GENERAL SOLUTION OF CONSTRICTION RESISTANCE WITHIN A COMPOUND DISK

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Abstract

A general solution for the thermal constriction resistance due to a flux applied over a circular portion of the upper surface of a compound disk is presented. The disk consists of two layers having different conductivities. Heat flow to a prescribed outer edge temperature and/or through a film coefficient over the lower surface to an ambient temperature is considered. The solution is derived with due consideration given to the compatibility required at the interface between the two materials. The results are presented in analytic form and encompass a wide variety of thermal and geometric parameters.

Nomenclature

a = contact radius
A = coefficient in series
B = coefficient in series
Bi = Biot modulus, $Bi \equiv hc/k_1$
c = disk radius
ch = hyperbolic cosine
C = coefficient in series
D = coefficient, defined in Eqs. (31) and (32)
f = function, defined in text
F = coefficient in series
h = heat transfer coefficient

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J_n = Bessel function of the first kind of order n
k = thermal conductivity
n = index in series summation
q = heat flux
Q = total heat flow rate
r = radial coordinate
R_c = constriction resistance
R_T = total resistance
s = thickness of top layer
sh = hyperbolic sine
t = total thickness of disk
th = hyperbolic tangent, th $\equiv$ sh/ch
T = temperature
u = normalized dimension, $u \equiv r^*/\varepsilon$
Y_o = Bessel function of the second kind of order zero
z = axial coordinate
α = geometric parameter, $\alpha \equiv t/c$
γ = geometric parameter, $\gamma \equiv s/c$
Δ = change in accompanying variable
ε = geometric parameter, $\varepsilon \equiv a/c$
κ = conductivity ratio, $\kappa \equiv k_1/k_2$
λ = eigenvalue
μ = exponent of flux distribution
ϕ = thermal parameter, defined by Eqs. (21) and (22)
ψ = thermal parameter, defined by Eq. (15)

Introduction

Modern aerospace technology often brings to the thermal designer new complex systems with their associated thermal problems which require novel solutions. The present paper examines the problem of determining the thermal constriction resistance due to a circular contact located on a finite, cylindrical substrate. The analysis includes consideration of substrates which are composed of two materials having different thermal conductivities. Both components of the substrate are geometrically represented as finite, concentric cylinders of equal radius, and perfect thermal contact is assumed at the interface separating the two substrate components. Solutions are presented for a wide range of boundary condition specifications, and, therefore, the analysis provides for great flexibility of application.

The thermal designer has been faced with the problem for many years,[1-3] and analytical solutions for the complete problem have not been available to provide the necessary assistance in thermal design. Applications of current relevance include the use of a high-thermal-conductivity coating to minimize thermal constriction resistance at mechanical joints as examined by Mikic,[3] prediction of the apparent thermal conductivity of aluminum-coated microspheres in microsphere superinsulations,[4,5] and the determination of the operating temperature of electronic devices mounted on thermal spreader plates.[1,2] The solutions presented in this paper resolve many of the uncertainties existing within the present state-of-the-art as it pertains to these applications.

Solutions have been presented previously for many applications of the single-component substrate problem. For the two-dimensional planar applications, these include the solutions of Oliveira and Forslund[6] and of Schneider et. al.[7] For the two-dimensional axisymmetric applications to substrates of large extent, the solutions of Yovanovich,[8] Strong et. al.,[9] and Schneider et. al.[10,11] are available. For the axisymmetric, cylindrical problem, the solutions of Kennedy[1] and of Yovanovich[12] are recommended.

The availability of solutions is greatly diminished when the complexity of a two-component substrate is added to the problem. The electrical analog to the thermal problem was examined by Simon et. al.,[2] but limited results were presented. A model for the thermal problem was proposed subsequently by Mikic and Carnasciali,[3] but a rigorous evaluation of the model was not possible because of the lack of an analytical solution, of accurate numerical predictions, and of extensive experimental results. In both cases, the extent of the cylindrical member was assumed to be infinite in the axial direction.

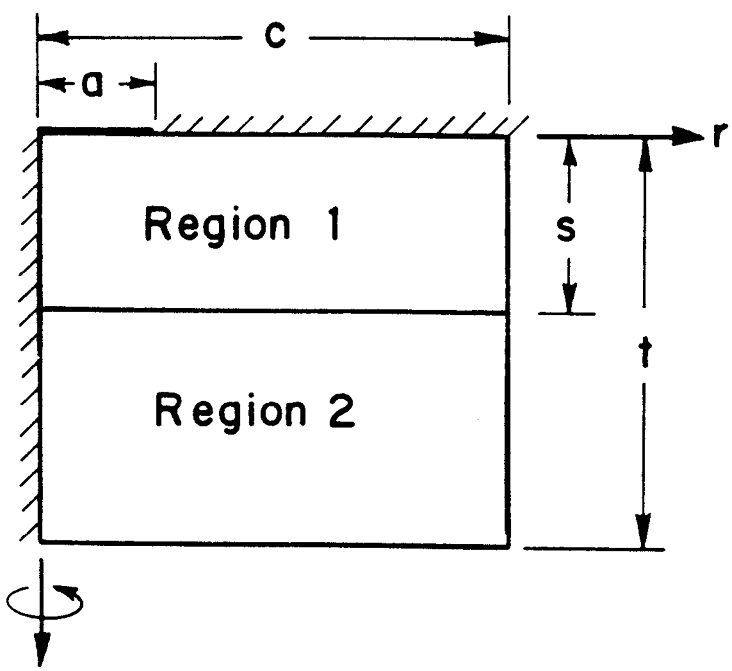

Fig. 1 Composite geometric characteristics.

In view of this discussion, it is evident that additional solutions are required, particularly for the finite, two-component substrate problem. The purpose of this paper is to obtain general solutions to this problem for several boundary condition specifications. Indeed, the axisymmetric problems which have been investigated previously then become limiting cases of the solution presented in this paper. The analysis considers the problem in which the heat flow enters the composite cylinder through a circular contact region on the end face of the cylinder and flows either radially outward to a Dirichlet-specified outside diameter or longitudinally to a Dirichlet-specified cylinder end. In the latter case, a Robin condition also can be applied as the second surface boundary condition. In all cases, the continuities of temperature and of heat flux are enforced over the interface separating the two materials of the composite substrate.

Problem Solution

Mathematical Statement of the Problem

The problem geometry for analysis purposes is the basic element illustrated in Fig. 1. Axisymmetric heat flow is considered, and a circular cylinder coordinate system is established as shown in the figure.

The thermal problem considers the steady conduction of heat within each of the two regions, denoted 1 and 2, with no heat generated within either of the materials. Over the plane defined by z = 0, the condition common to all aspects

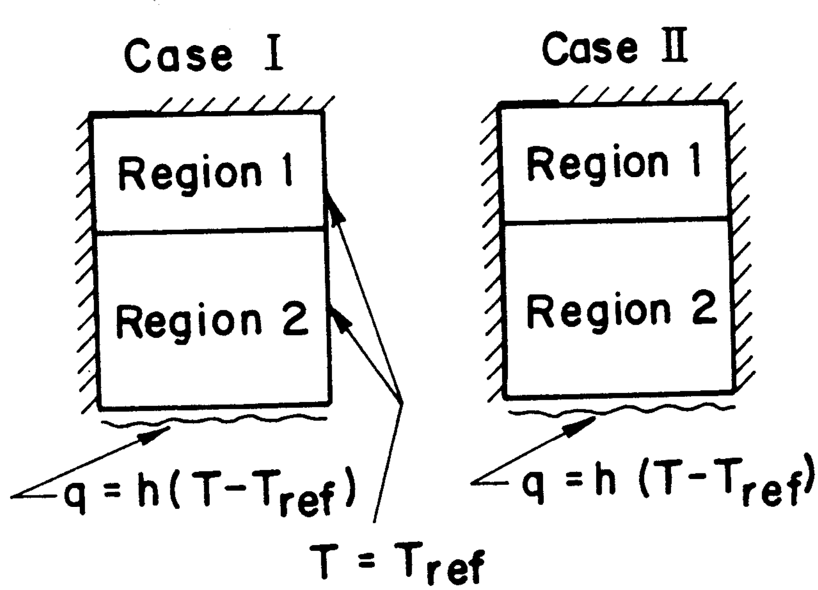

Fig. 2 Boundary condition specification.

of the analysis (over the contact an axisymmetric flux distribution will be prescribed) is that the remainder of this surface is considered to be impervious to heat transfer. Because of the axisymmetric nature of the problem, the centerline condition given at $r = 0$ will be zero temperature gradiant specification for all cases considered.

The remaining two boundaries will have their thermal descriptions given in consideration of two distinct cases. These are illustrated in Fig. 2 and are I) that for which the outer radial boundary is assigned a specified temperature, and II) that for which the outer radial boundary is insulated. The boundary condition applied to the lower surface in the axial direction will be a Robin condition relating the temperature and its gradient at this boundary. The use of this condition in two limiting cases can readily represent a temperature-specified surface and an adiabatic surface at this boundary.

The governing differential equation describing heat flow for the thermal problem described previously is Laplace's equation in two dimensions for each of the regions, respectively. These are

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_1}{\partial r}\right] + \frac{\partial^2 T_1}{\partial z^2} = 0 \quad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial T_2}{\partial r}\right] + \frac{\partial^2 T_2}{\partial z^2} = 0 \quad (2)$$

The boundary conditions common to both problem cases considered are the following:

$$z = 0;\ 0 \leqslant r < a,\ -k_1\frac{\partial T_1}{\partial z} = q(r)$$

$$a < r \leqslant c,\qquad \frac{\partial T_1}{\partial z} = 0 \quad (3a)$$

$$z = t;\ 0 \leqslant r \leqslant c,\ -k_2\frac{\partial T_2}{\partial z} = h(T_2 - \mathrm{Tref}) \quad (3b)$$

$$r = 0;\ 0 \leqslant z \leqslant t,\ \frac{\partial T_1}{\partial r} = \frac{\partial T_2}{\partial r} = 0 \quad (3c)$$

The remaining boundary condition over $r = c$ is applied for the particular case under consideration:

Case I

$$r = c;\ 0 \leq z \leq t,\ T_1 = T_2 = T\ \text{ref} \tag{4a}$$

Case II

$$r = c;\ 0 \leq z \leq t,\ \frac{\partial T_1}{\partial r} = \frac{\partial T_2}{\partial r} = 0 \tag{4b}$$

In addition to the preceding boundary conditions, the continuity of heat flux and of temperature at the interface delineating the two substrate materials must be satisfied. These are expressed mathematically as

$$z = s,\ 0 \leq r \leq c,\ T_1 = T_2 \tag{5a}$$

$$z = s,\ 0 \leq r \leq c,\ k_1 \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z} \tag{5b}$$

These equations also can be written in a nondimensionalized form. Defining a nondimensional temperature excess ratio,

$$T^*_i = (T_i - T\ \text{ref})/\Delta T_o,\quad i = 1, 2 \tag{6}$$

where ΔT_o is a reference temperature difference whose magnitude will be of no consequence in the ensuing analysis. The radial and longitudinal coordinates are normalized according to

$$r^* = r/c,\quad z^* = z/c \tag{7}$$

The governing differential equations now can be written as

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left[r^* \frac{\partial T^*_1}{\partial r^*} \right] + \frac{\partial^2 T^*_1}{\partial z^{*2}} = 0 \tag{8a}$$

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} \left[r^* \frac{\partial T^*_2}{\partial r^*} \right] + \frac{\partial^2 T^*_2}{\partial z^{*2}} = 0 \tag{8b}$$

with boundary conditions as given below:

$$z^* = 0;\ 0 \leq r^* \leq \varepsilon,\ \frac{\partial T^*_1}{\partial z^*} = -\frac{q(r)c}{\Delta T_o k_1}$$

$$\varepsilon \leq r^* \leq 1,\ \frac{\partial T^*_1}{\partial z^*} = 0 \tag{9a}$$

$$z^* = \alpha;\ 0 \leq r^* \leq 1,\ \frac{\partial T^*_2}{\partial z^*} = -\ Bi\ T^*_2 \qquad (9b)$$

$$r^* = 0;\ 0 \leq z^* \leq \alpha,\ \frac{\partial T^*_1}{\partial r^*} = 0 \qquad (9c)$$

Case I

$$r^* = 1,\ 0 \leq z^* \leq \alpha,\quad T^*_1 = T_2 = 0 \qquad (9d)$$

Case II

$$r^* = 1,\ 0 \leq z^* \leq \alpha,\ \frac{\partial T^*_1}{\partial r^*} = \frac{\partial T^*_2}{\partial r^*} = 0 \qquad (9e)$$

Finally, the compatibility conditions become

$$z^* = \gamma,\ 0 \leq r^* \leq 1,\ T^*_1 = T^*_2 \qquad (10a)$$

$$z^* = \gamma,\ 0 \leq r^* \leq 1,\ \kappa \frac{\partial T^*_1}{\partial z^*} = \frac{\partial T^*_2}{\partial z^*} \qquad (10b)$$

Analytical Solution

The general solution to the differential equations (8) is given by

$$T^*_i = (C_1 + C_2 z^*)\ (C_3 + C_4\ \ell n\ r^*)$$
$$+ \sum_{n=1}^{\infty} [A_n sh(\lambda_n z^*) + B_n ch(\lambda_n z^*)]\ [E_n J_o(\lambda_n r^*)$$
$$+ F_n Y_o(\lambda_n r^*)] \qquad (11)$$

for both materials (i = 1, 2). With the boundary conditions (9c), this solution can be reduced to

$$T^*_i = C_1 + C_2 z^* + \sum_{n=1}^{\infty} [A_n sh(\lambda_n z^*) + B_n ch(\lambda_n z^*)] J_o(\lambda_n r^*) \qquad (12)$$

since C_4 and F_n must be zero for all cases, and C_3 and E_n can be absorbed by the constants C_1 and C_2, and A_n and B_n, respectively.

Application of the conditions (9d) establishes the eigenvalues for the two specific cases considered in this paper. (In case I the constants C_1 and C_2 also are determined.) These are determined from the following relations:

Case I

$$J_o(\lambda_n) = 0, \quad C_1 = C_2 = 0 \tag{13a}$$

Case II

$$J_1(\lambda_n) = 0 \tag{13b}$$

leading to two sets of eigenvalues. In applying the solutions to be obtained below, the eigenvalues for the appropriate case must be used. It is noted that, for these two conditions, the eigenvalues obtained are independent of the particular temperature distribution, T_1 or T_2, under consideration. This would not be true, however, if a Robin condition were applied on the outer radial boundary.

Application of boundary condition (9b) to the distribution T^*_2 leads to

$$T^*_2 = C_1^{(2)} \left[1 - \frac{Biz^*}{(1 + \alpha Bi)}\right] + \sum_{n=1}^{\infty} A_n^{(2)} [sh(\lambda_n z^*) - \psi_n^{(2)} ch(\lambda_n z^*)] J_o(\lambda_n r^*) \tag{14}$$

where

$$\psi_n^{(2)} \equiv \frac{\lambda_n ch(\lambda_n \alpha) + Bi\ sh\ (\lambda_n \alpha)}{\lambda_n sh(\lambda_n \alpha) + Bi\ ch\ (\lambda_n \alpha)} \tag{15}$$

and where the superscript (2) is used to note the applicability of the superscripted quantities to the temperature distribution T^*_2 for region 2.

The coefficients in the expression for T^*_1 are related to those for T^*_2 though the compatibility requirements of Eq. (10) which must be enforced at the interface defined by $z^* = \gamma$. Applying the temperature equality relations at $z^* = \gamma$ results in the constraints

$$C_1^{(1)} + \gamma\ C_2^{(1)} = C_1^{(2)} \left\{1 - [\gamma Bi/(1 + \alpha Bi)]\right\}$$

$$sh(\lambda_n \gamma) A_n^{(1)} + ch(\lambda_n \gamma) B_n^{(1)} = A_n^{(2)} [sh(\lambda_n \gamma) - \psi_n^{(2)} ch(\lambda_n \gamma)] \tag{16}$$

Applying the continuity of heat flux requirement along $z^* = \gamma$ results in the constraints

$$\kappa C_2^{(1)} = [-C_1^{(2)} Bi/(1 + \alpha Bi)]$$

$$\kappa \, ch(\lambda_n \gamma) A_n^{(1)} + \kappa \, sh(\lambda_n \gamma) B_n^{(1)}$$

$$= A_n^{(2)} [ch(\lambda_n \gamma) - \psi_n^{(2)} sh(\lambda_n \gamma)] \quad (17)$$

Equations (16) and (17) represent a system of equations from which $C_1^{(1)}$, $C_1^{(2)}$, and $A_n^{(1)}$, $B_n^{(1)}$ can be determined in terms of $C_1^{(2)}$ and $A_n^{(2)}$, respectively. These relations are found to be

$$C_1^{(1)} = \left[1 - \left(\frac{\kappa - 1}{\kappa}\right) \frac{\gamma Bi}{(1 + \alpha Bi)}\right] C_1^{(2)} \quad (18a)$$

$$C_2^{(1)} = \left[- \left(\frac{1}{\kappa}\right) \frac{Bi}{(1 + \alpha Bi)}\right] C_1^{(2)} \quad (18b)$$

and

$$A_n^{(1)} = \Big\{ (1/\kappa) \, ch(\lambda_n \gamma) \, [ch(\lambda_n \gamma) - \psi_n^{(2)} sh(\lambda_n \gamma)]$$

$$- sh(\lambda_n \gamma) \, [sh(\lambda_n \gamma) - \psi_n^{(2)} ch(\lambda_n \gamma)] \Big\} A_n^{(2)} \quad (19a)$$

$$B_n^{(1)} = \Big\{ ch(\lambda_n \gamma) \, [sh(\lambda_n \gamma) - \psi_n^{(2)} ch(\lambda_n \gamma)]$$

$$- (1/\kappa) \, sh(\lambda_n \gamma) \, [ch(\lambda_n \gamma) - \psi_n^{(2)} sh(\lambda_n \gamma)] \Big\} A_n^{(2)} \quad (19b)$$

Using these expressions, the temperature distribution for both regions can be written in a general form as

$$T^* = C_1^{(2)} \left[1 - \frac{Bi z^*}{(1 + \alpha Bi)} - \phi_o\right]$$

$$+ \sum_{n=1}^{\infty} A_n^{(1)} \left[(1 - \phi_n) sh(\lambda_n z^*) - \psi_n^{(2)} \left(1 - \frac{th(\lambda_n \gamma)}{\psi_n^{(2)}} \phi_n\right)\right.$$

$$\left. ch(\lambda_n z^*)\right] J_o(\lambda_n r^*) \quad (20)$$

where

$$\phi_o = \left(\frac{\kappa - 1}{\kappa}\right) \frac{Bi(\gamma - z^*)}{(1 + \alpha Bi)} \; ; \quad 0 \leq z^* \leq \gamma$$

$$\phi_o = 0 \qquad ; \quad z^* > \gamma \tag{21}$$

and

$$\phi_n = \left(\frac{\kappa - 1}{\kappa}\right) ch(\lambda_n \gamma) \; [ch(\lambda_n \gamma) - \psi_n^{(2)} \, sh(\lambda_n \gamma)] ;$$

$$0 \leq z^* \leq \gamma$$

$$\phi_n = 0 ; \; z^* > \gamma \tag{22}$$

for n = 1, 2, 3, ...

The final set of constants, $C_1^{(2)}$ and $A_n^{(2)}$, must be determined through application of the boundary condition over the surface $z^* = 0$. Applying this condition, Eq. (9a), leads to the requirement that

$$\frac{(-Bi/\kappa)}{(1 + \alpha Bi)} C_1^{(2)} + \sum_{n=1}^{\infty} A_n^{(2)} \lambda_n (1 - \phi_n) \, J_o(\lambda_n r^*)$$

$$= \frac{-q(r^*)c}{\Delta T_o \, k_1} \; ; \quad 0 \leq r^* < \varepsilon$$

$$= 0 \qquad ; \quad \varepsilon < r^* \leq 1 \tag{23}$$

where $C_1^{(2)} = 0$ for case I. $C_1^{(2)}$ for case II can be determined through multiplying Eq. (23) by r^*dr^* and integrating. Noting that

$$\int_o^{\lambda_n} \lambda_n r^* \, J_o(\lambda_n r^*) d \, (\lambda_n r^*) = \lambda_n J_1(\lambda_n) = 0$$

As a result of Eq. (13) for the case II problem, the constant $C_1^{(2)}$ for this case can be determined to be

$$C_1^{(2)} = \frac{2\kappa \; (1 + \alpha Bi)}{Bi} \frac{c}{\Delta T_o k_1} \int_o^1 q(r^*) r^* dr^* \tag{24}$$

The remaining A_n are determined using the orthogonality property of Bessel functions. Multiplying Eq. (24) by $r^* J_o(\lambda_m r^*) dr^*$ and noting that

$$\int_0^1 r^* J_o(\lambda_n r^*) J_o(\lambda_m r^*) dr^*$$
$$= 0, \ \lambda_m \neq \lambda_n$$
$$= \frac{1}{2} [J_o^2(\lambda_n) + J_1^2(\lambda_n)], \ \lambda_m = \lambda_n \qquad (25)$$

for the case where the terms λ_n are roots of either $J_o(\lambda_n) = 0$ or $J_1(\lambda_n) = 0$ enables the explicit determination of each of the A_n. The expressions resulting from this procedure are given below:

$$A_n^{(2)} = \frac{-2}{\lambda_n(1 - \phi_n) f_1(\lambda_n)} \frac{c}{\Delta T_o k_1} \int_0^\varepsilon r^* q(r^*) J_o(\lambda_n r^*) \ dr^* \qquad (26)$$

where

$$f_1(\lambda_n) = J_1^2(\lambda_n) \quad \text{(case I)}$$
$$= J_o^2(\lambda_n) \quad \text{(case II)} \qquad (27)$$

Using Eq. (26) in the temperature distribution of Eq. (20) and rearranging leads to the solution of the general problem:

$$T^* = D_o \left[\frac{1}{Bi} + (\alpha - z^*) + f_2(\kappa,\gamma,z^*)\right]$$
$$+ \sum_{n=1}^{\infty} D_n \ [sh(\lambda_n z^*) + f_3 \ (n,\gamma,\alpha) ch(\lambda_n z^*)] \ J_o(\lambda_n r^*) \qquad (28)$$

where

$$f_2(\kappa,\gamma,z^*) = -\left(\frac{\kappa - 1}{\kappa}\right) (\gamma - z^*); \ 0 \leqslant z^* \leqslant \gamma$$
$$= 0 \qquad ; \ z^* > \gamma \qquad (29)$$

$$f_3(n,\gamma,\alpha) = \frac{\phi_n th(\lambda_n \gamma) - \psi_n^{(2)}}{1 - \phi_n} \qquad (30)$$

$$D_o = 0 \ \text{(case I)}$$
$$= 2\kappa \int_0^\varepsilon q^*(r^*) r^* dr^* \ \text{(case II)} \qquad (31)$$

$$D_n = \frac{-2}{\lambda_n f_1(\lambda_n)} \int_0^\varepsilon q^*(r^*) r^* \ J_o(\lambda_n r^*) dr^* \qquad (32)$$

and where

$$q^* \equiv q(r^*)/q_o; \quad \Delta T_o \equiv q_o\, c/k_1 \tag{33}$$

where q_o is the thermal flux at $r^* = 0$.

Thermal Constriction Resistance

An expression for the thermal constriction of the composite disk is derived in this section. This is performed by beginning with the definition for the total thermal resistance:

$$R_T \equiv [\bar{T}\ (\text{source}) - \bar{T}\ (\text{sink})]/Q \tag{34}$$

For use in this expression, the total heat-flow rate Q is given by

$$Q = 2\pi\, c^2\, q_o \int_o^{\varepsilon} q^*(r^*) r^* dr^* \tag{35}$$

and thermal flux distributions over the contact are considered in the form

$$q^*(r^*) = (1 - u^2)^{\mu} \tag{36}$$

where $u \equiv r^*/\varepsilon$, and where μ is a parameter.

In the flux distribution of Eq. (36), three values of the parameter μ are of special interest: $\mu = -\frac{1}{2}, 0, +\frac{1}{2}$. For the case where $\mu = -\frac{1}{2}$, the resulting distribution closely approximates an isothermal contact for values of ε in the range $0 < \varepsilon \lesssim 0.3$.[12] This distribution has a maximum value at the outer edge of the contact. The case where $\mu = 0$ corresponds to a uniform flux contact, and the case where $\mu = +\frac{1}{2}$ leads to a distribution peaking over the center of the contact and falling to zero at the contact edge.

In terms of the nondimensional temperature distribution, the total resistance can be written as

$$R_T = \frac{\Delta T_o\ \bar{T}^*(\text{source})}{2\pi\, c^2\, q_o \int_o^{\varepsilon} (1 - u^2)^{\mu}\ r^* dr^*} \tag{37}$$

From this expression, defining a nondimensional thermal resistance by $R_T^* = R_T k_1 a$ and employing the previously chosen definition for ΔT_o, the nondimensional resistance is given by

$$R_T^* = \left(\frac{\varepsilon}{2\pi}\right)\ \bar{T}^*(\text{source}) \Big/ \int_o^{\varepsilon} (1 - u^2)^{\mu}\ r^* dr^* \tag{38}$$

Evaluating $\bar{T}^*$(source), the average temperature over the contact, leads to

$$\bar{T}^*(\text{source}) = 2D_o \left[\frac{1}{Bi} + (\alpha - \gamma) + \frac{\gamma}{\kappa}\right] + \sum_{n=1}^{\infty} \frac{2D_n f_3(n,\gamma,\alpha) J_1(\lambda_n \varepsilon)}{(\lambda_n \varepsilon)} \tag{39}$$

Employing Eq. (39) in Eq. (38), the nondimensional total resistance can be expressed as

$$R_T^* = \frac{D_o[1/Bi+(\alpha-\gamma+\gamma/\kappa] + \sum_{n=1}^{\infty} D_n f_3(n,\gamma,\alpha) J_1(\lambda_n \varepsilon)/(\lambda_n \varepsilon)}{[\pi\varepsilon/(2\mu + 2)]} \tag{40}$$

where the integral in the denominator of Eq. (38) has been evaluated explicitly.

A constriction resistance now will be defined for the two cases considered in this paper. For the case in which the outer radial boundary is isothermal at T_{ref}, the constriction resistance will be identified as the total resistance, since in this case there is no single predominant direction of heat flow. For the case in which the outer radial boundary is insulated, however, the constriction resistance will be defined by

$$R_c^* \equiv R_T^* - R_{1-D}^* \tag{41}$$

where R_{1-D}^* is the one-dimensional resistance which would be realized if the entire heat-flow rate were uniformly applied over the upper surface corresponding to $z^* = 0$. This one-dimensional resistance includes the two material resistances and the film resistance and, for this case, represents the true constriction effect due to the localized thermal flux over the contact. For both cases, the nondimensional thermal resistance is given by

$$R_c^* = \frac{2(\mu + 1)}{\pi\varepsilon} \sum_{n=1}^{\infty} D_n f_3(n,\gamma,\alpha) \frac{J_1(\lambda_n \varepsilon)}{\lambda_n \varepsilon} \tag{42}$$

The evaluation of D_n for the flux distributions corresponding to $\mu = -\frac{1}{2}, 0, +\frac{1}{2}$ can be evaluated using the results of Yovanovich[12] and are presented in Table 1. It is noted here that the approximate eigenvalues for the case under consider-

Table 1 Specific evaluation of D_n

μ	D_n
$-\frac{1}{2}$	$\dfrac{-2\varepsilon\sin(\lambda_n\varepsilon)}{\lambda_n^2 f_1(\lambda_n)}$
0	$\dfrac{-2\varepsilon J_1(\lambda_n\varepsilon)}{\lambda_n^2 f_1(\lambda_n)}$
$+\frac{1}{2}$	$\dfrac{-2\varepsilon\sin(\lambda_n\varepsilon)}{\lambda_n^2 f_1(\lambda_n)}\left[\dfrac{1}{(\lambda_n\varepsilon)^2}-\dfrac{1}{(\lambda_n\varepsilon)\tan(\lambda_n\varepsilon)}\right]$

ation must be employed in the preceding expression as they are determined from Eq. (13).

Summary and Conclusions

A general expression has been developed for determining the thermal constriction resistance of circular contact areas supplying heat to compound right circular cylinders. The compound cylinder consists of two layers of material, each having a different thermal conductivity. Steady, axisymmetric heat conduction is considered.

Two specific cases have been considered explicitly in this paper. These are the cases in which the outer radial boundary is isothermal and in which the outer radial boundary is adiabatic. The nature of these two different cases is reflected through the determination of the eigenvalues appropriate for each case, as provided by Eq. (13). The coefficients for the series solution have been presented for three particular flux distributions. These flux distributions are defined by $q^* = (1 - u^2)^\mu$, where $\mu = -\frac{1}{2}$, 0, $+\frac{1}{2}$ are the three values considered.

The problem examined in this paper is a multiparameter problem including, in addition to q^* and μ, parameters Bi, κ, ε, α, γ. Because of the large number of geometric and thermal parameters, tabular or graphical expressions have not been presented in this paper. The results of interest in any particular investigation, however, may be obtained readily by programming the expression given in Eq. (42) for summation on a digital computer. The generality and flexibility of this expression render the solution obtained of great utility for use in thermal design.

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