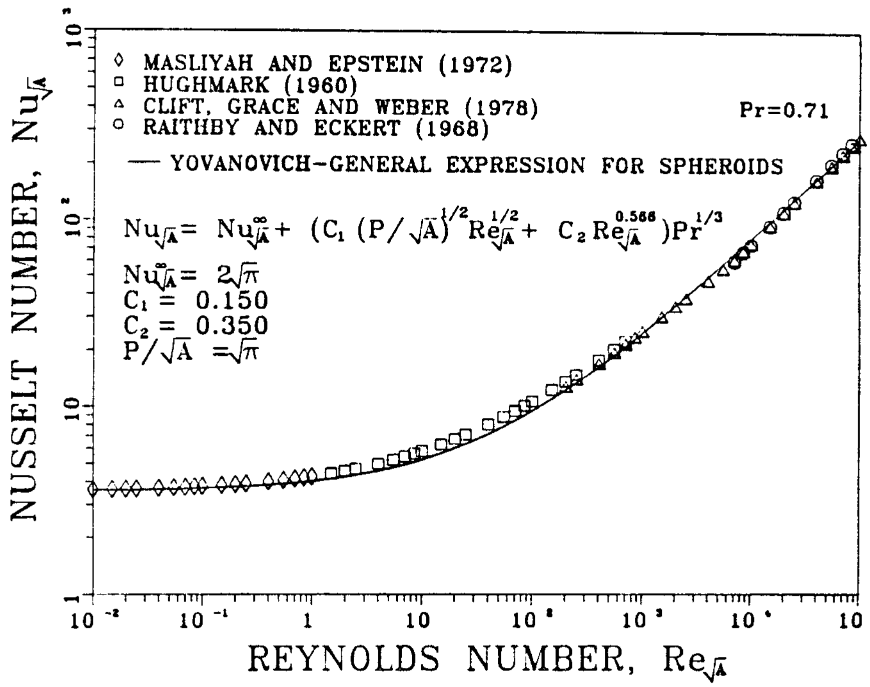

Fig. 5. Comparison of proposed correlation equation for spheres against several correlation equations for $0.01 \le Re_{\sqrt{A}} \le 10^5$ and $Pr = 0.71$.

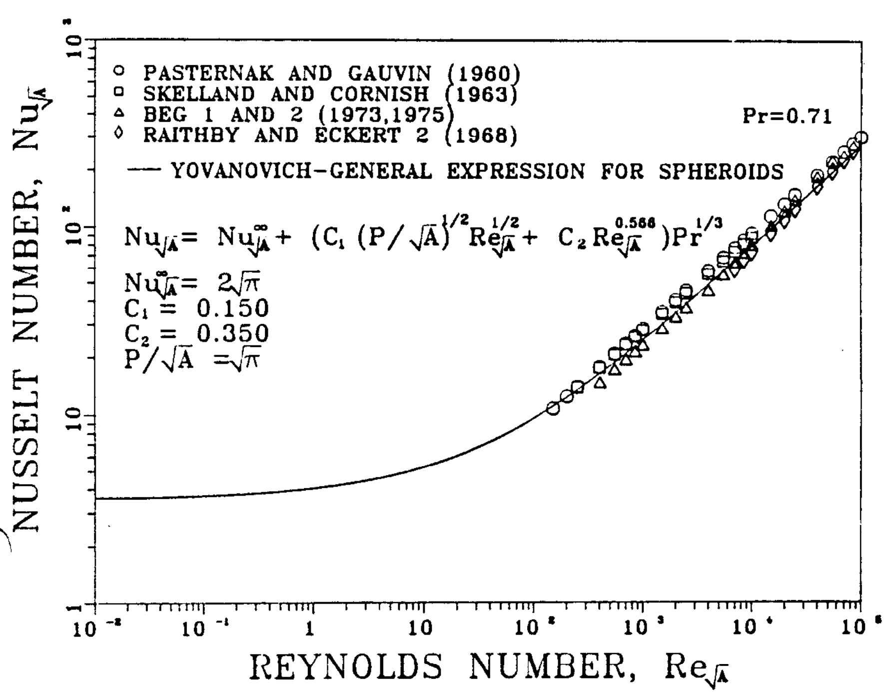

Fig. 6. Comparison of proposed correlation equation for spheres against several correlation equations for $0.01 \le Re_{\sqrt{A}} \le 10^5$ and $Pr = 0.71$.

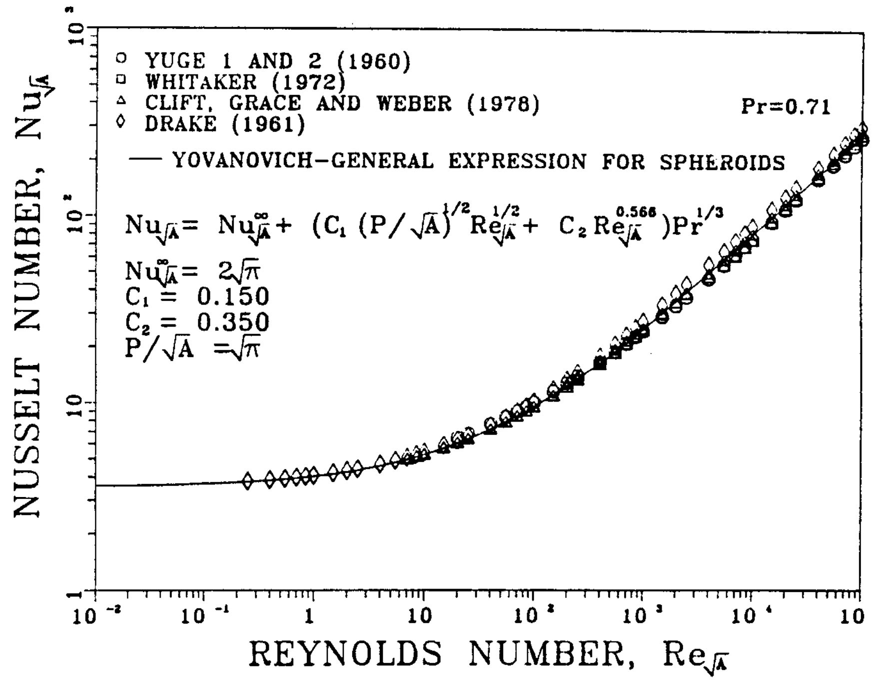

Fig. 7. Comparison of proposed correlation equation for spheres against correlation equations for spheroids for $0.01 \leq Re_{\sqrt{A}} \leq 10^5$ and $Pr = 0.71$.

Summary and Conclusions

A review of the important correlation coefficients proposed for laminar, axisymmetric, forced convection heat and mass transfer from isopotential spheroids into an extensive flowing air stream has been presented. The effect of various characteristic body lengths on the correlation equations has been discussed. It is shown that the characteristic body length of Pasternak and Gauvin which adequately correlates heat and mass transfer data at high values of the Reynolds number does not appear to be useful in the diffusive regime. On the other hand the diffusive body length proposed by Yovanovich which is the best body length for characterizing laminar natural convection heat and mass transfer is shown to be appropriate over the entire range of Reynolds number because it minimizes the effect of body aspect ratio.

A single, relatively simple, correlation equation based on the blending of the two Yuge correlation equations for isothermal spheres in air streams is in very good to excellent agreement with several other correlation equations developed for certain ranges of the Reynolds number.

The blended equation was converted to a new correlation equation based on the diffusive body length of Yovanovich.

By means of simple geometric arguments the correlation equation was modified for oblate and prolate spheroids.

The comparison between the predictions of the proposed correlation equation and those of numerous other authors is shown to be very good to excellent over the entire range of Reynolds number.

It is also shown that the difference in the area-average Nusselt number for the oblate spheroid ($AR = 0.2$) and the prolate spheroid ($AR = 5$) is less than 10% over the entire range of Reynolds number when the diffusive body length is used in the Nusselt and Reynolds numbers. The sphere data is found to lie between the oblate and prolate data. At a Reynolds number of 10 the difference between the oblate and prolate data is less than 2%.

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