THERMAL RESISTANCE OF HOLLOW SPHERES SUBJECTED TO ARBITRARY FLUX OVER THEIR POLES

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Abstract

An analytical study is presented for the general solution of the thermal resistance of hollow spheres with arbitrary flux over their contact areas. Resistances are presented for two specific flux distributions: uniform and approximate isothermal, for a practical range of the half-contact angle and the radii ratio. Comparisons of the numerical results with the well-known half-space and a two-zone model are made over a practical range of the parameters.

Nomenclature

A_c	= contact area
a	= inner sphere radius
b	= outer sphere radius
2c	= chord subtended by the contact area
$E(r,\varepsilon,n)$	= function defined by Eq. (23)
k	= thermal conductivity
n	= degree of Legendre and Chebyshev polynomials
P_n	= Legendre polynomial
Q	= heat flow rate
q	= heat flux
q_o	= heat flux defined by Eqs. (20) and (21)
R	= thermal resistance
R^*	= dimensionless sphere resistance ($kb\sin\alpha R$)
R_c	= constriction resistance

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R_w = wall resistance
$T(r,\theta)$ = temperature distribution
$\bar{T}$ = average temperature
T_n = Chebyshev polynomial
(r,θ) = spherical coordinates
α = contact half-angle
β = angle parameters defined by Eq. (31)
ε = radii ratio (a/b)
η_1, η_2 = oblate spheroidal coordinates, Fig. 4
μ = $\cos\theta$
ν = exponent on flux distribution, Eq. (19)

Introduction

The problem of determining the geometric, physical, and thermal parameters that influence the effective thermal conductivity of a bed of packed solid or hollow glass microspheres in a vacuum reduces, in the final analysis, to the study of the thermal resistance of a single sphere with multiple, small contact areas[1-6] that are subjected to uniform flux or isothermal boundary conditions. Chan and Tien[4] recently demonstrated that the resistances of body-centered cubic and face-centered cubic packed spheres can be related to the fundamental study of a single sphere with two opposed contact areas, as shown in Fig. 1. The analytical solution for the uniform flux condition is available,[2,4] but the analytical solution for the isothermal condition has not been obtained because of the great difficulty inherent in the mixed boundary-value problem. In Refs. 1-6, it was stated, but not proved, that the resistance of a solid sphere with two very small contact areas subjected to either uniform flux or isothermal conditions can be modeled as two constant flux or isothermal contact areas transferring or receiving heat from two insulated half-spaces thermally connected in series. Therefore, the total resistance of a solid sphere should be approximately $kcR = (16/3\pi^2)$ for the uniform flux condition and $kcR = (1/2)$ for the isothermal condition. Kaganer[2] obtained a numerical solution for the isothermal case and reported $kcR = 0.55$ when $\alpha = 11.54$ deg for the solid sphere. The question of the range of validity of the models has not been addressed.

The purpose of this paper is to obtain a general analytical solution of Laplace's equation in spherical coordinates for steady-state heat conduction through a hollow sphere with radii a, b ($a < b$) and thermal conductivity k. Heat enters and leaves the sphere by means of two small circular contact areas located at the poles and subtending the half-angle α. The inner and outer surfaces are assumed to be perfectly insulated. By means of the general solution, the two special conditions

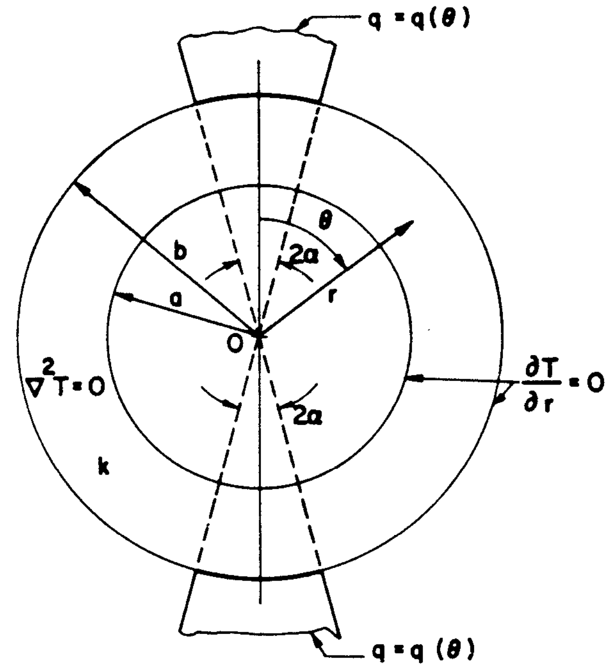

Fig. 1 Hollow sphere model.

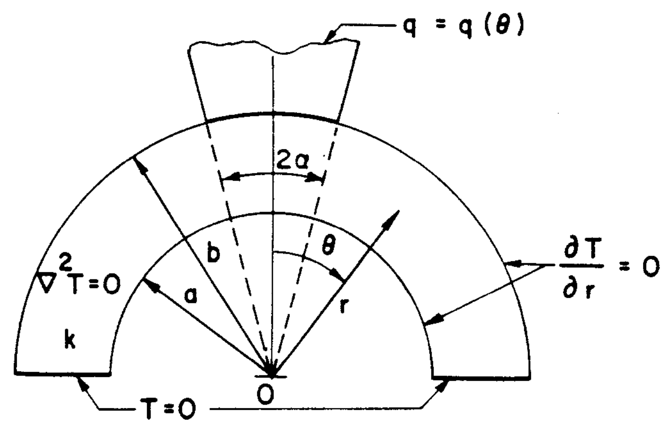

Fig. 2 Equivalent analytical model.

of a uniform flux and isothermal contact will be obtained and examined.

Problem Statement and Solution

The geometric and thermal boundary conditions are shown in Figs. 1 and 2. The governing differential equation in spherical coordinates is

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial T}{\partial \theta} = 0 \quad (1)$$

and the boundary conditions for the equivalent thermal problem depicted in Fig. 2 are

$$r = a, \; 0 \leq \theta \leq \pi/2, \; \frac{\partial T}{\partial r} = 0 \quad (2)$$

$$r = b,\ 0 \leq \theta \leq \alpha,\ q = +k\frac{\partial T}{\partial r} = q(\theta) \tag{3a}$$

$$\alpha < \theta \leq \pi/2,\ q = +k\frac{\partial T}{\partial r} = 0 \tag{3b}$$

$$\theta = 0,\ a \leq r \leq b,\ \frac{\partial T}{\partial \theta} = 0 \tag{4}$$

$$\theta = \pi/2,\ a \leq r \leq b,\ T = 0 \tag{5}$$

Because of symmetry, the appropriate boundary condition in the diametral plane $\theta = \pi/2$ is isothermal, and therefore zero temperature was chosen for mathematical convenience.

The temperature distribution within the spherical wall is of the following form:

$$T(r,\theta) = \sum_{n=0}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]P_n(\cos\theta) \tag{6}$$

where $P_n(\cos\theta)$ are Legendre polynomials of the first kind of degree n. The Legendre polynomials of the second kind, $Q_n(\cos\theta)$, are inadmissible because they become singular at $\theta = 0$ and will not satisfy the third boundary condition. Equation (6) satisfies the third boundary condition for all n = positive integers. A further restriction on the degree n is obtained by the fourth boundary condition, which requires

$$T(r,\pi/2) = \sum_{n=o}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]P_n(o) = 0 \tag{7}$$

Since Legendre polynomials of the first kind have the following property:

$$Pn(o) = 0,\ n = 1,3,5,7,\ldots,\ \text{odd integers} \tag{8a}$$

$$Pn(o) \neq 0,\ n = 2,4,6,8,\ldots,\ \text{even integers} \tag{8b}$$

we must exclude Legendre polynomials of even degree. Therefore, the solution is

$$T(r,\theta) = \sum_{n,odd}^{\infty}\left[A_n r^n + B_n r^{-(n+1)}\right]P_n(\cos\theta) \tag{9}$$

and the derivative with respect to r is

$$\frac{\partial T(r,\theta)}{\partial r} = \sum_{n,odd}^{\infty}\left[nA_n r^{n-1} - (n+1)\ B_n r^{-(n+2)}\right]P_n(\cos\theta) \tag{10}$$

The first boundary condition will be satisfied for all θ if we put

$$B_n = \frac{n}{n+1} A_n a^{2n+1} \tag{11}$$

Finally, the coefficients A_n can be determined by means of the second boundary condition. Equation (10) [using Eq. (11)] applied along $r = b$ yields

$$\sum_{n,odd}^{\infty} kb^{n-1} \left[1 - \varepsilon^{2n+1}\right] A_n P_n(\cos\theta) = q(\theta), \quad 0 \leq \theta \leq \alpha$$

$$= 0, \quad \alpha \leq \theta \leq \pi/2 \tag{12}$$

where $\varepsilon = a/b$ is the radii ratio.

The orthogonality property of Legendre polynomials can be used to obtain the general expression for the coefficients A_n. After multiplying both sides of Eq. (12) by $P_m(\mu)d\mu$, where $\mu = \cos\theta$, and integrating from $\theta = 0(\mu = 1)$ to $\theta = \pi/2(\mu = o)$, we obtain

$$A_n = \frac{1}{k} \frac{b^{1-n}}{(1-\varepsilon^{2n+1})} \frac{(2n+1)}{n} \int_{\cos\alpha}^{1} q(\theta) P_n(\mu) d\mu \tag{13}$$

having put $q(\theta) = 0$ in the range $0 \leq \mu \leq \cos\alpha$.

After substitution of Eqs. (11) and (13) into Eq. (9), we obtain the general temperature distribution within the hollow sphere valid for any flux distribution over the contact area:

$$T(r,\theta) = \frac{b}{k} \sum_{n,odd}^{\infty} \left(\frac{2n+1}{n}\right) \frac{\int_{\cos\alpha}^{1} q(\theta) P_n(\mu) d\mu}{\left[1 - \varepsilon^{2n+1}\right]} \left[\left(\frac{r}{b}\right)^n + \left(\frac{n}{n+1}\right) \varepsilon^{2n+1} \left(\frac{r}{b}\right)^{-(n+1)} \right] P_n(\mu) \tag{14}$$

Sphere Resistance

The expression for the thermal resistance of the sphere will be determined by means of the following definition:

$$QR = \bar{T}(\text{source}) - \bar{T}(\text{sink}) \tag{15}$$

where Q is the total heat flow rate through the sphere, and $\bar{T}$ is the contact area average temperature. The total resistance of the sphere is twice the resistance of the equivalent problem shown in Fig. 2. Since the sink in Fig. 2 is at zero temperature, the total resistance can be obtained from

$$QR = \frac{2}{A_c} \iint_{A_c} T(b,\theta)\, dA_c \tag{16}$$

where the elemental contact area is $dA_c = 2\pi b^2 \sin\theta\, d\theta$, and the total area is $A_c = 2\pi b^2 (1 - \cos\alpha)$. The total heat flow rate through the sphere is

$$Q = \iint_{A_c} q(\theta)\, dA_c = 2\pi b^2 \int_{\cos\alpha}^{1} q(\theta)\, d\mu \tag{17}$$

Substituting Eqs. (14) and (17) into Eq. (16) yields the following general expression for the thermal resistance of a hollow sphere valid for any flux distribution over the contact area:

$$kcR = \frac{\sin\alpha}{\pi \int_{\cos\alpha}^{1} q(\theta)\, d\mu} \times \sum_{n,\text{odd}}^{\infty} \frac{\left[1 + \frac{n}{n+1}\varepsilon^{2n+1}\right] \int_{\cos\alpha}^{1} P_n(\mu)\, d\mu \int_{\cos\alpha}^{1} q(\theta) P_n(\mu)\, d\mu}{\left[\frac{n}{2n+1}\right] [1 - \cos\alpha] \quad [1 - \varepsilon^{2n+1}]} \tag{18}$$

where $c = b \sin\alpha$ is the chord subtended by the contact area. More will be said later about the use of the dimension c to nondimensionalize the sphere resistance. Equation (18) shows that the sphere resistance is a function of the radii ratio ε, the contact half-angle α, the boundary condition, and the thermal conductivity of the sphere. To obtain an explicit relationship between the resistance and the boundary condition, a particular class of problems will be examined next.

Temperatures and Resistances for Two Specific Cases

This section will deal with the sphere temperature distributions and the corresponding sphere resistances for a class of problems corresponding to the following contact area flux distribution:

$$q = q_o(\cos\theta - \cos\alpha)^{\nu} \tag{19}$$

where q_o is some convenient heat flux level which is to be determined by the parameter ν.

Since not all possible values of ν are mathematically tractable, two special cases will be examined. These correspond to 1) $\nu = 0$, which gives $q = q_o$, uniform flux; and 2) $\nu = -\frac{1}{2}$, which gives $q = q_o(\cos\theta - \cos\alpha)^{-\frac{1}{2}}$, a flux distribution that has its minimum value at the center of the contact area $\theta = 0$. The second flux distribution has the same form as the flux distribution over an isothermal circular contact situated on the surface of an insulated half-space[7] and should, therefore, be a good approximation to the still unresolved mixed boundary-value problem for the sphere.

The heat flux levels, temperature distributions, and the sphere resistances are given below for these two cases. The heat flux levels can be shown to be

$$q_o = \frac{Q}{4\pi b^2\sqrt{1-\cos\alpha}}, \quad \nu = -\tfrac{1}{2} \tag{20}$$

$$q_o = \frac{Q}{2\pi b^2\sqrt{1-\cos\alpha}}, \quad \nu = 0 \tag{21}$$

For $\nu = 0$, corresponding to a uniform flux, we have

$$\frac{kT(r,\theta)}{bq_o} = \sum_{n,\text{odd}}^{\infty} \frac{E(r,\varepsilon,n)}{n}\left[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)\right] P_n(\mu) \tag{22}$$

for the temperature distribution, with q_o given by Eq. (21), and the function $E(r,\varepsilon,n)$ is defined as

$$E(r,\varepsilon,n) = \frac{\left(\frac{r}{b}\right)^n + \left(\frac{n}{n+1}\right)\varepsilon^{2n+1}\left(\frac{r}{b}\right)^{-(n+1)}}{1-\varepsilon^{2n+1}} \tag{23}$$

The sphere resistance for $\nu = 0$ is

$$kcR = \frac{\sin\alpha}{\pi(1-\cos\alpha)^2}\sum_{n,\text{odd}}^{\infty}\frac{E(b,\varepsilon,n)\left[P_{n-1}(\cos\alpha)-P_{n+1}(\cos\alpha)\right]^2}{n(2n+1)} \tag{24}$$

For $\nu = -\frac{1}{2}$, the approximate isothermal case, we obtain

$$\frac{kT(r,\theta)}{bq_o} = \sum_{n,odd}^{\infty} \frac{2E(r,\varepsilon,n)}{n} \frac{\left[T_n(\cos\alpha) - T_{n+1}(\cos\alpha)\right]}{\sqrt{1-\cos\alpha}} P_n(\mu) \qquad (25)$$

with q_o given by Eq. (20), and where T_n is the Chebyshev polynomial of the first kind of degree n. An alternate and useful form of these polynomials is[8,9]

$$T_n(x) = \cos(n \cos^{-1} x) = \cos n\alpha \qquad (26)$$

The sphere resistance for $\nu = -\frac{1}{2}$ is

$$kcR = \frac{\sin\alpha}{\pi[1-\cos\alpha]^2} \sum_{n,odd}^{\infty} E(b,\varepsilon,n) \times$$

$$\frac{\left[\cos n\alpha - \cos(n+1)\alpha\right]\left[P_{n-1}(\cos\alpha) - P_{n+1}(\cos\alpha)\right]}{n(2n+1)} \qquad (27)$$

Numerical Results and Discussion

Computer programs were written to evaluate numerically the expressions for the total sphere resistance. These values are reported in Table 1 as functions of the two boundary conditions, the half-contact angle, and the radii ratio. An isothermality study was conducted for the case of $\nu = -\frac{1}{2}$, examining the temperature variation with α and ε over the contact surface. In Fig. 3, we observe that α has a small effect upon the temperature distribution when $\alpha \leq 20$ deg, and the sphere is solid. The maximum difference is about 2% at $\alpha = 20$ deg, less than 1% at $\alpha = 5$ deg, and negligible when $\alpha \leq 1$ deg. Figure 4 presents the contact area temperature distribution when $\varepsilon = 0.9$, corresponding to a thin-wall sphere. It can be seen that the proximity of the inner adiabatic wall to the contact area reduces its isothermality for the $\nu = -\frac{1}{2}$ flux distribution. However, when $\alpha \leq 5$ deg, the maximum temperature difference is less than 2%. Figure 5 shows the effect of the half-contact angle upon the temperature distribution for $\varepsilon = 0.99$, which corresponds to an ultrathin-wall sphere. It can be seen that the maximum difference is about 10% for $\alpha = 10$ deg and about 3% for $\alpha = 1$ deg. It is apparent from Figs. 3-5 that the flux distribution corresponding to $\nu = -\frac{1}{2}$ is an excellent approximation to the isothermal contact area provided that $\alpha < 1$ deg and $\varepsilon \leq 0.99$. This range falls within the realm of values encountered in most applications.

Table 1 presents the numerical values of the dimensionless total sphere resistance, Eqs. (24) and (27), over the practical

Table 1 Dimensionless sphere resistance vs α and ε ($R^* = ckR$)

	α = 0.05 deg		α = 0.10 deg		α = 1.00 deg		α = 5.00 deg	
ε/ν	0	-1/2	0	-1/2	0	-1/2	0	-1/2
0	0.5411	0.5001	0.5422	0.5016	0.5534	0.5127	0.5821	0.5405
0.2	0.5412	0.5001	0.5422	0.5016	0.5536	0.5129	0.5831	0.5415
0.4	0.5412	0.5002	0.5424	0.5017	0.5551	0.5144	0.5908	0.5492
0.6	0.5415	0.5005	0.5430	0.5023	0.5610	0.5203	0.6200	0.5784
0.8	0.5429	0.5019	0.5458	0.5051	0.5890	0.5484	0.7577	0.7157
0.9	0.5468	0.5058	0.5536	0.5129	0.6668	0.6260	1.1263	1.0813
0.92	0.5491	0.5081	0.5581	0.5174	0.7118	0.6710	1.3318	1.2842
0.94	0.5532	0.5121	0.5662	0.5256	0.7927	0.7519	1.6896	1.6364
0.96	0.5622	0.5211	0.5842	0.5436	0.9702	0.9290	2.4339	2.3662
0.98	0.5932	0.5522	0.6464	0.6056	1.5693	1.5248	4.7372	4.6171
0.99	0.6651	0.6241	0.7899	0.7491	2.8788	2.8204	9.4051	9.1720

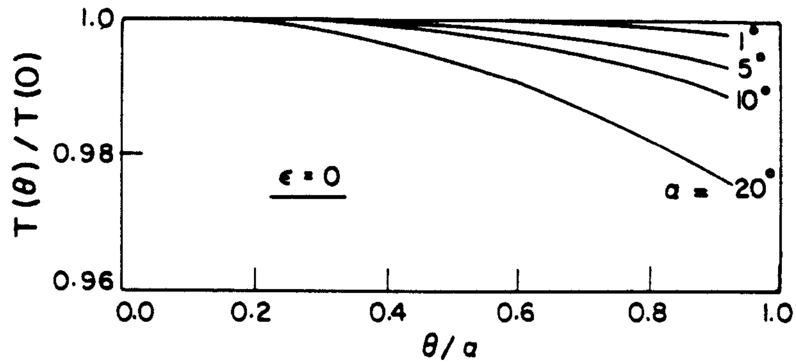

Fig. 3 Solid sphere contact temperature distribution.

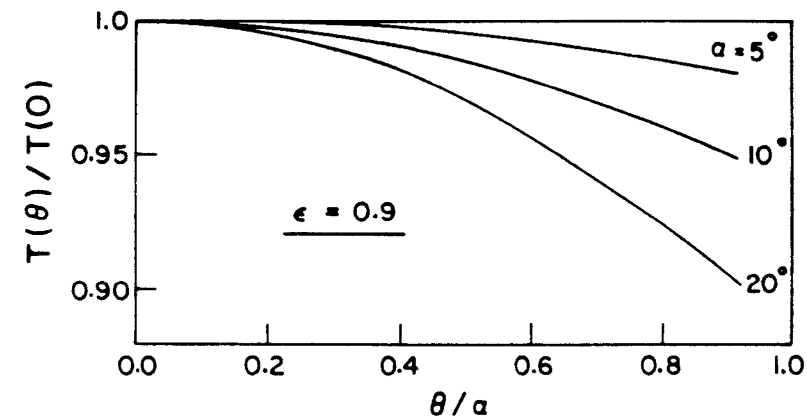

Fig. 4 Hollow sphere contact temperature distribution.

range of α and ε. We observe that, when $\varepsilon = 0$ and $\alpha = 0.05$ deg, $R^* = 0.5411$ (0.5001), corresponding to $\nu = 0$ ($-\frac{1}{2}$). This value differs by only 0.13% (0.02%) from the half-space solution. The effect of the inner spherical wall is of negligible importance for $\varepsilon \lesssim 0.9$, where the resistance has increased 1.18% (1.16%) over the solid sphere resistance. At $\varepsilon > 0.9$, the effect of the spherical wall is significant, and, when $\varepsilon = 0.99$, the total resistance of a hollow sphere is 22.9% (24.8%) larger than that of a solid sphere.

It also can be seen from Table 1 that, when $\varepsilon = o$ and $\alpha \lesssim 1$ deg, the total sphere resistance is very close to the half-space solutions. When $\alpha = 1$ deg, the sphere resistance is 2.41% (2.54%) greater than the corresponding half-space solution. It should be noted, however, that, when $\alpha \lesssim 1$ deg and $\varepsilon > 0.9$, the wall resistance is equal to or much greater than the constriction resistance. For example, when $\alpha = 1$ deg and $\varepsilon = 0.99$, the total sphere resistance is 5.33 (5.64) times the corresponding half-space solution.

Normalization of the Sphere Resistance

During the early stages of this investigation, it was observed that the normalization of the total sphere resistance with respect to the outer sphere radius b gave values that ranged monotonically from about 621.3 at $\alpha = 0.05$ deg to about 6.7 at $\alpha = 5$ deg when $\varepsilon = 0$ and $\nu = 0$. Multiplying the dimensionless resistance by $\sin\alpha$ yielded new values, which now ranged between 0.5411 and 0.5821 for the same range of α. Therefore, $b\sin\alpha = c$ is a more characteristic dimension for the solid and hollow spheres, as seen in Table 1.

It should be noted that $b\sin\alpha$ is equal to $1/\sqrt{\pi}$ times the square root of the projected contact area. The projected and the actual contact areas are related closely for small values of α. The square root of the ratio of the projected area to the actual area differs from unity by only 1.5% at $\alpha = 20$ deg, and the difference is negligible when $\alpha \leq 10$ deg. The importance of the square root of the contact area has been demonstrated for arbitrary planar contacts on a half-space.[10,11]

Comparison of the Numerical Results with a Two-Zone Model

A simple two-zone model has been developed[6] for the prediction of the total thermal resistance of hollow spheres. This model assumes that the sphere can be reduced to two constriction zones associated with the contact areas plus one wall zone, as depicted in Fig. 6. Each constriction zone is bounded by the contact area denoted by η_1 and an imaginary boundary denoted by η_2, shown in Fig. 7. This zone is used to account for the constriction resistance. The wall zone is bounded by the inner and outer adiabatic boundaries and the surfaces denoted by $\theta = \beta$ and $\theta = (\pi - \beta)$. The shaded region shown in Fig. 7 is not taken into account in this model, and, therefore,

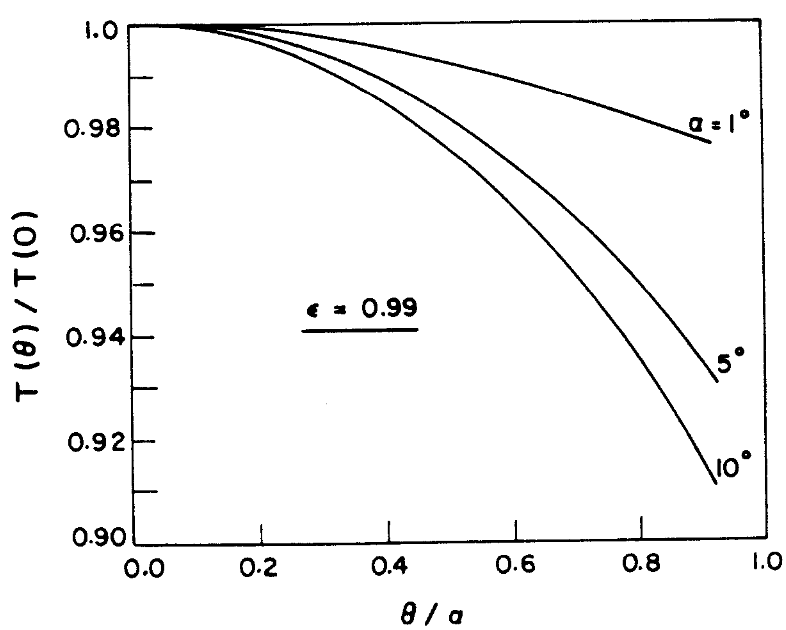

Fig. 5 Ultrathin wall contact temperature distribution.

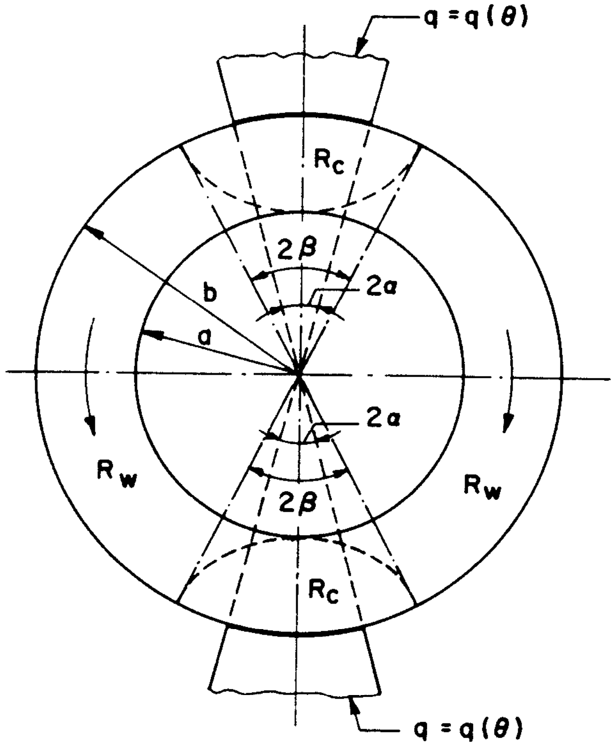

Fig. 6 Two-zone model.

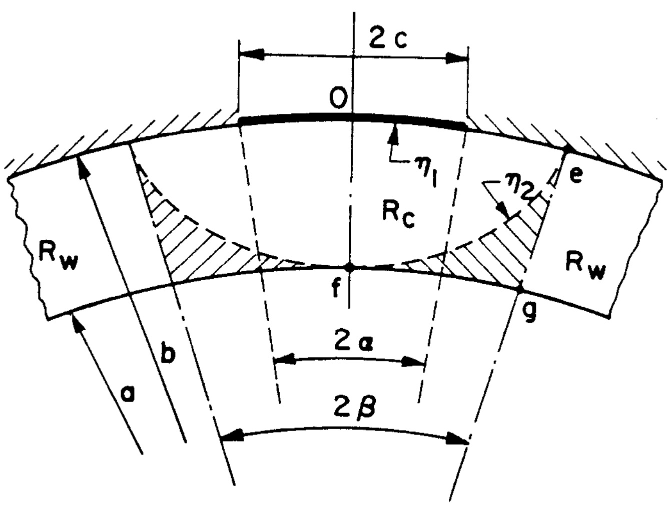

Fig. 7 Constriction zone.

the total resistance predicted by it will be slightly less than the actual resistance when α is small.

The total sphere resistance is equal to approximately twice the constriction resistance plus the wall resistance, i.e.,

$$R = 2R_c + R_w \tag{28}$$

Oblate spheroidal coordinates were employed to obtain the constriction resistance, assuming that the contact areas are isothermal. The model yields

$$2kb\sin\alpha\, R_c = (1/\pi)\arctan[(1-\varepsilon)/\sin\alpha] \tag{29}$$

using the dimensions $\overline{fo} = b(1-\varepsilon)$ and $c = b \sin\alpha$ shown in Fig. 7.

The wall resistance is based upon the assumption that the planes $\theta = \beta$ and $\theta = (\pi - \beta)$ are isothermal and is given by

$$kb\sin\alpha\, R_w = [\sin\alpha/\pi(1-\varepsilon)]\, \ell n[1/\tan(\beta/2)] \tag{30}$$

The angle β is obtained from the geometry

$$\sin\beta = [\sin^2\alpha + (1-\varepsilon)^2]^{\frac{1}{2}} \tag{31}$$

and is seen to be related to the half-contact angle and the radii ratio or wall thickness.

Equation (28) was used to compute the values of the total sphere resistance given in Tables 2 and 3. It is evident from Tables 2 and 3 that the values of R^* predicted by the two-zone model are in good agreement with those predicted by Eq. (27) up to $\alpha = 1$ deg and $\varepsilon = 0.99$. The agreement improves as α and ε decrease. It can be seen in Table 2 that the constriction resistance is the dominant resistance up to $\alpha = 0.1$ deg; at $\alpha = 1$ deg it accounts for about 73% of the total resistance. On the other hand, when $\varepsilon = 0.99$, the constriction accounts for 76, 60, and 6% of the total resistance at α =

Table 2 Comparison of Eqs. (27) and (28) at $\varepsilon = 0.9$

	α=0.05 deg	α=0.10 deg	α=1.00 deg
β, deg	5.74	5.74	5.83
$2R_c^*$	0.4972	0.4944	0.4450
R_w^*	0.0083	0.0166	0.1654
R^*[Eq. (28)]	0.5055	0.5110	0.6104
R^*[Eq. (27)]	0.5058	0.5129	0.6260
% diff.	-0.05	-0.37	-2.48

Table 3 Comparison of Eqs. (27) and (28) at $\varepsilon = 0.99$

	α=0.05 deg	α=0.10 deg	α=1.00 deg
β, deg	0.58	0.58	1.15
$2R_c^*$	0.4723	0.4450	0.1656
R_w^*	0.1471	0.2935	0.5551
R^*[Eq. (28)]	0.6194	0.7385	2.7207
R^*[Eq. (27)]	0.6241	0.7491	2.8204
% diff.	-0.75	-1.42	-3.54

0.05, 0.1, and 1 deg, respectively. Since the two-zone model yields a simple, accurate expression that shows the relative importance of the constriction and wall resistances, it is recommended that it be used to predict the resistance of packed spheres.

Conclusions

A general solution has been presented to predict the total resistance of a single sphere subject to arbitrary flux distributions. Two specific flux distributions have been examined; expressions for the temperature distribution and thermal resistances are presented, and tabulated values of the resistances are presented, over the practical range of interest of the parameters. The normalized results were compared with the well-known half-space solutions, as well as the two-zone model proposed by Yovanovich.

Acknowledgment

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