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GENERAL EXPRESSIONS FOR CONSTRICTION RESISTANCES DUE TO ARBITRARY FLUX DISTRIBUTIONS AT NON-SYMMETRIC, COAXIAL CONTACTS

by
M. MICHAEL YOVANOVICH
University of Waterloo
Waterloo, Ontario, Canada

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GENERAL EXPRESSIONS FOR CONSTRICTION RESISTANCES DUE TO ARBITRARY FLUX DISTRIBUTIONS AT NON-SYMMETRIC, COAXIAL CONTACTS

M. Michael Yovanovich*

Thermal Engineering Group
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada

Abstract

A theoretical analysis is presented for obtaining a general expression for predicting thermal constriction resistances of coaxial cylindrical contacts as a function of arbitrary contact area heat flux distributions. Another general expression is developed for heat flux distributions proportional to $(1 - u^2)^\mu$ where $\mu > -1$. Three special cases: $\mu = -1/2$, o and 1/2 are considered, and the results are compared with those of Roess and Mikic. Approximate expressions are given for evaluating the constriction resistance for the three special cases when the contact areas are relatively small.

Nomenclature

- a = contact area radius
- b = heat flux tube radius
- C_o = average contact plane temperature, Eq. (12)
- C_n = coefficients in temperature distributions, Eq. (22)
- J_ν = Bessel function of the first kind of order ν
- K = scaling parameter, Eq. (4)
- k = thermal conductivity
- ℓ = distance from the contact plane
- Q = total heat flow rate
- q_o = heat flux $(Q/\pi b^2)$
- R = thermal resistance
- R_c = constriction resistance
- R_c^* = dimensionless constriction resistance $(4a\ k\ R_c)$
- R_m = heat flux tube resistance, Eq. (15)
- r = radial coordinates
- r_i = Roess coefficients, Eq. (38), $(i = 1, 3, 5, \ldots)$
- T = temperature
- $\bar{T}$ = average temperature, Eq. (10)
- T_c = average contact area temperature, Eq. (13)
- u = dimensionless radial position (r/a)
- z = axial coordinate

Greek Symbols

- α = coefficient, Eq. (36)
- β = coefficient, Eq. (36)
- Γ = gamma function, Eq. (28)
- ε = radii ratio (a/b)
- λ_n = separation constants, Eq. (9)
- π = pi
- ψ = constriction parameter, Eq. (28)

Subscripts

- c = constriction
- m = material
- t = total

Superscript

- γ = exponent, Eq. (37)
- μ = exponent on flux distribution, Eq. (7)

Introduction

The problem of thermal constriction resistance of symmetric, coaxial cylindrical contacts, Fig. 1a, has been examined by a number of investigators[1,2]. It can be demonstrated by means of symmetry arguments that the boundary condition over the contact area shown in Fig. 1a must be isothermal. Roess[1] developed an approximate infinite series expression of the constriction resistance for the isothermal case by seeking a solution for the equivalent case of a heat flux distribution which varies as $(1 - u^2)^{-1/2}$ over the contact area, but is zero outside the contact. Mikic[2] also obtained another approximate infinite series expression for the isothermal case by assuming the same heat flux distribution, as well as an infinite series expression for the case of uniform heat flux over the contact area. These solutions are not valid for the often encountered case of non-symmetric, coaxial contacts shown in Figs. Ib and lc.

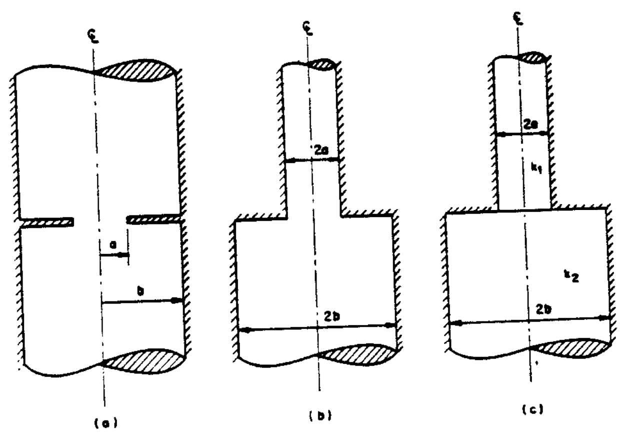

Fig. 1 Typical symmetric and non-symmetric contacts

*Professor, Member AIAA

Figure 1b shows a system consisting of one circular cylinder (πa^2) in perfect contact with a larger circular cylinder (πb^2). The thermal conductivity of both cylinders is the same. There will be a constriction resistance associated with the change in the heat flow area. The following simple thought experiment will show that the solutions for the boundary conditions considered by Mikic[2] do not apply in this case.

It is obvious that steady, linear heat flow in the smaller cylinder results in a uniform temperature distribution independent of the radius, and a uniform heat flux over the cross section of the cylinder. For the larger cylinder, on the other hand, a uniform heat flux over the contact area corresponds to a contact area temperature which is non-uniform. In fact, this temperature is maximum at the center and decreases to the edge. Thus a uniform heat flux will not be compatible for the smaller and larger cylinders. A uniform temperature over the contact area is the result of a heat flux distribution which is minimum at the center and increases to the edge. Thus a uniform temperature over the contact area will not be compatible for the smaller and larger cylinders. It is clear from these arguments that the actual heat flux over the contact area lies between q = constant and $q \alpha (1 - u^2)^{-1/2}$. Furthermore the temperature of the contact area must be non-uniform. For this reason we must seek solutions which are different from those developed by Mikic[2].

Figure 1c shows a smaller cylinder of one material in elastic contact with a larger cylinder of another substance. If the end of the smaller cylinder is curved, the contact pressure will not be uniform. It will be a maximum at the center, decreasing to the edge of the contact area. Under this condition the microscopic constriction resistance due to surface roughness will be a minimum at the center and a maximum at the edge of the contact. Thus the heat flux for a constant temperature drop will be inversely proportional to the microscopic resistance. For the larger cylinder, the macroscopic constriction resistance will have to be determined for a heat flux distribution which is maximum at the center and minimum or zero at the edge. This is another reason for seeking a general expression which predicts the thermal constriction resistance of cylindrical coaxial contacts as a function of the prescribed heat flux over the contact area. This paper deals with such an analysis.

Problem Statement and Solutions

Consider the steady flow of heat Q through a long right circular heat flux tube of radius b having uniform thermal conductivity k, Fig. 2. All

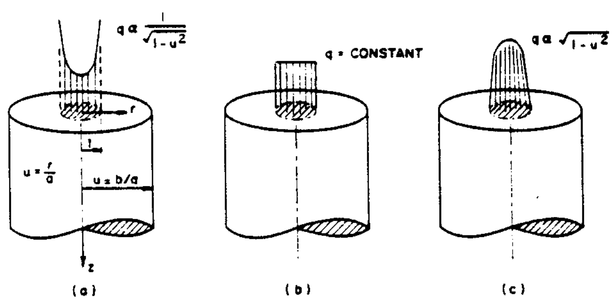

Fig. 2 Contact area flux distributions

the heat flowing through the tube enters it through a circular contact area of radius $a (a < b)$ located on the centerline of the heat flux tube. The portion of the end of the tube located outside the contact area is taken to be perfectly insulated. The lateral boundary of the tube is also assumed to be impervious to heat transfer. At distances far from the contact area the heat flows uniformly and the temperature gradient along the tube length is constant.

We wish to determine the temperature distributions and the corresponding thermal constriction resistances of the heat flux tube described in the previous section as a function of the prescribed heat flux distributions over the contact area. All other boundary conditions are unchanged.

Circular cylinder coordinates will be employed in the solution of the thermal problem and the origin of the coordinates (r,z) will be placed in the center of the contact area. Since the problem as stated is axially symmetric, the temperature will depend upon the radial and axial positions only.

The solutions we seek must satisfy Laplace's equation:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (1)$$

as well as the following boundary conditions:

$$r = 0 \quad z \geq 0 \quad T \text{ is finite} \qquad (2)$$

$$r = b \quad z \geq 0 \quad \frac{\partial T}{\partial r} = 0 \qquad (3)$$

$$z = 0 \quad 0 \leq r < a \quad \frac{\partial T}{\partial z} = -K \left(\frac{b}{a}\right)^2 \frac{q_o}{k} f(u) \qquad (4)$$

$$z = 0 \quad a < r \leq b \quad \frac{\partial T}{\partial z} = 0 \qquad (5)$$

$$z \to \infty \quad 0 \leq r \leq b \quad \frac{\partial T}{\partial z} = -\frac{q_o}{k} \qquad (6)$$

where $q_o = Q/\pi b^2$.

Boundary conditions (2) and (3) require that the solutions be finite along the axis of the tube, and that they yield zero heat flux at the tube boundary. Boundary condition (4) relates the temperature gradient in the axial direction over the contact area as a function of the heat flux distribution over that area. In Eq. (4), K is a scaling parameter which depends upon the heat flux distribution only. It can be shown that $K = (\mu + 1)$ when

$$f(u) = (1 - u^2)^\mu \qquad (7)$$

In Eqs. (4) and (7), $u = r/a$ and therefore $0 \leq u \leq 1$. Boundary conditions (5) and (6) state that the solutions must yield zero heat flux outside the contact area, as well as a uniform heat flux far from the contact area. By means of separation of variables and superposition of solutions, it can be shown that the following expression:[3]

$$T = -\frac{q_o}{k} z + C_o + \sum_{n=1}^{\infty} C_n e^{-\lambda_n z} J_o(\lambda_n r) \cdot \qquad (8)$$

satisfies the governing differential equation, Eq. (1), as well as the boundary conditions given by Eqs. (2) and (6). In Eq. (8), $J_o(\lambda_n r)$ is the

Bessel function of the first kind of zero order whose argument is $(\lambda_n r)$.

The λ_n are the separation constants which can be determined by means of Eq. (3). Taking the derivative of Eq. (8) with respect to r and setting it equal to zero when r = b, it can be seen that λ_n must be the roots of

$$J_1 (\lambda_n b) = 0 \tag{9}$$

where $J_1 (\lambda_n b)$ is the Bessel function of the first kind of order one. The roots of Eq. (9) are tabulated in mathematical handbooks[4,5] and are not given here. It should be noted that the constraint imposed by Eq. (9) upon the separation constants is independent of the heat flux distribution over the contact area and will be valid for all solutions.

The coefficients C_n appearing under the summation sign will be determined by means of Eqs. (4) and (5) in a subsequent section.

Average Temperatures

Equation (8) gives the temperature at any r and z in terms of C_o and C_n. An examination of Eq. (8) shows that in any section of the heat flux tube $(z \geq 0)$, the temperature decreases monotonically from the axis to the insulated boundary. The temperature decreases monotonically with increasing z. It is evident that the maximum temperature within the heat flux tube occurs in the center of the contact area. In fact the temperature of the contact area is always a maximum r = 0 and a minimum at r = a. The exception is the equivalent isothermal case solved by both Roess[1] and Mikic[2].

For the subsequent analysis it is necessary that an average temperature in any plane $z = \ell$ be defined as follows:

$$\bar{T} (z = \ell) = \frac{1}{\pi b^2} \int_o^b T \, 2\pi r dr \tag{10}$$

After substitution of Eq. (8) into Eq. (10), performing the integrations with respect to r and utilizing the results of Eq. (9), it can be shown that

$$\bar{T} (z = \ell) = - \frac{q_o \ell}{k} + C_o \tag{11}$$

Obviously the average temperature in the contact plane can be obtained by setting $\ell = 0$ in Eq. (11), yielding the interesting result:

$$\bar{T} (z = 0) = C_o \tag{12}$$

The average contact area temperature is defined as

$$T_c = \frac{1}{\pi a^2} \int_o^a T \, 2\pi r dr \tag{13}$$

Upon substitution of Eq. (8) into Eq. (13) with z = 0, and performing the integration one obtains for the average contact area temperature

$$T_c = C_o + 2 \sum_{n=1}^{\infty} \frac{C_n J_1 (\lambda_n a)}{(\lambda_n a)} \tag{14}$$

which is a function of C_o, the average contact plane temperature, and the C_n.

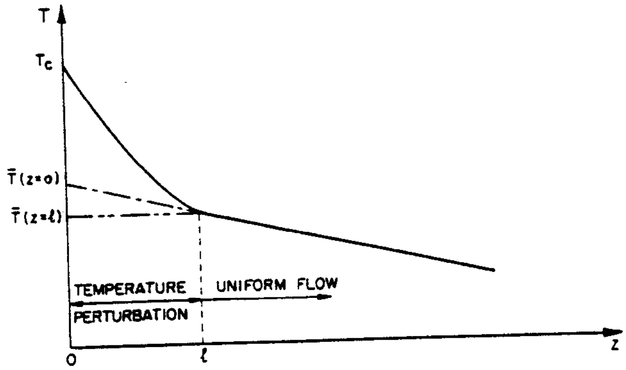

Fig. 3 Axial flux tube temperature distribution

Constriction Resistance

Figure 3 shows a plot of the average temperatures as a function of z as well as the average contact area temperature. The temperature difference $\bar{T} (z = 0) - \bar{T} (z = \ell)$ is due to the thermal resistance of the heat flux tube itself and is obviously given by

$$Q R_m = \frac{q_o \ell}{k} \tag{15}$$

The temperature drop $T_c - \bar{T} (z = \ell)$ is due to the thermal constriction resistance and the thermal resistance of the heat flux tube. If we assume that these two resistances are additive, then we can write

$$T_c - \bar{T} (z = \ell) = Q R_t = Q [R_c + R_m] \tag{16}$$

Subtracting Eq. (15) from Eq. (16) gives the following relationship:

$$T_c - \bar{T} (z = 0) = Q R_c \tag{17}$$

Eq. (17) can be considered as the defining equation of the thermal constriction resistance.

After substitution of Eqs. (12) and (14) in Eq. (17), and dividing through by Q, we obtain the following expression for the constriction resistance:

$$R_c = \frac{2}{Q} \sum_{n=1}^{\infty} \frac{C_n J_1 (\lambda_n a)}{(\lambda_n a)} \tag{18}$$

Thus we see that it depends upon the coefficients C_n which are functions of the heat flux distribution over the contact area.

Along z = 0 we have according to Eq. (8):

$$\frac{-\partial T}{\partial z} = \frac{q_o}{k} + \sum_{n=1}^{\infty} C_n \lambda_n J_o (\lambda_n r) \tag{19}$$

Multiplying both sides of Eq. (19) by $r J_o (\lambda_m r) dr$ and integrating with respect to r from r = 0 to r = b, we obtain

$$\int_o^b \frac{-\partial T}{\partial z} r J_o (\lambda_m r) dr = \frac{Q}{k \pi b^2} \int_o^b r J_o (\lambda_m r) dr$$

$$+ \sum_{n=1}^{\infty} C_n \lambda_n \int_o^b r J_o (\lambda_m r) J_o (\lambda_n r) dr \tag{20}$$

The first term on the right hand side of Eq. (20) is equal to zero because $J_1(\lambda_n b)$ and $J_1(0)$ are both zero.

Using the orthogonality property of Bessel functions, the second term of Eq. (20) is equal to zero whenever $\lambda_m \neq \lambda_n$, otherwise we have

$$\frac{C_n (\lambda_n b)^2 J_o^2 (\lambda_n b)}{2\lambda_n} \tag{21}$$

We can now solve for C_n:

$$C_n = \frac{2 \lambda_n a^2}{(\lambda_n b)^2 J_o^2 (\lambda_n b)} \int_o^1 \frac{-\partial T}{\partial z} u J_o (\lambda_n au) \, du \tag{22}$$

An alternate expression for C_n, after substitution of Eq. (4) into Eq. (22), is

$$C_n = \frac{2 K b^2 q_o \lambda_n}{k (\lambda_n b)^2 J_o^2 (\lambda_n b)} \int_o^1 u f(u) \times J_o (\lambda_n b \varepsilon u) \, du \tag{23}$$

with $\varepsilon = a/b$ and $u = r/a$.

The general expression for the dimensionless constriction resistance defined as $4ak R_c = R_c^*$ can be determined by substitution of Eq. (23) into Eq. (18), with the total heat flow rate given by

$$Q = 2\pi K b^2 q_o \int_o^1 u f(u) \, du \tag{24}$$

Therefore,

$$R_c^* = \frac{8}{\pi} \frac{1}{\int_o^1 u f(u) \, du} \sum_{n=1}^{\infty} \frac{J_1 (\lambda_n b\varepsilon)}{(\lambda_n b)^2 J_o^2 (\lambda_n b)} \times \int_o^1 u f(u) J_o (\lambda_n b\varepsilon u) \, du \tag{25}$$

with the $(\lambda_n b)$ given by Eq. (9). This expression is valid for any continuous heat flux distribution $f(u)$ over the contact area. Eq. (25) has been called the thermal constriction parameter ψ_c, and it is clearly a function of the geometry of the heat flux tube as well as the boundary condition over the contact area.

Constriction Parameter ψ_c for $f(u) = (1 - u^2)^\mu$

Another general expression for the thermal constriction parameter ψ_c defined by Eq. (25) can be obtained for the case where the heat flux distribution $f(u)$ is given by $(1 - u^2)^\mu$ with μ limited to values equal to or greater than minus one.

The integral outside the summation sign in Eq. (25) yields[5]

$$\int_o^1 u (1 - u^2)^\mu \, du = \frac{1}{2(\mu + 1)} \tag{26}$$

whereas the integral inside the summation sign gives[5]

$$\int_o^1 u (1 - u^2)^\mu J_o (\lambda_n b \varepsilon u) \, du = \frac{2^\mu \Gamma(\mu + 1)}{(\lambda_n b\varepsilon)^{\mu+1}} J_{\mu+1} (\lambda_n b\varepsilon) \tag{27}$$

where $\Gamma(\mu + 1)$ is the Gamma function and $J_{\mu+1} (\lambda_n b\varepsilon)$ is the Bessel function of the first kind of order $(\mu + 1)$.

Substituting Eqs. (26) and (27) into Eq. (25) results in another general expression for the dimensionless constriction parameter:

$$\psi_c = \frac{16}{\pi} (\mu + 1) \, 2^\mu \, \Gamma(\mu + 1) \left(\frac{1}{\varepsilon}\right) \sum_{n=1}^{\infty} \frac{J_1 (\lambda_n b\varepsilon) J_{\mu+1} (\lambda_n b\varepsilon)}{(\lambda_n b)^3 J_o^2 (\lambda_n b) (\lambda_n b\varepsilon)^\mu} \tag{28}$$

Thus we see that $\psi_c = \psi(\mu, \lambda_n b, \varepsilon)$ where μ and $(\lambda_n b)$ represent the influence of the boundary conditions, while ε represents the effect of the geometry or relative size of the contact area.

Special Cases of ψ_c for $\mu = -1/2, 0, 1/2$

Three particular cases will be considered to illustrate the utility of Eq. (28).

I $\mu = -1/2$

When $\mu = -1/2$, the heat flux distribution is a minimum over the center of the contact area and increases with increasing u. At the edge of the contact area, the flux becomes unlimited. For this case, Eq. (28) becomes

$$\psi_c (\mu = -1/2) = \frac{8}{\pi} \sqrt{2} \, \Gamma(1/2) \left(\frac{1}{\varepsilon}\right) \sum_{n=1}^{\infty} \frac{J_1 (\lambda_n b\varepsilon) J_{1/2} (\lambda_n b\varepsilon)}{(\lambda_n b)^3 J_o^2 (\lambda_n b) (\lambda_n b\varepsilon)^{-1/2}} \tag{29}$$

A further simplification can be made to Eq. (29) if we use the following[5]:

$$\Gamma(1/2) = \sqrt{\pi}$$

$$J_{1/2} (\lambda_n b\varepsilon) = \sqrt{\frac{2}{\pi}} \frac{\sin (\lambda_n b\varepsilon)}{(\lambda_n b\varepsilon)^{1/2}} \tag{30}$$

With Eq. (30), the constriction parameter reduces to

$$\psi_c (\mu = -1/2) = \frac{8}{\pi} \left(\frac{1}{\varepsilon}\right) \sum_{n=1}^{\infty} \frac{J_1 (\lambda_n b\varepsilon) \sin(\lambda_n b\varepsilon)}{(\lambda_n b)^3 J_o^2 (\lambda_n b)} \tag{31}$$

II $\mu = 0$

When $\mu = 0$, the heat flux distribution is constant over the entire contact area and for this case Eq. (28) becomes

$$\psi_c\ (\mu = 0) = \frac{16}{\pi}\left(\frac{1}{\varepsilon}\right)\sum_{n=1}^{\infty}\frac{J_1^2\ (\lambda_n\ b\varepsilon)}{(\lambda_n\ b)^3\ J_o^2\ (\lambda_n\ b)} \tag{32}$$

III $\mu = 1/2$

When $\mu = 1/2$, the heat flux distribution is a maximum over the center of the contact area and decreases with increasing u to a value of zero at the edge of the contact area. For this case Eq. (28) yields

$$\psi_c\ (\mu = 1/2) = \frac{24}{\pi}\sqrt{2}\ \Gamma\left(\frac{3}{2}\right)\left(\frac{1}{\varepsilon}\right)\sum_{n=1}^{\infty}\frac{J_1\ (\lambda_n\ b\varepsilon)\ J_{3/2}\ (\lambda_n\ b\varepsilon)}{(\lambda_n\ b)^3\ J_o^2\ (\lambda_n\ b)\ (\lambda_n\ b\varepsilon)^{1/2}} \tag{33}$$

This expression can be simplified if we make use of the following[5]:

$$\Gamma(3/2) = \frac{\sqrt{\pi}}{2}$$
$$J_{3/2}(x) = (2/\pi x)^{1/2}\left[\frac{\sin x}{x} - \cos x\right] \tag{34}$$

Thus Eq. (33) takes the form:

$$\psi_c\ (\mu = 1/2) = \frac{24}{\pi}\left(\frac{1}{\varepsilon}\right)\sum_{n=1}^{\infty}\frac{J_1\ (\lambda_n\ b\varepsilon)\ \sin\ (\lambda_n\ b\varepsilon)}{(\lambda_n\ b)^3\ J_o^2\ (\lambda_n\ b)}$$
$$\times\left\{\frac{1}{(\lambda_n\ b\varepsilon)^2} - \frac{1}{(\lambda_n\ b\varepsilon)\ \tan\ (\lambda_n\ b\varepsilon)}\right\} \tag{35}$$

Values of ψ_c for these three cases have been computed and are presented in Table 1 along with the values given by Roess[1] corresponding to $\mu = -1/2$.

The constriction parameter expressions, Eqs. (29), (32) and (35), converge quickly for $0.1 < \varepsilon < 0.8$. For values of $\varepsilon < 0.1$, the convergence is much slower and hundreds of terms must be evaluated before convergence occurs[6]. As $\varepsilon \to 0$, thousands of terms are required. For this reason, a numerical solution[7] was obtained for the limiting case of $\varepsilon = 0$. It was shown that $\psi_c = 1$, 1.0808 and 1.1252 for the three cases: $\mu = -1/2$, 0 and 1/2, respectively. For the very important range, $0 \leq \varepsilon < 0.1$, an approximate expression can be developed by assuming a linear relationship between ψ_c and ε. Using the results of reference (7) for $\varepsilon = 0$ and the values of ψ_c for $\varepsilon = 0.1$ in Table 1, one can relate ψ_c to ε in the following manner:

$$\psi_c = \alpha\ (1 - \beta\varepsilon) \tag{36}$$

where the coefficients α and β are given in Table 2 for the particular values of μ.

The values of ψ_c given by Eq. (36) differ by less than 0.1% from those evaluated by means of Eqs. (29), (32) and (35).

For values of ψ_c corresponding to $\varepsilon \leq 0.3$, it is recommended that one use the following approximation:

$$\psi_c = \alpha(1 - \varepsilon)^{\gamma} \tag{37}$$

where α and γ are given in Table 2. These values of ψ_c differ by less than 1% from those evaluated by means of Eqs. (29), (32) and (35).

Comparison with the Results of Other Investigators

The results of this paper will be compared with those of Roess[1] and Mikic[2].

Roess in an extensive unpublished work determined the thermal constriction resistance for the case $\mu = -1/2$. His analysis is highly mathematical necessitating the use of contour integration, theory of residues, theory of hypergeometric functions, elliptical integrals as well as the theory of Bessel functions. He presented his results for this particular case in the form of an infinite series:

$$R_c = \frac{1}{4ka}\{1 - r_1\varepsilon + r_3\varepsilon^3 + r_5\varepsilon^5 + r_7\varepsilon^7 + r_9\varepsilon^9 + r_{11}\varepsilon^{11} + \ldots\} \tag{38}$$

The coefficients r_1, r_3, r_5, etc. are presented in Table 3.

The product of $4ka\ R_c$ or the thermal constriction parameter given by Roess for various values of ε are also shown in Table 1. The agreement between the values of Roess and those presented in this paper is excellent over the range of ε running from 0.1 to 0.6. Roess did not give values of ψ for $\varepsilon > 0.6$. It was also shown by Roess that when $\varepsilon \leq 0.3$, this heat flux distribution results in a contact area temperature distribution which for all practical purposes can be considered to be uniform. He further determined that for $\varepsilon = 0.4$ and 0.5, the temperature at the edge of the contact exceeded that of the center by 3.86% and 9.90%, respectively. For $\varepsilon > 0.5$, the temperature distribution is quite non-uniform.

For $\varepsilon \leq 0.1$, the infinite series given by Eq. (38) reduces to the first two terms because the remaining terms are negligibly small. It can be seen that Roess' coefficient r_1 agrees to within 1 part in 141 with the value of β corresponding to $\mu = -1/2$ in Table 2.

Mikic[2] examined the two cases $\mu = -1/2$ and 0 and obtained expressions identical to Eqs. (29) and (32) which were generated by means of the gen-

Table 1 Some values of ψ_c for $\mu = -1/2, 0, 1/2$

ε	.10	.20	.30	.40	.50	.60	.70	.80
$\psi_c(\mu = -1/2)$	.8580	.7201	.5851	.4556	.3341	.2231	.1262	.04819
$\psi_c(\mu = 0)$	.9397	.8008	.6649	.5337	.4092	.2936	.1895	.1008
$\psi_c(\mu = 1/2)$	.9842	.8450	.7085	.5763	.4500	.3316	.2234	.1284
ψ_c(Roess)[1]	.8594	.7205	.5854	.4558	.3342	.2232	—	—

Table 2 Values of α, β, γ for Eqs. (36) and (37)

μ	-1/2	0	1/2
α	1	1.0808	1.1252
β	1.4197	1.4111	1.4098
γ	1.5	1.35	1.30

Table 3 Roess Coefficients for Eq. (38)

r_1	r_3	r_5	r_7	r_9	r_{11}
1.4093	0.2959	0.05254	.02105	0.01107	0.006312

eral expression, Eq. (28). The values of ψ_c were presented in graphical form for $0.1 \leq \varepsilon \leq 0.6$. He had recommended in his thesis that the results corresponding to $\mu = -1/2$ could be used to approximate the isothermal case provided $\varepsilon < 0.6$.

Summary and Conclusions

A general expression has been developed for determining the thermal constriction resistance of circular contact areas supplying heat to right circular cylinders as a function of the heat flux prescribed over the contact area. A second general expression was developed for heat flux distributions given by $(1 - u^2)^\mu$. Three special cases: $\mu = -1/2$, 0 and 1/2 were considered and compared with published expressions. There was excellent agreement between the expressions developed here and those of Roess and Mikic. Equivalent expressions are presented to allow evaluation of the constriction parameter for values of $\varepsilon \leq 0.1$ as well as $\varepsilon \leq 0.3$. It is recommended that further work be done to examine the effect of $\mu > 1/2$, as well as the two narrow ranges $-1/2 < \mu < 0$ and $0 < \mu < 1/2$.

Acknowledgements

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