AIAA'85

AIAA-85-0069

CIRCULAR AND ANNULAR CONSTRICTION RESISTANCES WITHIN A COMPOUND THERMAL SPREADER FOR COOLING MICROELECTRONIC DEVICES

H.J. Saabas, N.J. Fisher and M.M. Yovanovich,
University of Waterloo, Ontario, Canada

AIAA 23rd Aerospace Sciences Meeting

January 14-17, 1985/Reno, Nevada

CIRCULAR AND ANNULAR CONSTRICTION RESISTANCES WITHIN A COMPOUND THERMAL SPREADER FOR COOLING MICROELECTRONIC DEVICES

H.J. Saabas*, N.J. Fisher*, M.M. Yovanovich†
Thermal Engineering Group
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, N2L 3G1

Abstract

Analytical solutions of the steady-state temperature distributions as well as the one-dimensional and two-dimensional (constriction) resistances were obtained for a compound thermal spreader which has a circular heat source (device) on one face and an annular sink on the other face. The resistances were found to be functions of several thermal and geometric parameters. The results of a parametric study are given in graphical form for a range of the parameters which are of some interest to the device thermal analyst.

Nomenclature

a = device radius
$A_n^{(i)}$ = Fourier coefficients (i=1,2)
b = sink inner radius
$B_n^{(i)}$ = Fourier coefficients (i=1,2)
c = thermal spreader radius
$C_1^{(i)}, C_2^{(i)}$ = Fourier coefficients (i=1,2)
$J_o(\cdot), J_1(\cdot)$ = Bessel functions of the first kind of order zero and one
k_1, k_2 = thermal conductivities
$q_1(r)$ = device heat flux distribution
q_o = sink heat flux
Q = heat flow rate
r, z = polar coordinates
R_{1D} = one-dimensional resistance
R_c = constriction resistance
R_T = total resistance ($R_T = R_{1D} + R_c$)
R^* = dimensionless resistance ($R^* = 4k_1 aR$)
t_1, t_2, t = thickness of each component ($t = t_1 + t_2$)
T_1, T_2 = temperature distributions
$\bar{T}_{so}, \bar{T}_{si}$ = average temperatures of heat source and sink

Greek Symbols

γ = b/c
δ_n = roots of $J_1(\delta_n) = 0$
ϵ = a/c
κ = k_1/k_2
λ_n = separation constants where $J_1(\lambda_n c) = 0$
π = pi
τ_1, τ_2, τ = t_1/c, t_2/c, ($\tau = \tau_1 + \tau_2$)

Introduction

There are two primary objectives in the thermal design of microelectronic equipment. The first is to ensure, during worst case operating conditions, that all component temperatures are maintained below specified maximum functional limits. The second objective is to ensure, during nominal operating conditions, that the relatively lower component temperature reliability requirement is satisfied. Successful thermal management requires an accurate determination of the individual thermal resistances inherent in the package, including the often crucial thermal contact and constriction resistances associated with the microelectronic devices found within modern high speed computers.

Thermal Constriction (Spreading) Resistance

Heat transfer from a semiconductor junction is often modelled as a small singly-connected planar heat source of some simple shape (circular, square, rectangular, etc.) in perfect contact with a chip which is assumed to be a semi-infinite body [1,2]. Furthermore, it is assumed that the thermal energy generated by the semiconductor is released uniformly over the junction area. As shown in Fig. 1, heat flows from the junction through the chip, spreads in some complex manner and leaves the chip through some interface located at a distance which is large relative to the characteristic dimension of the junction area. This spreading of the heat flow lines gives rise to a spreading (or constriction) resistance. The maximum temperature within the junction area will occur at or near the centroid of the junction area, depending upon whether it is symmetric or nonsymmetric. The average junction temperature will always be below the centroidal temperature for all geometries [3,4].

The spreading (or constriction) resistance is defined as the temperature difference between the average junction temperature and some reference temperature, divided by the total heat flow rate from the junction. It should be noted that an

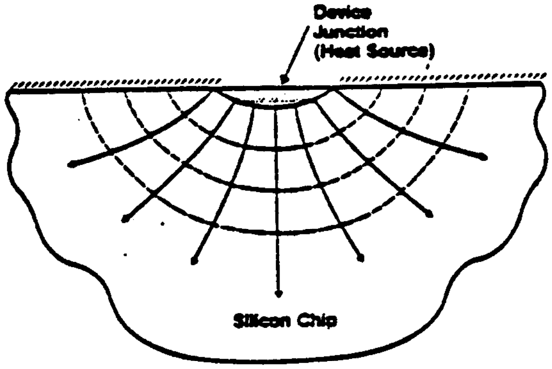

Fig. 1 Device junction on half-space

---

*Graduate Research Assistant
†Professor, Associate Fellow, AIAA

4) $z=t_1$, $o \le r \le c$, $T_1 = T_2$ (5a)

$$k_1 \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z} \quad (5b)$$

5) $z=t_1+t_2$, $o \le r < b$, $\frac{\partial T_2}{\partial z} = o$ (6a)

$b < r \le c$, $\frac{\partial T_2}{\partial z} = \frac{-q_o}{k_2}$ (6b)

with $q_o = Q/\pi(c^2-b^2)$ and Q is the total heat flow rate from the device.

By the method of separation of variables and the superposition principle, the two temperature distributions can be expressed as

$$T_1 = C_1^{(1)} + C_2^{(1)}z + \sum_{n=1}^{\infty}[A_n^{(1)}\cosh(\lambda_n z) + B_n^{(1)}\sinh(\lambda_n z)]J_o(\lambda_n r) \quad (7)$$

and

$$T_2 = C_1^{(2)} + C_2^{(2)}z + \sum_{n=1}^{\infty}[A_n^{(2)}\cosh(\lambda_n z) + B_n^{(2)}\sinh(\lambda_n z)]J_o(\lambda_n r) \quad (8)$$

where $J_o(\cdot)$ is the Bessel function of the first kind of order zero. The Bessel function of the second kind of order zero, $Y_o(\cdot)$, was eliminated to satisfy the boundedness condition along the axis $r=o$.

The separation constants λ_n appearing in both equations are the positive roots of

$$J_1(\lambda_n c) = o \quad (9)$$

where $J_1(\cdot)$ is the Bessel function of the first kind of order one.

The first two terms of Eqs. (7) and (8) represent the linear temperature distribution corresponding to $\lambda=o$. The terms under the summation sign represent the spreading or constriction of the heat flow lines due to the circular device (heat source) and the annular area (heat sink).

The temperature gradients in the z-direction will be required and are therefore given:

$$\frac{\partial T_1}{\partial z} = C_2^{(1)} + \sum_{n=1}^{\infty}\lambda_n[A_n^{(1)}\sinh(\lambda_n z)+B_n^{(1)}\cosh(\lambda_n z)]J_o(\lambda_n r) \quad (10)$$

and

$$\frac{\partial T_2}{\partial z} = C_2^{(2)} + \sum_{n=1}^{\infty}\lambda_n[A_n^{(2)}\sinh(\lambda_n z)+B_n^{(2)}\cosh(\lambda_n z)]J_o(\lambda_n r) \quad (11)$$

Since the linear and two-dimensional temperature distributions are independent, the boundary conditions can be applied separately to obtain the Fourier coefficients appearing in Eqs. (7) and (8).

For the linear temperature distribution it can be shown that $C_1^{(1)}$ is the average temperature of the surface $z=o$, i.e.,

$$\bar{T}_o = \frac{1}{\pi c^2}\int_o^c T(r,o)2\pi r dr = C_1^{(1)} \quad (12)$$

Also $C_2^{(1)}$ is determined by

$$q = -k_1\frac{\partial T_1}{\partial z} = -k_1 C_2^{(1)} = \frac{Q}{\pi c^2} \quad (13)$$

so that

$$C_2^{(1)} = -\frac{Q}{k_1\pi c^2} \quad (14)$$

The perfect contact conditions along $z=t_1$ require

$$k_1C_2^{(1)} = k_2C_2^{(2)} \quad (15)$$

so that

$$C_2^{(2)} = -\frac{Q}{k_2\pi c^2} \quad (16)$$

and

$$C_1^{(1)} + C_2^{(1)}t_1 = C_1^{(2)} + C_2^{(2)}t_1 \quad (17)$$

so that

$$C_1^{(2)} = \bar{T}_o - \frac{Qt_1}{\pi c^2}[\frac{1}{k_1} - \frac{1}{k_2}] \quad (18)$$

The linear temperature distributions are therefore

$$T_1 = \bar{T}_o - \frac{Qz}{k_1\pi c^2} \qquad o \le z \le t_1 \quad (19)$$

and

$$T_2 = \bar{T}_o - \frac{Qt_1}{\pi c^2}[\frac{1}{k_1} - \frac{1}{k_2}] - \frac{Qz}{k_2\pi c^2} \qquad t_1 \le z \le t_1+t_2 \quad (20)$$

The one-dimensional thermal resistance of the compound thermal spreader can be determined by means of the following definition:

$$QR_{1D} = \bar{T}_o - \bar{T}_t \quad (21)$$

where

$$\bar{T}_t = \frac{1}{\pi c^2}\int_o^c T(r,t)2\pi r dr$$

Therefore,

$$R_{1D} = \frac{1}{\pi c^2}[\frac{t_1}{k_1} + \frac{t_2}{k_2}] \quad (22)$$

For the two-dimensional temperature distributions the perfect contact conditions along $z=t_1$ require

$$A_n^{(1)}\cosh(\lambda_n t_1) + B_n^{(1)}\sinh(\lambda_n t_1) = A_n^{(2)}\cosh(\lambda_n t_1) + B_n^{(2)}\sinh(\lambda_n t_1) \quad (23)$$

and $k_1[A_n^{(1)}\sinh(\lambda_n t_1) + B_n^{(1)}\cosh(\lambda_n t_1)] =$

$$k_2[A_n^{(2)}\sinh(\lambda_n t_1) + B_n^{(2)}\cosh(\lambda_n t_1)] \quad (24)$$

$$^*A_n^{(1)}\cosh(\delta_n\tau_1) + {}^*B_n^{(1)}\sinh(\delta_n\tau_1) =$$

$$^*A_n^{(2)}\cosh(\delta_n\tau_1) + {}^*B_n^{(2)}\sinh(\delta_n\tau_1) \qquad (42)$$

$$\kappa[{}^*A_n^{(1)}\sinh(\delta_n\tau_1) + {}^*B_n^{(1)}\cosh(\delta_n\tau_1)] =$$

$$^*A_n^{(2)}\sinh(\delta_n\tau_1) + {}^*B_n^{(2)}\cosh(\delta_n\tau_1) \qquad (43)$$

The total constriction resistance, Eq. (36), becomes

$$R_c^* = 2\sum_{n=1}^{\infty} {}^*A_n^{(1)} \frac{J_1(\delta_n\epsilon)}{(\delta_n\epsilon)} +$$

$$\frac{2\gamma^2}{(1-\gamma^2)} \sum_{n=1}^{\infty} [{}^*A_n^{(2)}\cosh(\delta_n\tau) + {}^*B_n^{(2)}\sinh(\delta_n\tau)] \frac{J_1(\delta_n\gamma)}{(\delta_n\gamma)} \qquad (44)$$

The simultaneous solution of Eqs. (40) through (43) with Eq. (44) gives us the total dimensionless constriction which is seen to be dependent upon ϵ, γ, κ, τ_1 and τ or

$$R_c^* = R_c^*(\epsilon,\gamma,\kappa,\tau_1,\tau) \qquad (45)$$

Parametric Study and Results

As shown in the previous section the dimensionless total constriction resistance depends upon several geometric and thermal parameters. To illustrate the effect of these parameters for a typical microelectronics application without having to resort to numerous graphs, another parameter was introduced which is the ratio of the total constriction resistance for a unit disk $\tau=c=1$ to the total constriction resistance of the same unit disk of conductivity $k_2 = k_1$ and full sink area, $b=0$, thus

$$R^* = \frac{R_c^*(\epsilon,\gamma,\kappa,\tau_1,\tau=1)}{R_c^*(\epsilon,\gamma=0,\kappa=1,\tau_1=\tau)} \qquad (46)$$

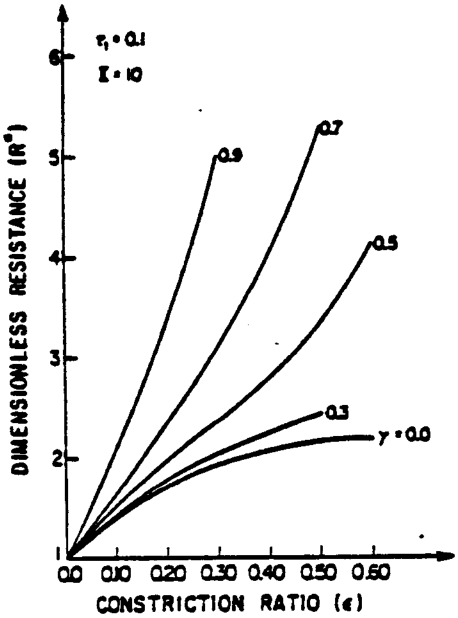

Fig. 4 Effect of several geometric parameters upon the dimensionless resistance for silicon on kovar

This parameter, called the dimensionless resistance, was computed for $\kappa=10$ which corresponds to the case of silicon in contact with kovar for an interesting range of the parameters ϵ, γ and τ_1. The results of the study are given in Figs. 4 through 8.

We observe that when $\tau_1=0.1$, R^* is considerably greater than one for all values of $\gamma \geq 0$ when $\epsilon \geq 0.3$. This is due primarily to the constriction resistance of the thermal sink and the lower conductivity of the kovar. As τ_1 increases, the dimensionless resistance R^* decreases approaching

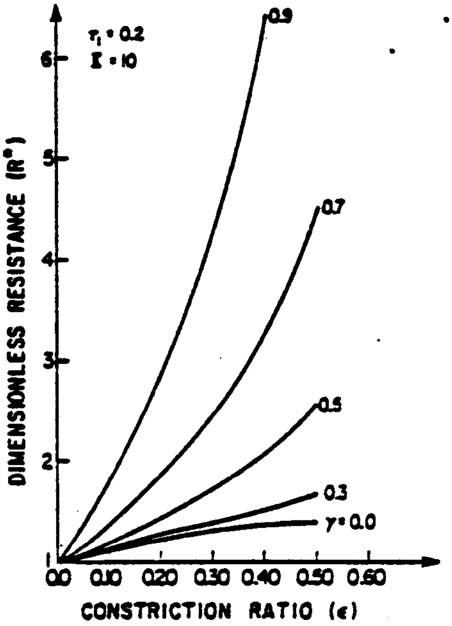

Fig. 5 Effect of several geometric parameters upon the dimensionless resistance for silicon on kovar

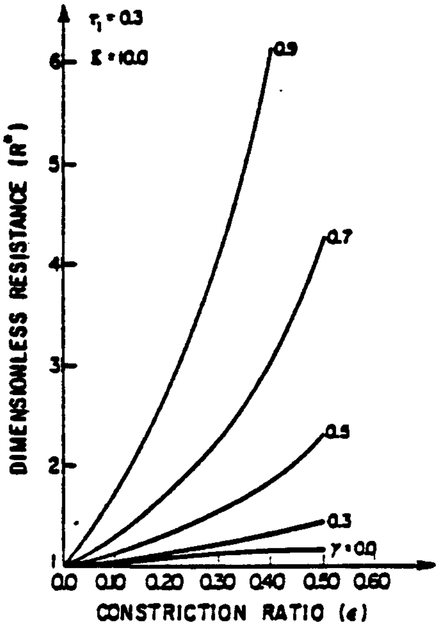

Fig. 6 Effect of several geometric parameters upon the dimensionless resistance for silicon on kovar

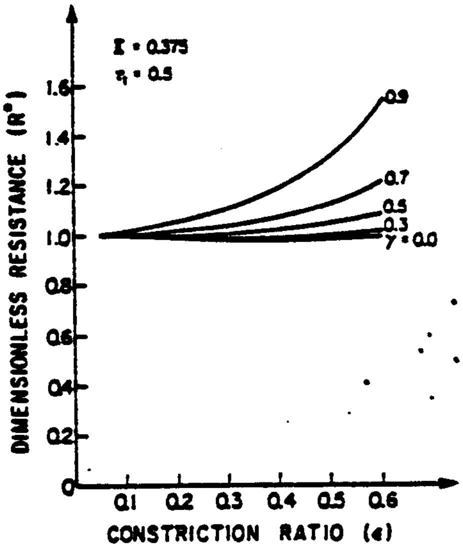

Fig. 11 Effect of copper or gold substrate upon the dimensionless resistance when $\tau_1=0.1$

Fig. 12 shows the effect of τ_1 upon R^* when $\kappa=0.375$ and $\gamma=0.9$ for a range of values of ϵ. It can be seen that when $\tau_1>0.2$, $R^*>1$ for all ϵ, but when $\tau_1<0.2$, $R^*<1$ for $\epsilon<0.5$. The greatest decrease in R^* occurs when $0.2<\epsilon<0.3$ for $\tau_1=0.1$.

The graphical results given in Figs. 4 through 12 illustrate how the analytical solution can be used to demonstrate the effect of the several geometric and thermal parameters which influence the total dimensionless constriction resistance of a compound thermal spreader.

Conclusions

An analytical solution has been presented for steady conduction through a compound thermal spreader. By means of the expression developed for the dimensionless total constriction resistance a parametric study was undertaken to demonstrate how the various parameters influence the resistance for typical ranges of the parameters such as conductivity ratio, relative device size, relative sink size and the relative thicknesses of the chip and substrate.

Acknowledgements

H.J.S. and N.J.F. acknowledge the Natural Sciences and Engineering Research Council (NSERC) of Canada for post graduate scholarships. M.M.Y. also acknowledges the financial support of NSERC under operating grant A-7455. Both H.J.S. and N.J.F. acknowledge the advice and assistance of Dr. I.J. McGee of the Applied Mathematics Department, University of Waterloo.

References

1. A.E. Bergles, R.C. Chu and J.H. Seely, "Survey of Heat Transfer Techniques Applied to Electronic Packages," Proceedings of the Technical Program, National Electronic Packaging and Production Conference, Anaheim, CA., March 1977.

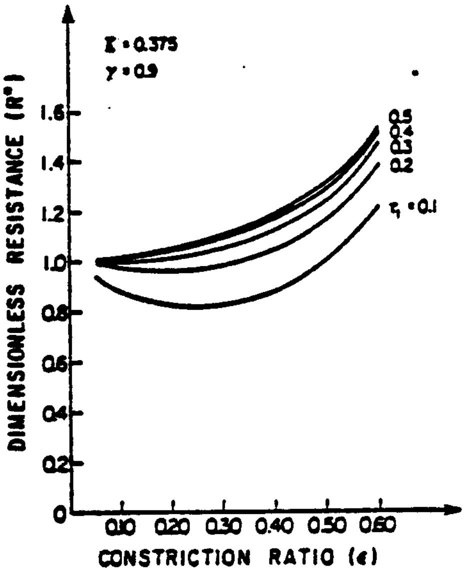

Fig. 12 Effect of sink when silicon is on a copper or gold substrate

2. A.D. Kraus, R.C. Chu and A. Bar-Cohen, "Thermal Management of Microelectronics: Past, Present, and Future," <u>Computers in Mechanical Engineering</u>, October 1982, pp. 69-79.

3. M.M. Yovanovich and S.S. Burde, "Centroidal and Area Average Resistances of Nonsymmetric, Singly Connected Contacts," <u>AIAA Journal</u>, Vol. 15, No. 10, October 1977, pp. 1523-1525.

4. M.M. Yovanovich, S.S. Burde and J.C. Thompson, "Thermal Constriction Resistance of Arbitrary Planar Contacts with Constant Flux," AIAA Progress in Astronautics and Aeronautics," <u>Thermophysics of Spacecraft and Outer Planet Entry Probes</u>, Vol. 56, edited by A.M. Smith, New York, 1977, pp. 127-139.

5. M.M. Yovanovich, "Thermal Constriction Resistance of Contacts on a Half-Space: Integral Formulation," AIAA Progress in Astronautics and Aeronautics, <u>Radiative, Transfer and Thermal Control</u>, Vol. 49, edited by A.M. Smith, New York, 1976, pp. 397-418.

6. D.P. Kennedy, "Spreading Resistance in Cylindrical Semiconductor Devices," <u>Journal of Applied Physics</u>, Vol. 31, 1960, pp. 1490-1497.

7. L.C. Roess, "Theory of Spreading Conductance," Appendix to N.D. Weills and E.A. Ryder, "Thermal Resistance Measurements on Joints Formed Between Stationary Metal Surfaces," presented at ASME Heat Transfer Meeting, Milwaukee, Wisc., 1948.

8. B.B. Mikic, "Thermal Contact Resistance," Sc.D. Thesis, Department of Mechanical Engineering, MIT, Cambridge, MA., 1966.

9. M.M. Yovanovich, "General Expressions for Circular Constriction Resistances for Arbitrary Flux Distribution," AIAA Progress in Aeronautics and Astronautics, <u>Radiative Transfer and Thermal Control</u>, Vol. 49, edited by A.M. Smith, New York, 1976, pp. 381-396.