

THERMAL CONTACT CONDUCTANCE OF TURNED SURFACES

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Abstract

This paper presents an analytical work performed to determine the thermal resistance to heat transfer at the interface formed by the contact of a hard smooth flat surface with a softer turned surface. The results are valid for surfaces in a vacuum environment when there is negligible radiation heat transfer across the gaps. The thermal analysis was based upon steady heat flow in a two-dimensional heat channel and the contact analysis was based upon plastic deformation of a ridge formed by the turning process. A dimensionless group consisting of contact conductance, harmonic mean thermal conductivity of the contacting surfaces and the distance between adjacent contacting ridges correlates the available data if surface roughness is also taken into account.

Nomenclature

a	=	half-width of contact strips
A_a	=	apparent contact area
b	=	half-width of heat channel
C	=	coefficient in Eq. (35)
d_1	=	average diameter of innermost heat channel
D	=	diameter of contacting cylinders
h_c	=	contact conductance
h_{cr}	=	contact conductance due to contact spots
h_{cw}	=	contact conductance due to contact strips
J_0	=	Bessel function
k	=	thermal conductivity

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k_m	= harmonic mean thermal conductivity $k_m = 2k_1k_2/(k_1 + k_2)$
l	= length of heat channel
N	= number of heat channels
P_a	= apparent contact pressure
P_m^*	= maximum yield pressure
P	= dimensionless contact pressure
Q	= heat flow rate
R_c	= constriction resistance
T	= temperature
T_a	= average contact temperature, Eq. (13)
T_c	= contact strip temperature
u	= transformation, Eq. (42)
x, y	= Cartesian coordinates
α	= contact angle
γ	= dimensionless area ratio, Eq. (36)
δ	= distance between spirals
θ	= transformation, Eq. (42)
σ	= root-mean-square of surface roughness
η, ψ	= elliptic coordinates
Ψ	= geometric factor, Eq. (23)

Subscripts

1,2	= solids 1 and 2, respectively
i	= ith heat channel

Introduction

The thermal resistance to steady heat flow across interfaces formed by contacting solids is currently of great interest to aerospace engineers, especially when the interfaces are placed in a vacuum environment. Many investigators have studied (analytically and experimentally) various aspects of this rather complex problem. The complexity is due to the fact that there are essentially two related problems which have to be studied: the thermal and the mechanical. The thermal problem is solved when one is able to predict the interface resistance from a knowledge of certain physical (thermal conductivities of the contacting solids and interstitial substance) and geometric (number, shape, size and placement of the contact spots) characteristics. The mechanical problem is solved when one is capable of predicting the required geometric characteristics from a knowledge of the geometry (surface roughness and waviness) of the contacting surfaces and certain physical characteristics (modulus of elasticity, maximum yield pressure and apparent contact pressure).

Several important problems dealing with nominally flat rough surfaces and smooth wavy surfaces have been extensively studied and the solutions can be found in the open literature.¹⁻⁶ Some aspects of these results will be applied in this study which will consider the heat transfer across an interface formed when a hard smooth flat surface contacts a softer turned surface. Certain investigators^{3,7,11} have presented theoretical works dealing with an ideal two-dimensional heat channel, but, to-date, no one has examined the complete thermal-mechanical problem. This paper will be limited to the study of thermal contact conductance in a vacuum only, since it serves as a logical starting point for the more difficult analysis required to handle interstitial substances such as fluids.

Statement of the problem

A solid metallic cylinder whose surface has first been made flat and then turned on a lathe is brought into contact with a second metallic cylinder of different material whose surface is smooth and flat. A contact area (consisting of individual contact spots) resembling a long spiral of effective width $2a$ (Fig.1) is formed as a result of the plastic deformation of the ridge formed during the turning process (Fig. 2). It is the softer turned surface which undergoes the plastic deformation. The distance between adjacent ridges (or spirals) δ will depend upon the turning rate and the rate of tool advance. This dimension will be much larger than the effective width of the spiral which actually consists of differently shaped contact spots. Some of the spots are relatively far apart while others are very close to each other. Those close together will merge to form larger ones as the contact load is increased. In fact at very high loads, the majority of the contact spots will have merged to form a quasi-continuous strip of width $2a$.

When the interface described above is placed in a vacuum and steady linear heat flow occurs in both cylinders, a pseudo-tem-

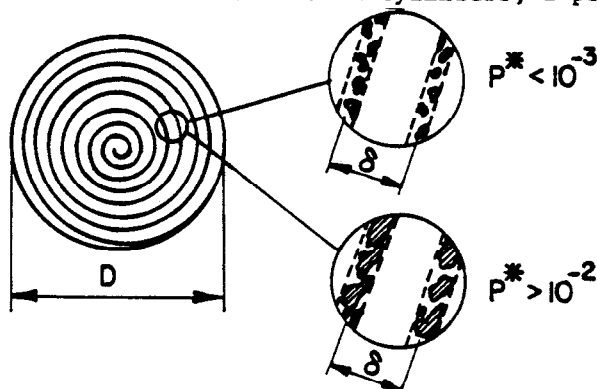


Fig. 1 Contact areas for turned surfaces.

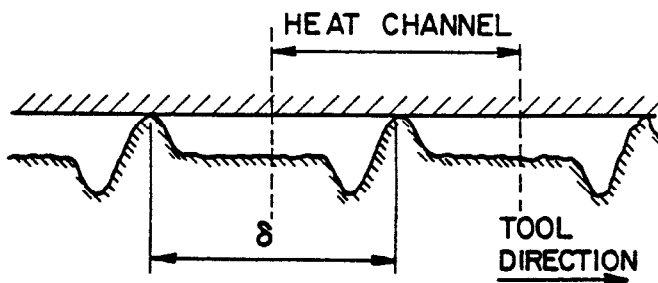


Fig. 2 Profiles of contacting surfaces.

perature drop appears at the interface (Fig. 3). This temperature drop is a direct measure of the thermal resistance occurring at the imperfect interface. The objective, then, is to predict this thermal resistance or its reciprocal, the thermal conductance. For interfaces placed in a vacuum and negligible radiation heat transfer across the gaps, there will be one thermal path available for heat transfer across the interface and that is conduction through the contact spots.

The thermal problem will be modeled as N concentric circular heat channels thermally connected in parallel contacting N other concentric circular heat channels. The first set of heat channels are thermally connected in series with the second set of heat channels. Once the thermal constriction resistance of a typical heat channel has been determined, it is relatively simple to determine the total resistance of the first set of heat channels and finally the total resistance of the two sets together.

Analytical solution

Constriction Resistance of a Single Heat Channel

Fig. 4 shows the model used in this analysis as well as the model used by other investigators.^{3,7} Only one half of a typ-

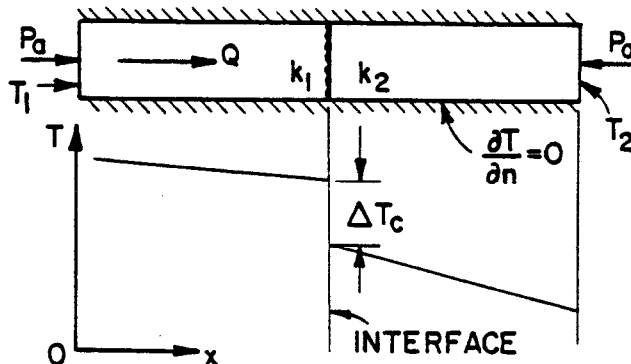


Fig. 3 Temperatures in contacting solids.

following transformation equations:

$$x = a \cosh \eta \cos \psi \quad y = a \sinh \eta \sin \psi \quad (6)$$

where a is the half-width of the contact area assumed to be a very long rectangular strip. The parameter η determines the elliptic isothermal surfaces in the neighborhood of the contact area, and $\eta = 0$ represents the isothermal contact area while $\eta = \eta_1 > 0$ represents the elliptic isothermal surface located far from the contact area, (Fig. 4). The parameter ψ is an angular measure determined from $\psi = \arctan (y/x)$. Employing the transformations of Eqs. (6), Eq. (1) transforms to

$$d^2 T / d\eta^2 = 0 \quad (7)$$

where the temperature field depends only upon one parameter η . The original differential equation has been transformed into a much simpler equation and its solution is

$$T = C_1 \eta + C_2 \quad (8)$$

and the corresponding boundary conditions are

$$T = T_c \quad \eta = 0 \quad (9)$$

$$T = T_1 < T_c \quad \eta = \eta_1 \quad (10)$$

The mixed boundary conditions which are difficult to satisfy when Cartesian coordinates are used, are automatically satisfied when elliptic-cylinder coordinates are employed.

Upon substitution of Eqs. (9) and (10) into Eq. (8) and after evaluating the two constants of integration, one obtains the following expression for the temperature distribution in the region of interest:

$$(T_c - T) / (T_c - T_1) = \eta / \eta_1 \quad (11)$$

This is a simple linear temperature distribution in terms of η , but a complex two-dimensional field in terms of x and y . Equation (11) can also be written in closed form as

$$\frac{T_c - T}{T_c - T_1} = \frac{\cosh^{-1} (x/a \cos \psi)}{\cosh^{-1} (b/a \cos \psi)} \quad (12)$$

where the first expression of Eq. (6) was used to obtain the relationship between η and ψ .

Equation (12) will now be used to evaluate the average interface temperature which is defined as

$$T_a = \frac{1}{b} \int_0^a T dx + \frac{1}{b} \int_a^b T dx \quad (13)$$

After substitution of Eq. (12) in Eq. (13) the average interface temperature is found to be

$$T_a = T_c - \frac{(T_c - T_1)}{b \cosh^{-1}(b/a \cos \psi)} \int_a^b \cosh^{-1}(x/a \cos \psi) dx \quad (14)$$

or

$$T_a = T_c - \frac{(T_c - T_1) a \cos \psi}{b \cosh^{-1}(b/a \cos \psi)} \int_{b/a \cos \psi}^{b/a} \cosh^{-1} u du \quad (15)$$

where $u = x/a \cos \psi$. It should be noted that the average interface temperature depends upon the contact area temperature, the temperature difference between the contact area and an isothermal surface located at the far boundary of the region of interest as well as certain geometric characteristics.

The effective temperature difference which is required to overcome the thermal constriction resistance will be defined as

$$T_c - T_a = \frac{(T_c - T_1) a \cos \psi}{b \cosh^{-1}(b/a \cos \psi)} \int_{b/a \cos \psi}^{b/a} \cosh^{-1} u du \quad (16)$$

The steady heat flow rate through the heat channel can be evaluated at the contact area

$$Q = 2 \int_0^l \int_0^a -k \frac{\partial T}{\partial y}(x, 0) dx dz \quad (17)$$

where l is the effective length of the heat channel measured into the paper. By means of Eq. (6), Eq. (17) can be transformed into an expression dependent upon η and z :

$$Q = 2 \int_0^l \int_{\alpha}^{\pi/2} -k \frac{\partial T}{\partial \eta}(\eta = 0) d\psi dz \quad (18)$$

where α is the contact angle, Fig. 4. Eq. (18) can be integrated to yield an expression for the total heat flow rate

$$Q = \frac{2kl(\pi/2 - \alpha)(T_c - T_1)}{\cosh^{-1}(b/a \cos \alpha)} \quad (19)$$

The thermal constriction resistance of a heat channel is defined as the effective temperature difference divided by the total heat flow rate. Therefore,

$$R_c = (T_c - T_a)/Q \quad (20)$$

and after substitution of Eqs. (16) and (19) into (20) one obtains

$$R_c = \frac{(a \cos \alpha / b)}{k l \pi (1 - 2\alpha / \pi)} \int_{1/\cos \alpha}^{b/a \cos \alpha} \cosh^{-1} u \, du \quad (21)$$

where k is the thermal conductivity and the geometric characteristics of the interface are a, b, α , and l . Equation (21) can be written as

$$R_c = \Psi / k l \quad (22)$$

where the dimensionless geometric factor Ψ defined as

$$\Psi = \frac{(a \cos \alpha / b)}{\pi (1 - 2\alpha / \pi)} \int_{1/\cos \alpha}^{b/a \cos \alpha} \cosh^{-1} u \, du \quad (23)$$

is seen to depend only upon α and a/b .

Equation (22) is the total thermal constriction resistance of a typical heat channel. The resistance is a function of the thermal conductivity as well as certain geometric characteristics.

Thermal Resistance of Multiple Channels in Parallel

The total thermal resistance of the spiral arrangement shown in Fig. 1 will now be modelled as a number of concentric circular heat channels thermally connected in parallel. The constriction resistance of the i th channel is

$$R_{ci} = \Psi_i / k_1 l_i \quad (24)$$

where k_1 is the thermal conductivity of the solid which has been separated into N heat channels. l_i is the effective length of the heat channel and Ψ_i is the corresponding geometric factor. The total resistance for solid 1 is, therefore,

$$1/R_{tcl} = \sum_{i=1}^N 1/R_{ci} = k_1 \sum_{i=1}^N l_i / \Psi_i \quad (25)$$

when N heat channels are connected in parallel. If the heat channels are geometrically similar, the geometric factor Ψ_i will be the same for each heat channel, and Eq. (25) becomes

$$1/R_{tcl} = (k_1/\Psi_1) \sum_{i=1}^N l_i \quad (26)$$

where Ψ_1 is now the geometric factor corresponding to solid 1 and

$$\sum_{i=1}^N l_i$$

is the total effective length of the contact.

Effective Length of Contact

In order to evaluate the total resistance given by Eq. (26), it is necessary to relate the total effective length of the contact to the apparent contact area and the spacing between adjacent spirals. Since the contact has been modeled as concentric circular heat channels with common spacing δ , (Fig. 5) the effective length of each channel, starting from the center of the contact area, is

$$\begin{aligned} l_1 &= \pi d_1 \\ l_2 &= \pi d_1 + 2\pi\delta \\ &\vdots \\ l_i &= \pi d_1 + 2\pi(i-1)\delta \end{aligned} \quad (27)$$

where d_1 is the average diameter of the innermost spiral and it is of the order of δ . The total effective length of the contact is the sum of all the heat channel lengths

$$\sum_{i=1}^N l_i = \pi \{ N d_1 + 2\delta \sum_{i=1}^{(N-1)} i \} \quad (28)$$

The second term within the bracket can be summed and it is equal to

$$\delta N(N-1) \quad (29)$$

The total number of heat channels, to a first approximation, is $D/2\delta$ where D is the diameter of the apparent contact area.

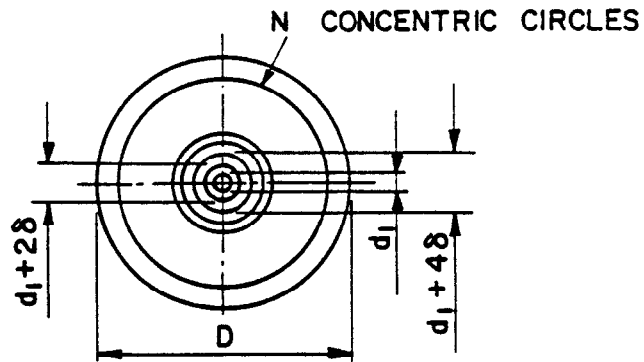


Fig. 5 Contact model for conductance theory.

Upon substitution of Eq. (29) in Eq. (28) we obtain as the total effective length of contact

$$\sum_{i=1}^N l_i = \pi [N d_1 - N \delta + \frac{D^2}{4\delta}] = \frac{\pi D^2}{4\delta} \quad (30)$$

where $\pi D^2/4$ is the apparent contact area.

Thermal Contact Conductance of the Spiral

The total contact resistance, Eq. (26), can now be written as

$$1/R_{tcl} = (k_1/\psi_1 \delta) (\pi D^2/4) \quad (31)$$

The definition of contact resistance, $R_c = \Delta T_c/Q$, and that of contact conductance, $h_c = \Delta T_c/(Q/Aa)$, allows one to equate the resistance and conductance in the following manner:

$$h_c = 1/(R_c Aa) \quad (32)$$

Thus Eq. (32) with Eq. (31) shows that the thermal contact conductance of N heat channels in solid 1 can be written as

$$h_{c1} = k_1/\psi_1 \delta \quad (33)$$

A similar expression can be written for the second set of N heat channels in solid 2.

The over-all contact conductance for two sets of N dissimilar heat channels thermally connected in series is

$$1/h_{cw} = 1/h_{c1} + 1/h_{c2} = \delta [\psi_1/k_1 + \psi_2/k_2] \quad (34)$$

where ψ_1 and ψ_2 are the geometric factors, defined by Eq. (23), corresponding to solid 1 and solid 2, respectively. The additional subscript w will be used to denote thermal conductance due to the spirals under ideal conditions, i.e., when the spirals or strips are continuous (all the contact spots have merged).

Contact Model

If one is to be able to use Eq. (34) to predict the thermal conductance of an ideal turned surface, it is necessary to have a relationship between the geometric parameter a/b , the apparent contact pressure and the maximum yield pressure of the softer turned surface. If one assumes that during the first compression cycle the contacting ridges undergo plastic deformation, then one can write the following simple expression which is a force balance at the interface:

$$2aLP_m = (\pi D^2/4)P_a \quad (35)$$

where $2aL$ is the contact area formed because of plastic deformation of the spirals, P_m is the maximum yield pressure and P_a is the apparent contact pressure. This simple model will be a good approximation of the real situation for the first loading cycle.

Replacing L , the total effective length of the contact area, by Eq. (30), one obtains

$$a/b = P^* \quad (36)$$

where $P^* = P_a/P_m$. The effective contact width may be smaller than that predicted by Eq. (36), and in order to take this into account, write

$$a/b = P^*/\gamma \quad (37)$$

where $\gamma \geq 1$ depending upon how closely the contact model agrees with the real situation. Equation (37) can now be substituted into Eq. (23) to obtain a relationship between conductance, applied load and material strength.

Over-all Conductance Including Surface Roughness

All surfaces possess surface roughness prior to the turning process. This roughness will not change during the turning process or will be altered somewhat (increased). The ridges formed during turning will, therefore, not be perfect and will not form a continuous line during the initial contact. The

contact will consist of discrete contact spots formed on a line corresponding to the lay of the ridges when the load is very light. The discrete contact spots will be relatively far apart and, therefore, the total conductance will be influenced by the surface roughness. The presence of the contact spots within the spiral contact and their influence on the over-all conductance can be taken into consideration by superposing⁸ upon Eq. (34) the conductance due to surface roughness. For light and moderate loads an expression such as the one developed by Cooper, Mikic and Yovanovich⁵ is recommended.

$$h_{cr} = C(k_m/\sigma)(P^*)^{0.985} \quad (38)$$

In Eq. (38), C is a geometric parameter depending upon the slope of the contacting asperities (surface roughness). Typical values of C are 0.036, 0.175 and 0.290 for lapped surfaces, average rough and very rough flat surfaces, respectively. The other parameters appearing in Eq. (38) are: σ , the standard deviation of the profile heights of the asperities; k_m , the harmonic mean thermal conductivity; and P^* , the dimensionless contact pressure.

As the contact pressure becomes very large, more contact spots appear and the characteristic distance between them decreases becoming zero for many contact spots. At these very high loads most of the contact spots will have merged to form a quasi-continuous spiral. At these high contact pressures Eq. (34) should be adequate, but for the lower pressures the over-all conductance will be more accurately predicted by

$$1/h_c = (1/h_{cw}) + (1/h_{cr}) \quad (39)$$

provided that Eq. (38) can predict the roughness conductance at moderately high pressures.

Comparison of Theory and Some Experimental Data

The theory presented in this paper will be compared with the results of the experimental work performed at the University of Poitiers, France. Under the direction of Cordier, Roiron,⁹ Bardon,¹⁰ and Fouché¹¹ systematically studied various aspects of heat transfer across the interface formed by the contact of a smooth flat surface and a softer turned surface. In their experimental work conducted under ambient and vacuum conditions, they sought an empirical correlation for the thermal contact conductance. The one correlation by Fouché¹¹ developed for ambient as well as vacuum conditions failed to predict the thermal conductance with any degree of accuracy. The vacuum test data of Bardon¹⁰ will be compared with the present

theory. In his work Bardon placed stainless steel (18-8) into contact with an alloy of magnesium-zirconium (0.7% zirconium). The stainless steel cylinder was prepared in the following manner: A collar fabricated from identical material was placed around the cylinder; then both were made as flat as possible by turning. After turning, the pieces were lapped and finally polished. After these operations the collar was removed and the stainless steel surface was observed to be smooth and optically flat. The softer material was made flat by turning it on a lathe such that the depth of cut was just sufficient to eliminate all the high spots. During this operation the tool advance was 0.02 mm. The surface roughness was measured to be about 12×10^{-6} inches (rms). After this initial flattening process the alloy material was again turned on a lathe using a tool steel cutter having an angle of 60° . The cylinders were turned at 250 rpm. The tool advance and depth of cut were: a) 0.5 mm and 0.05 mm; b) 0.25 mm and 0.05 mm; and finally c) 0.125 mm and 0.025 mm, respectively, for the three interfaces studied. The diameter of all these pieces was 25.4 mm.

The thermal tests were conducted in a vacuum system similar to systems used by other investigators²⁻⁴ and the technique used was the same. The tests were done in a vacuum of 10^{-4} mm Hg. The system was maintained at a pressure of 10^{-4} mm Hg for four days prior to the actual testing so that all surfaces were thoroughly de-gassed. The heat flux based upon the apparent contact area was about 5000 BTU/hr.sq.ft. The load on the interface ranged from about 150 to 1500 psi. The pseudotemperature drop across the interface ranged from a low value of 0.5°F corresponding to the high loads up to a maximum value of about 6°F corresponding to the light loads. Temperature readings were taken every 45 minutes and the calculated thermal conductance ranged from about 800 to 12,000 BTU/hr.sq.ft. $^\circ\text{F}$. Fouché conducted hardness tests and found the yield pressure of magnesium-zirconium to be 44,000 psi.

To facilitate the comparison of this theory with the test data of Bardon, Eq. (39) has been written in the following dimensionless form:

$$(\delta h_c / k_m)^{-1} = \pi^{-1} (k_m / k_1) [\ln(2\gamma / P^*) - 1] \\ \times [1 + (k/k_2) / (1 - 2\alpha/\pi)] + C^{-1} (\sigma/\delta) (1/P^*)^{0.985} \quad (40)$$

For the interfaces investigated by Bardon, Eq. (40) becomes

$$(\delta h_c / k_m)^{-1} = 0.68 [\ln(2\gamma / P^*) - 1] \\ + (2.78 \times 10^{-5} / \delta) (1/P^*)^{0.985} \quad (41)$$

The geometric and physical characteristics of the contacting surfaces used in the test program are listed in Table 1.

Table 1 Geometric and physical characteristics of the surfaces

Stainless steel	Magnesium-zirconium
$k_1 = 10 \text{ BTU/hr ft } ^\circ\text{F}$	$k_2 = 59 \text{ BTU/hr ft } ^\circ\text{F}$
$\sigma = 0$	$\sigma = 10^{-6} \text{ ft}$
$\alpha = 0$	$\alpha = 30^\circ$
$P_m = 360,000 \text{ psi}$	$P_m = 44,000 \text{ psi}$
	$\delta_1 = 1/610 \text{ ft}$
	$\delta_2 = 1/1220 \text{ ft}$
	$\delta_3 = 1/2440 \text{ ft}$

A comparison between experimental results and the predicted values calculated by means of Eq. (34) with $\gamma = 1$ and by means of Eq. (41) with $\gamma = 1$ are presented in (Figs. 6-8). Equation (34) is based upon the assumption that there are no surface roughness effects, while Eq. (41) takes into consideration roughness effects. The first interface tested δ_1 (Fig. 6) had a total spiral length of about 3.3 ft and could be approximated

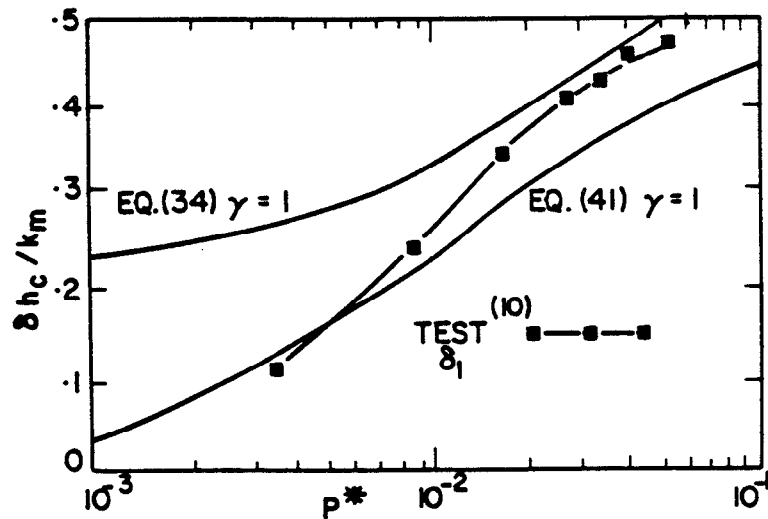


Fig. 6 Theory vs test data.

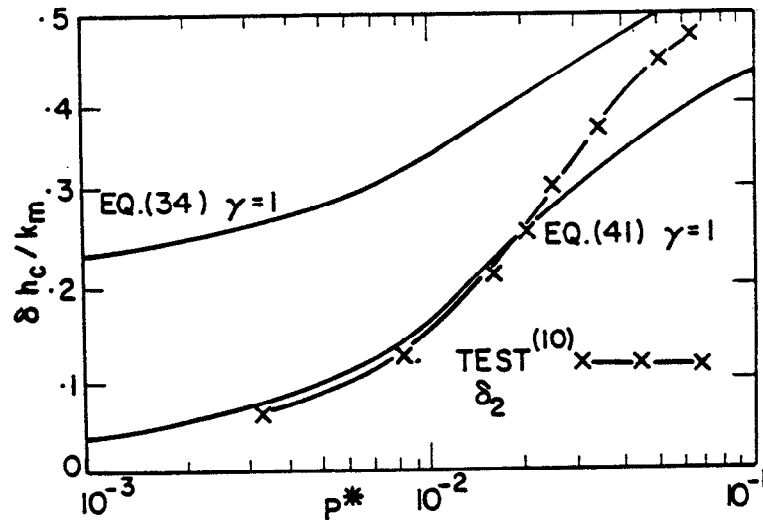


Fig. 7 Theory vs test data.

by 25 heat channels connected in parallel. There is good agreement between Eq. (34) and test data in the load range $P^* = 1.2 \times 10^{-2}$ to 5.2×10^{-2} , whereas in the load range $P^* = 3 \times 10^{-3}$ to 1.2×10^{-2} there is good agreement between Eq. (41) and the data. The second interface tested δ_2 (Fig. 7) had a total spiral length of about 6.6 ft and could be approximated by 50 heat channels in parallel. There is excellent agreement between Eq. (41) and the data from $P^* = 3.4 \times 10^{-3}$ up to $P^* = 3.5 \times 10^{-2}$, and then the test data falls on the values predicted by Eq. (34). The last interface tested δ_3 (Fig. 8) had a total spiral length of about 13.2 ft and could be approximated by 100 heat channels in parallel. For this interface there

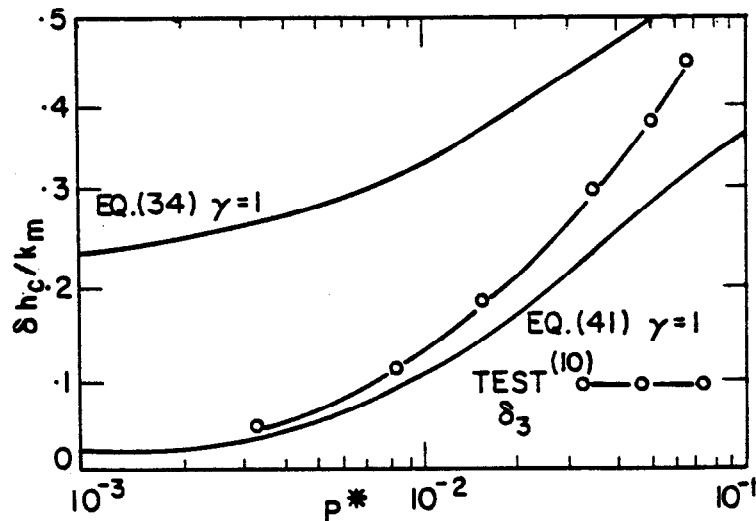


Fig. 8 Theory vs test data.

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