Journal of Thermophysics and Heat Transfer
Vol. 20, No. 4, October–December 2006

Analytical Model for Convection Heat Transfer From Tube Banks

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DOI: 10.2514/1.15453

The main objective of this study is to investigate heat transfer from tube banks in crossflow under isothermal boundary conditions. Because of the complex nature of fluid flow and heat transfer in a tube bank, the heat transfer from a tube in the first row of an in-line or staggered bank is determined first. For this purpose, a control volume is selected from the leading row of a tube bank and an integral method of boundary layer analysis is employed to determine the average heat transfer from the front stagnation point to the separation point, whereas the heat transfer from the separation point to the rear stagnation point is determined by an empirical correlation. To include the effect of the remaining rows, an empirical correlation is employed. The models for in-line and staggered arrangements are applicable for use over a wide range of Reynolds and Prandtl numbers as well as longitudinal and transverse pitch ratios.

Nomenclature

a = dimensionless longitudinal pitch $\equiv S_L/D$
b = dimensionless transverse pitch $\equiv S_T/D$
CV = control volume
c = dimensionless diagonal pitch $\equiv S_D/D$
c_p = specific heat of fluid, $\mathrm{J/kg \cdot K}$
D = tube diameter, m
F_a = arrangement factor
h = average heat transfer coefficient, $\mathrm{W/m^2 \cdot K}$
k = thermal conductivity, $\mathrm{W/m \cdot K}$
L = tube length, m
N = total number of tubes in bank $\equiv N_T N_L$
N_L = number of tubes in longitudinal direction
N_T = number of tubes in transverse direction
Nu_D = Nusselt number based on tube diameter $\equiv Dh/k_f$
Pr = Prandtl number $\equiv \nu/\alpha$
Q = total heat transfer rate, W
Re_D = Reynolds number based on tube diameter $\equiv DU_{\max}/\nu$
S_D = diagonal pitch, m
S_L = longitudinal distance between two consecutive tubes, m
S_T = transverse distance between two consecutive tubes, m
s = distance along curved surface of tube measured from forward stagnation point, m
T = temperature, °C
U_{app} = approach velocity, m/s
$U_{\max}$ = maximum velocity in minimum flow area, m/s
$U(s)$ = velocity in inviscid region just outside boundary layer m/s
u = s component of velocity in boundary layer, m/s
v = η component of velocity in boundary layer, m/s
α = thermal diffusivity, $\mathrm{m^2/s}$
ΔT_{lm} = log mean temperature difference, °C
δ_T = thermal boundary layer thickness, m
δ = hydrodynamic boundary layer thickness, m
η = distance normal to and measured from surface of tube, m
λ = pressure gradient parameter
θ = angle measured from stagnation point, radians
ζ = ratio of thermal to hydrodynamic boundary layer thickness

Subscripts

a = ambient
f = fluid
o = outlet
p = pressure
T = thermal
w = wall

I. Introduction

Heat transfer in flow across a bank of tubes is of particular importance in the design of heat exchangers. Heat exchangers are found in numerous industrial applications, such as steam generation in a boiler or air cooling in the coil of an air conditioner. Tube banks, used in heat exchangers, are usually arranged in an in-line or staggered manner and are characterized by the dimensionless transverse, longitudinal, and diagonal pitches, shown in Figs. 1 and 2.

This study is one of the first attempts to develop analytical models for the heat transfer from tube banks (in-line and staggered). These models are developed in terms of longitudinal and transverse pitch ratios and Reynolds and Prandtl numbers. Depending upon the application, they are classified as compact or widely spaced tube banks. Typically, one fluid moves over the tubes, while the other fluid, at a different temperature and pressure, passes through the tubes. In this study, the authors are specifically interested in the convection heat transfer associated with crossflow over the tubes.

Based on the pertinent data available up to 1933, Colburn [1] proposed a simple correlation for heat transfer for flow across banks of staggered tubes as follows:

$$Nu_D = 0.33 Re_D^{0.6} Pr^{1/3} \tag{1}$$

This correlation works well for 10 or more rows of tubes in a staggered arrangement and for $10 < Re_D < 40{,}000$. Then Huge [2], Pierson [3], Omohundro et al. [4], Bergelin et al. [5–7], Jones and Monroe [8], Gram et al. [9], Žukauskas [10], Aiba et al. [11,12], and Žukauskas and Ulinskas [13] reported extensive experimental data for heat transfer and fluid friction during viscous flow across in-line and staggered banks of tubes under both isothermal and isoflux boundary conditions. Grimison [14] correlated the experimental data of Huge [2] and Pierson [3] for both arrangements and gave a correlation of the form

Presented as Paper 958 at the 43rd AIAA Aerospace Meeting and Exhibit, Reno, Nevada, 5–8 January 2005; received 7 January 2005; revision received 23 March 2006; accepted for publication 26 March 2006. Copyright © 2006 by the American Institute of Aeronautics and Astronautics, Inc. All rights reserved. Copies of this paper may be made for personal or internal use, on condition that the copier pay the $10.00 per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923; include the code $10.00 in correspondence with the CCC.

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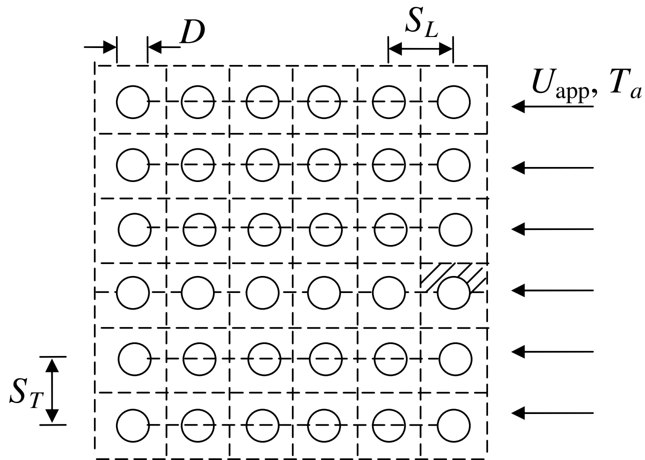

Fig. 1 Schematic of an in-line arrangement.

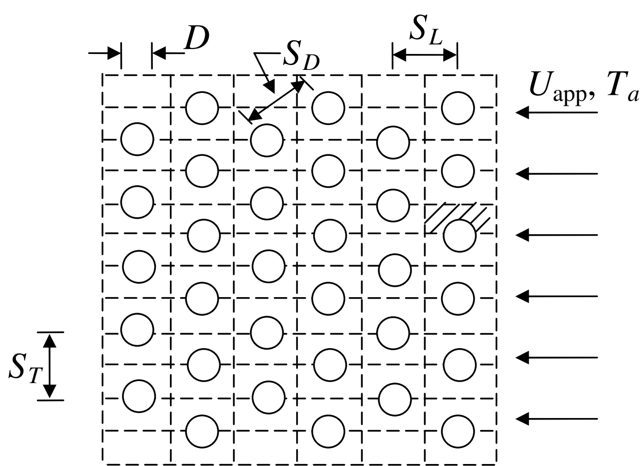

Fig. 2 Schematic of a staggered arrangement.

$$Nu_D = C\,Re_D^n \tag{2}$$

His empirical relation is valid for air only and pertains to tube banks having ten or more rows of tubes in the flow direction. For fewer rows, Kays and London [15] gave a correction factor C_2 such that

$$Nu_D|_{N_L<10} = C_2 Nu_D|_{N_L\geq 10} \tag{3}$$

The values of C, C_2, and n are given, in tabular form, in most heat transfer text books (e.g., Holman [16] and Kreith and Bohn [17]) for both in-line and staggered arrangements. Grimison [14] also correlated the test measurements of Pierson [3] and Huge [2] using a second method and derived the following expression:

$$Nu_D = 0.32 F_a Re_D^{0.61} Pr^{0.31} \tag{4}$$

where F_a was presented graphically by Grimison [14] for the various values of Re_D, subject to the dimensionless transverse and longitudinal pitches.

Hausen [18] modified slightly the above correlation and presented an empirical formula for the tubes F_a instead of the graphical representation by Grimison [14]. For an in-line arrangement

$$Nu_D = 0.34 F_a Re_D^{0.61} Pr^{0.31} \tag{5}$$

can be used with

$$F_a = 1 + (a + 7.17/a - 6.52) \times [0.266/(b-0.8)^2 - 0.12]\sqrt{1000/Re_D} \tag{6}$$

For a staggered arrangement

$$Nu_D = 0.35 F_a Re_D^{0.57} Pr^{0.31} \tag{7}$$

can be employed with

$$F_a = 1 + 0.1a + 0.34/b \tag{8}$$

Žukauskas [10] gave the following experimental correlation for the average Nusselt number for a tube bank consisting of 16 or more rows:

$$Nu_D = FCRe_D^n Pr^m \tag{9}$$

where the coefficients C, m, n and the parameter F are given in tabular form (Kreith and Bohn [17]). Launder and Massey [19], Fujii and Fujii [20], Dhaubhadel et al. [21], Wung and Chen [22], and Murray [23] presented numerical solutions of local heat transfer for the tube bank problem for a wide range of longitudinal and transverse pitches: Reynolds and Prandtl numbers.

Beale [24] conducted a detailed numerical study of fluid flow and heat transfer in tube banks. Using complex function theory, he obtained a potential flow solution in the form of a power series. He presented his results in the form of skin friction, pressure drop, and heat transfer for different thermal boundary conditions. Later Beale and Spalding [25,26] extended the previous work for laminar fully developed crossflow and heat transfer in tube-bank heat exchangers. They obtained a wide range of results for in-line square, rotated square, and equilateral triangle configurations.

Wilson and Bassiouny [27] developed a mathematical model to simulate the laminar and turbulent flow fields inside tube banks. They solved the conservation equations of mass, momentum, and energy using an implicit finite volume procedure. They found that pressure drop and friction factor increased with the longitudinal pitch. They recommended the use of a longitudinal pitch ratio $\mathcal{S}_L \leq 3$ to obtain the best performance and to achieve a high degree of compactness in an in-line arrangement, whereas $\mathcal{S}_L \leq 1.5$ was needed to reduce friction and enhance Nu_D in the staggered arrangement.

Mandhani et al. [28] solved the fluid flow and energy equations numerically to obtain detailed temperature fields and the distribution of Nusselt number on the surface of a typical cylinder in a cylinder bundle for the steady incompressible flow of Newtonian fluids. They found that the surface averaged value of Nusselt number increases with decreasing values of porosity and increasing values of Prandtl and Reynolds numbers. Their results were found in satisfactory agreement with previous numerical and experimental data for a single cylinder and for the tube banks.

The preceding literature review shows that almost all studies are experimental/numerical and no comprehensive analytical model exists for any in-line or staggered arrangement that can be used for a wide range of parameters. The empirical models were developed for specific fluids, values of longitudinal and transverse pitch ratios, and for a specific range of Reynolds numbers. Because of the complex nature of heat transfer in tube banks, the user cannot interpolate/extrapolate these correlations for other values of longitudinal and transverse pitch ratios, or for other fluids or for Reynolds numbers other than the specified range, often found in existing tube banks. New comprehensive models, developed in this study, can be used for the following:

$$1.25 \leq a, b \leq 3.0, \qquad 10^3 \leq Re_D \leq 2\times 10^5, \qquad 1 \leq Pr \leq 1000$$

II. Analysis

Consider a uniform flow of an incompressible Newtonian fluid past a tube bank as shown in Figs. 1 and 2. The ambient temperature is assumed to be T_a. The surface temperature of the tube wall is $T_w(>T_a)$. The flow is assumed to be laminar, steady, and two dimensional. In determining heat transfer from tube banks, the ambient temperature of the incoming fluid is taken as the determining temperature whereas the diameter of the tube is taken as the characteristic length in the definition of Reynolds and Nusselt numbers. These numbers are defined as $Re_D = DU_{\max}/\nu$ and $Nu_D = Dh/k_f$, where $U_{\max}$ is used as a reference velocity in the calculations of fluid flow and heat transfer for both types of arrangements, and is given by

$$U_{\max} = \max[(b/b-1)U_{\text{app}}, (b/c-1)U_{\text{app}}] \tag{10}$$

where $c = \sqrt{a^2 + (b/2)^2}$ is the dimensionless diagonal pitch in the case of the staggered arrangement.

A. Heat Transfer from a Tube Bank

The total heat transfer rate from the tube bank depends upon the average heat transfer coefficient, the total surface area for heat transfer, and the inlet and outlet fluid temperatures, and is given by

$$Q = h(N\pi DL)\Delta T_{\text{lm}} \tag{11}$$

where the log mean temperature difference is given by

$$\Delta T_{\text{lm}} = \frac{(T_w - T_a) - (T_w - T_o)}{\ln[(T_w - T_a)/(T_w - T_o)]} \tag{12}$$

where T_o is the outlet temperature of the fluid and can be obtained from using energy balance:

$$T_o = T_w - (T_w - T_a)\cdot \exp\left(-\frac{\pi D N h}{\rho U_{\text{app}} N_T S_T c_p}\right) \tag{13}$$

The only unknown quantity in Eq. (11) is the average convection heat transfer coefficient for the tube bank that depends on the geometry, the maximum velocity ($U_{\max}$) in the minimum free cross section between two tubes, and the arrangement of tubes, as well as the physical properties of the fluid (ρ, μ, c_p, and k_f). The functional relationship for the average dimensionless heat transfer can be written as

$$Nu_D = f(Re_D, Pr, b, a) \tag{14}$$

Extensive experimental investigations (Žukauskas and Ulinskas [13]) indicate that the average heat transfer from a tube in a tube bank is also dependent on its location in the bank. The difference in heat transfer from tubes of the first and inner rows depends on the Reynolds number, the number of tubes, the longitudinal and transverse pitch ratios, and the arrangement of the tubes in a bank. The increase in heat transfer is also observed due to flow blockage by the upstream tubes. For $Re_D > 10^3$, they observed that heat transfer from the inner tube rows starts to increase as a result of higher turbulence, which is generated by the first tube rows. When the number of rows in a bank along the streamwise direction is large, the higher heat transfer rate of the remaining rows must be taken into account. So the average heat transfer from the whole bank can be written as

$$Nu_D = C_1 Nu_{Df} \tag{15}$$

where Nu_{Df} is the dimensionless average heat transfer from a tube in the first row of a bank of smooth tubes in crossflow and C_1 is the coefficient that accounts for the dependence of the average heat transfer on the number of rows of a tube bank. The coefficient C_1 is derived from the experimental data of Žukauskas and Ulinskas [13] for $Re_D > 10^3$ for both arrangements and is given by

$$C_1 = \begin{cases} [1.23 + 1.47N_L^{1.25}]/[1.72 + N_L^{1.25}] & \text{in-line} \\ [1.21 + 1.64N_L^{1.44}]/[1.87 + N_L^{1.44}] & \text{staggered} \end{cases} \tag{16}$$

For $N_L \geq 16$, the value of C_1 is 1.43 for the in-line arrangement and 1.61 for the staggered arrangement.

B. Heat Transfer from a Single Tube in a Tube Bank

The average heat transfer coefficient of a single tube taken from the first row of an in-line or staggered tube bank can be determined by an integral method of boundary layer analysis. In this study, the von Kármán–Pohlhausen integral method is used to solve the momentum and energy equations for an isothermal boundary condition. A fourth-order velocity profile in the hydrodynamic boundary layer and a third-order temperature profile in the thermal boundary layer are used. For this purpose, a control volume is considered from the first row of an in-line or staggered arrangement as shown in Figs. 1 and 2. The width of the control volume is taken as unity for convenience, and the length and height, in dimensionless form, are taken as a and $b/2$ ($\equiv S_T/2D$), respectively. Because the flow is symmetrical about the horizontal centerline, the solution has been obtained for half of the flow domain (i.e., for ABCEFG in Fig. 3). The top and bottom surfaces of the control volume can be regarded as impermeable, adiabatic, and shear free (no mass transfer and shear work transfer across the boundary). The heat transfer between the tube and stream is Q and the wall temperature is T_w. The governing equations, velocity and temperature distributions for the CV, are the same as described by Khan et al. [29] for a single isolated cylinder. The potential flow velocity outside the boundary layer was obtained by using complex variable theory and following Suh et al. [30] it can be written as (see Appendix)

$$U = U_{\max} f(\theta) \tag{17}$$

where

$$f(\theta) = \sin\theta - 2\sin^2\left(\frac{\pi}{2a}\right)\left\{\frac{\cosh[(\pi/a)\sin\theta]\sin\theta}{\cosh[(\pi/a)\sin\theta] - \cos[(\pi/a)\cos\theta]} + \sinh\left(\frac{\pi}{a}\sin\theta\right)\frac{\sinh[(\pi/a)\sin\theta]\sin\theta + \cos\theta\sin[(\pi/a)\cos\theta]}{[\cosh((\pi/a)\sin\theta) - \cos((\pi/a)(\cos\theta)]^2}\right\} \tag{18}$$

for an in-line arrangement and

$$\begin{aligned} f(\theta) = \sin\theta - 2\sin^2\left(\frac{\pi}{4a}\right)\Bigg\{&\frac{\cosh[(\pi\sin\theta)/2a]\sin\theta}{\cosh[(\pi\sin\theta)/2a] - \cos[(\pi\cos\theta)/2a]} - \sinh\left(\frac{\pi\sin\theta}{2a}\right)\frac{\sinh[(\pi\sin\theta)/2a]\sin\theta + \sin[(\pi\cos\theta)/2a]\cos\theta}{[\cosh((\pi\sin\theta)/2a) - \cos((\pi\cos\theta)/2a)]^2} \\ &+ \frac{\cosh(\pi(\sin\theta - 2b)/2a)\sin\theta}{\cosh(\pi(\sin\theta - 2b)/2a) - \cos(\pi(\cos\theta - 2a)/2a)} \\ &- \sinh\left(\pi\frac{\sin\theta - 2b}{2a}\right)\frac{\sinh(\pi(\sin\theta - 2b)/2a)\sin\theta + \sin(\pi(\cos\theta - 2a)/2a)\cos\theta}{[\cosh(\pi(\sin\theta - 2b)/2a) - \cos(\pi(\cos\theta - 2a)/2a)]^2}\Bigg\} \end{aligned} \tag{19}$$

for the staggered arrangement.

The following boundary conditions are specified for the control volume of Fig. 3.

1) On the curved surfaces of the tube, $u = 0$, $v = 0$, and $T = T_w$.

2) Along the top and bottom of the control volume and on the side-wall regions between tubes, $v = 0$, $\tau_w = 0$, and $Q = 0$.

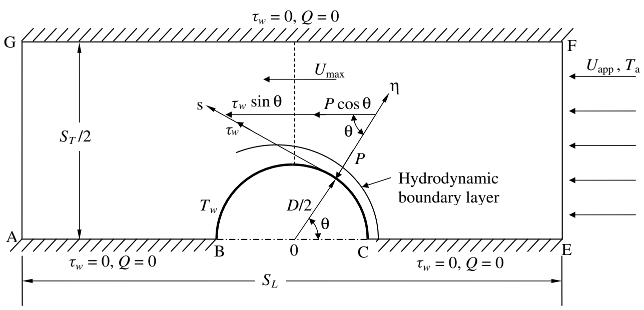

Fig. 3 Control volume for prediction of heat transfer from a tube bank.

3) At large distances upstream of the CV, $u = U_{\text{app}}$ and $T = T_a$.

Following Khan et al. [29] and assuming the presence of a thin thermal boundary layer δ_T along the tube surface in the CV, the energy integral equation for the isothermal boundary condition can be written as

$$\frac{\mathrm{d}}{\mathrm{d}s}\int_0^{\delta_T}(T - T_a)u\,\mathrm{d}\eta = -\alpha\frac{\partial T}{\partial\eta}\bigg|_{\eta=0} \tag{20}$$

Using a fourth-degree polynomial velocity profile and a third-degree polynomial temperature profile that satisfy all the boundary conditions (Khan et al. [29]) and assuming $\zeta = \delta_T/\delta < 1$ for $Pr \geq 1$, Eq. (20) can be integrated to give

$$\delta_T(\mathrm{d}/\mathrm{d}s)[U(s)\delta_T\zeta(\lambda + 12)] = 90\alpha \tag{21}$$

where $U(s)$ is given by Eq. () and λ is obtained from the momentum integral equation and the definition of momentum boundary layer thickness. The values of λ are obtained corresponding to each position along the tube surface and are fitted by the least squares method and given by

$$\begin{aligned}\lambda &= 7.36 - 3.74\theta + 27.95\theta^2 - 96.64\theta^3 + 157.83\theta^4 \\ &\quad - 135.87\theta^5 + 58.65\theta^6 - 10.10\theta^7\end{aligned} \tag{22}$$

Integrating Eq. (21) with respect to s, one can obtain local thermal boundary layer thicknesses

$$\left(\frac{\delta_T(\theta)}{D}\right)\cdot Re_D^{1/2}Pr^{1/3} = \sqrt[3]{\frac{90I_1}{(\lambda + 12)^2 f(\theta)^2}}\sqrt{\frac{\lambda}{2g(\theta)}} \tag{23}$$

where $g(\theta)$ is the derivative of the function $f(\theta)$ with respect to θ and I_1 is given by

$$I_1 = \int_0^\theta f(\theta)(\lambda + 12)\mathrm{d}\theta \tag{24}$$

The local heat transfer coefficient can be written as

$$h(\theta) = 3k_f/2\delta_T \tag{25}$$

Thus the dimensionless local heat transfer coefficient can be written as

$$\frac{\mathrm{Nu}_{Df}(\theta)|_{\text{isothermal}}}{\mathrm{Re}_D^{1/2}\mathrm{Pr}^{1/3}} = \frac{3}{2}\sqrt[3]{\frac{(\lambda + 12)^2 f(\theta)^2}{90I_1}}\sqrt{\frac{2g(\theta)}{\lambda}} \tag{26}$$

The average heat transfer coefficient is defined as

$$h = \frac{1}{\pi}\int_0^\pi h(\theta)\mathrm{d}\theta = \frac{1}{\pi}\int_0^{\theta_s} h(\theta)\mathrm{d}\theta + \frac{1}{\pi}\int_{\theta_s}^\pi h(\theta)\mathrm{d}\theta \tag{27}$$

In dimensionless form, the heat transfer coefficient can be written as

$$Nu_{Df} = hD/k_f = Nu_{Df1} + Nu_{Df2} \tag{28}$$

The first term on the right-hand side gives the dimensionless average heat transfer coefficient of the tube from the front stagnation point to the separation point, and can be obtained, using Eqs. (23–25), for different pitch ratios and then correlated to obtain a single expression in terms of the Re_D and Pr numbers for both in-line and staggered arrangements. This expression can be written as

$$Nu_{Df1} = C_2 Re_D^{1/2}Pr^{1/3} \tag{29}$$

where C_2 is a constant that depends upon the longitudinal and transverse pitches, arrangement of the tubes, and thermal boundary conditions. For the isothermal boundary condition, it is given by:

$$C_2 = \begin{cases}[-.016 + .6a^2]/[0.4 + a^2] & \text{in-line} \\ (0.588 + 0.004b)(0.858 + 0.04b - 0.008b^2)^{1/a} & \text{staggered}\end{cases} \tag{30}$$

Equation (30) is valid for $1.25 \leq a \leq 3$ and $1.25 \leq b \leq 3$ for both arrangements.

The second term on the right-hand side of Eq. (28) gives the dimensionless average heat transfer coefficient of the tube from the separation point to the rear stagnation point. The integral analysis is unable to predict these heat transfer coefficients. The experiments (Žukauskas and Žiugžda [31], Fand and Keswani [32], and Nakamura and Igarashi [33], among others) show that the heat transfer from the rear portion of the cylinder increases with the Reynolds numbers. From a collection of all known data, Van der Hegge Zijnen [34] demonstrated that the heat transferred from the rear portion of the cylinder to the air can be determined from

$$Nu_{Df2} = 0.001Re_D \tag{31}$$

Thus, the total heat transfer coefficient from a single tube in the first row can be written as

$$Nu_{Df} = C_2 Re_D^{1/2}Pr^{1/3} + 0.001Re_D \tag{32}$$

III. Results and Discussion

According to Žukauskas and Ulinskas [13], tube banks with $b \times a \leq 1.25 \times 1.25$ are considered compact, and with $b \times a \geq 2 \times 2$ they are said to be widely spaced. For both compact and wide tube banks, Incropera and DeWitt [35] solved a problem of a staggered tube bank that is used for space heating. In this study, that problem is chosen for comparing the results of the present analysis. Incropera and DeWitt [35] assumed steady state conditions, negligible radiation effects, and negligible effect of change in fluid temperature on fluid properties. They used the data given in Table 1 to calculate an air-side convection coefficient and heat transferred by the tube bank.

Table 1 Data used by Incropera and Dewitt [35] for a staggered tube bank

Quantity	Dimension
Tube diameter, mm	16.4
Longitudinal pitch, mm	20.5, 34.3
Transverse pitch, mm	20.5, 31.3
Number of tubes, staggered	8×7
Tube surface temperature, °C	70
Air properties:	
Approach velocity, m/s	6
Thermal conductivity, $\mathrm{W/m \cdot K}$	0.0253
Density, $\mathrm{kg/m^3}$	1.217
Specific heat, $\mathrm{J/kg \cdot K}$	1007
Kinematic viscosity, $\mathrm{m^2/s}$	14.82×10^{-6}
Prandtl number	0.701
Ambient temperature, °C	15

Table 2 Comparison of results for a compact tube bank (1.25 × 1.25)

	Nu_D	h, W/m^2 · K	T_o, °C	Q, kW
Incropera and DeWitt [35]	152.0	234.0	38.5	28.4
Present analysis	186.8	288.3	39.2	25.5

Table 3 Comparison of results for a wide tube bank (1.9 × 2.1)

	Nu_D	h, W/m^2 · K	T_o, °C	Q, kW
Incropera and DeWitt [35]	87.9	135.6	25.5	19.4
Present analysis	113.15	175.15	27.5	20.1

Incropera and DeWitt [35] solved this problem by using Žukauskas and Ulinskas [13] correlations, whereas the present analysis uses an analytical model. The results are shown in Table 2 for a compact bank and in Table 3 for a widely spaced bank. Table 2 shows that the present analysis gives higher heat transfer rate (around 18%) than Incropera and DeWitt [35], whereas Table 3 shows that the present heat transfer rate is 22% higher than Incropera and DeWitt [35]. The reason for higher heat transfer rates in the present case might be due to the fact that Incropera and DeWitt [35] used the same constants in their correlation for both cases, whereas the present models are sensitive to pitch ratios. The comparison of Tables 2 and 3 shows that heat transfer of a bank decreases with increasing pitch ratio.

Average heat transfer from a single tube in the first row of symmetrical in-line tube banks with 1.25 × 1.25 and 2.0 × 2.0 pitch ratios is shown in Figs. 4 and 5. In both cases, the average heat transfer increases with the Reynolds numbers and the behavior approximates a linear dependence on the logarithmic scale. The comparison of both figures show that the heat transfer increases in symmetrical in-line tube banks with their transverse and longitudinal pitch ratios. Turbulence generated by the first rows penetrates the boundary layer developed on the tube more effectively than in compact symmetrical in-line tube banks. In compact banks, turbulence decays as a result of the flow being compressed between longitudinal rows [13]. The present results are compared with the

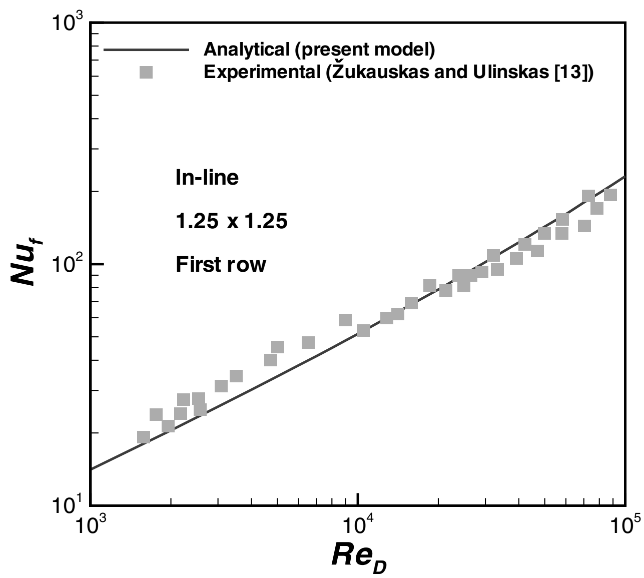

Fig. 4 Average heat transfer from the first row of a 1.25 × 1.25 in-line tube bank.

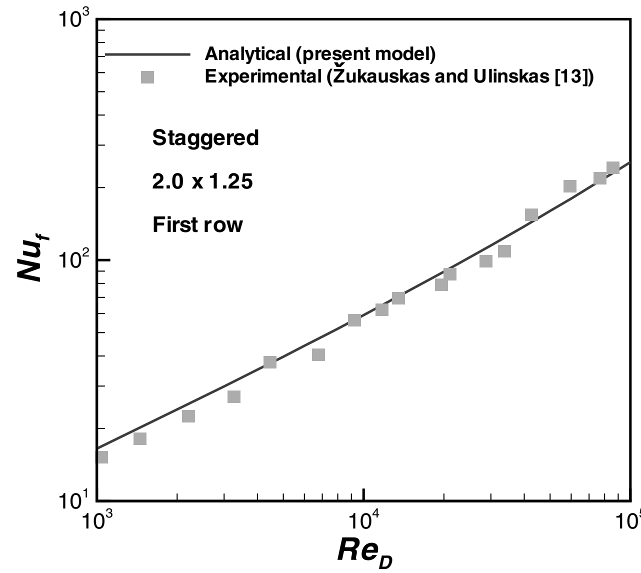

Fig. 6 Average heat transfer from the first row of a 2.0 × 1.25 staggered tube bank.

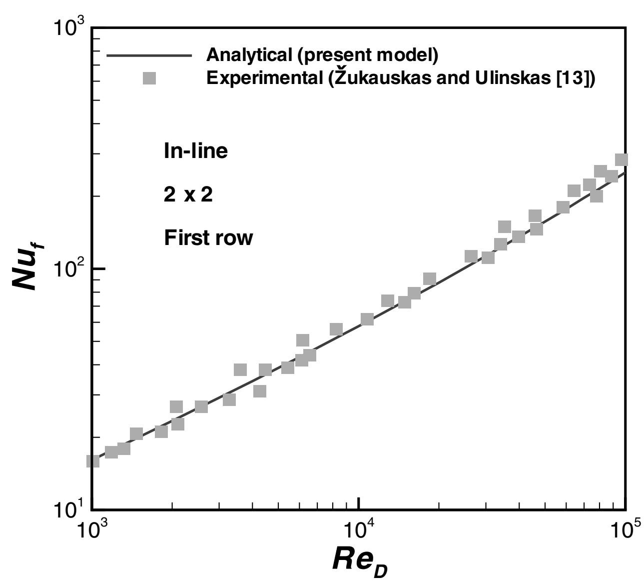

Fig. 5 Average heat transfer from the first row of a 2.0 × 2.0 in-line tube bank.

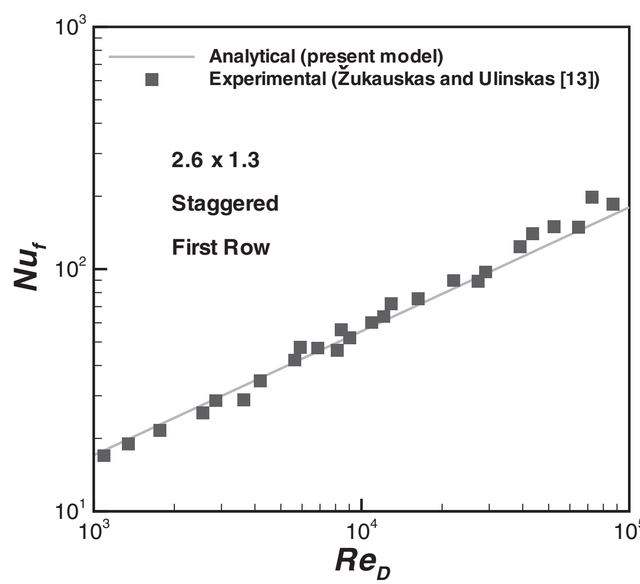

Fig. 7 Average heat transfer from the first row of a 2.6 × 1.3 staggered tube bank.

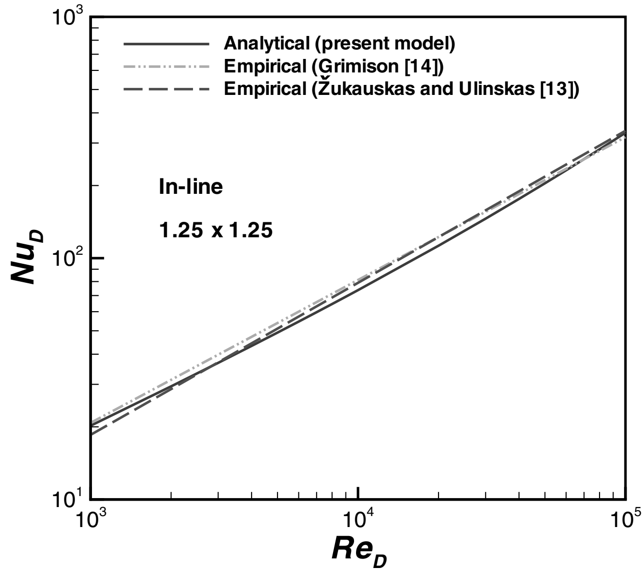

Fig. 8 Average heat yransfer from an in-line tube bank.

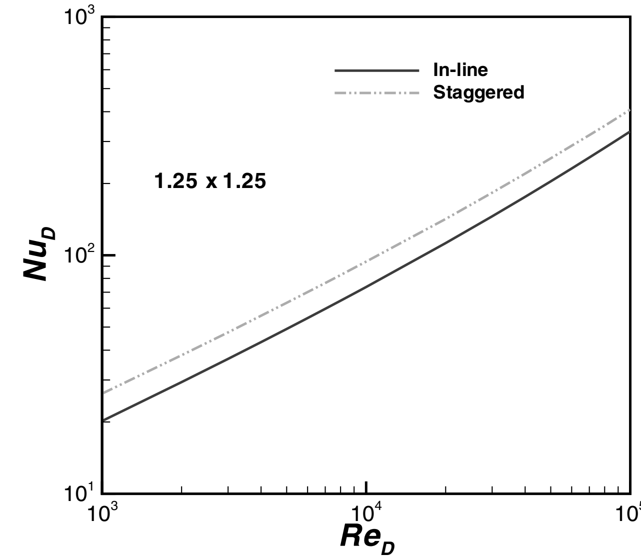

Fig. 10 Comparison of average heat transfer from in-line and staggered tube banks.

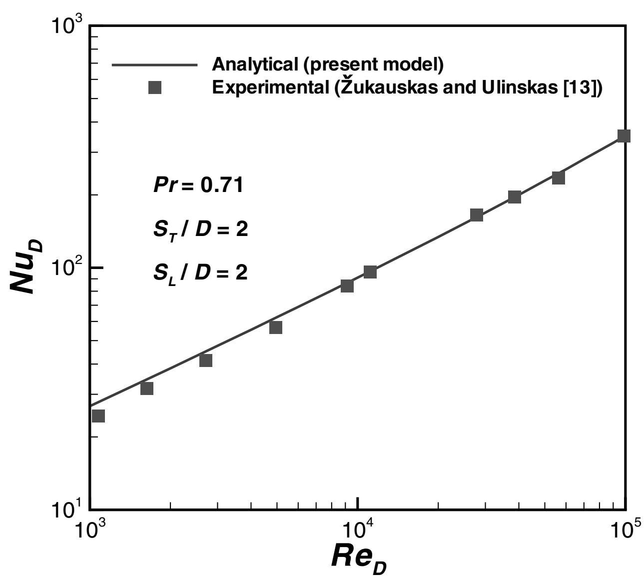

Fig. 9 Average heat transfer from a staggered tube bank.

experimental data of Žukauskas and Ulinskas [13]. The agreement is found to be good in both cases.

Figures 6 and 7 show the average heat transfer from a single tube in the first row of asymmetrical staggered tube banks with 2.0×1.25 and 2.6×1.3 pitch ratios. They show significant variation in the heat transfer at large Reynolds numbers for a large change in the transverse pitch ratio and a small change in the longitudinal pitch ratio. These results are also found to be in good agreement with the experimental data of Žukauskas and Ulinskas [13]. Similar results were obtained by Bergelin et al. [6,7] for the flow of air.

Average heat transfer values for the entire bank can be determined from Eq. (15) depending upon the number and the type of arrangement of tubes. For a compact in-line bank 1.25×1.25 with $N_L \geq 16$, the average heat transfer values are plotted versus Re_D in Fig. 8. On the logarithmic scale, heat transfer values increase linearly with the Reynolds numbers. The present values are compared with the empirical correlations of Grimison [14] and Žukauskas and Ulinskas [13]. Both correlations are found to be in good agreement with the analytical results. Figure 9 shows the heat transfer from a widely spaced staggered tube bank (2.0×2.0). The results are compared with the experimental data of Žukauskas and Ulinskas [13] for the same pitch ratios. Good agreement is found between the analytical and experimental results.

The comparison of the average heat transfer from both in-line and staggered tube banks for the same transverse and longitudinal pitch ratios is shown in Fig. 10. For the same pitch ratio, the heat transfer is found to be higher in a staggered bank than in an in-line bank. This is due to the fact that in a staggered bank the path of the main flow is more tortuous and a greater portion of the surface area of downstream tubes remains in this path.

IV. Conclusion

Heat transfer from tube banks in crossflow is investigated analytically, and simplified models of heat transfer for both arrangements (in-line and staggered) are presented. The coefficient C_1 in Eq. (16) is derived from the experimental data of Žukauskas and Ulinskas [13] and C_2 in Eq. (29) is determined by fitting the analytical results obtained for various pitch ratios in both arrangements. The results obtained from this investigation are as follows:

1) Both models can be applied over a wide range of parameters and are suitable for use in the design of tube banks.

2) The average heat transfer coefficients for tube banks in crossflow depend on the number of longitudinal rows, longitudinal and transverse pitch ratios, and Reynolds and Prandtl numbers.

3) Compact banks (in-line or staggered) indicate higher heat transfer rates than widely spaced ones.

4) The staggered arrangement gives higher heat transfer rates than the in-line arrangement.

Appendix

I. Cylindrical Tubes in an In-Line Arrangement

Following Suh et al. [30], the complex potential for in-line arrays, subjected to uniform flow, can be written as

$$w(z) = U_{\text{app}} z + \sum_{j,k=-\infty}^{\infty} \frac{\mu}{2\pi[(z - kS_L) - ijS_T]}$$
$$= U_{\text{app}} z + \frac{\mu}{2\pi} \sum_{j,k=-\infty}^{\infty} \frac{1}{[(z - kS_L) - ijS_T]} \tag{33}$$

where j and k are the number of rows and columns. Using

$$\sum_{j=-\infty}^{\infty} \frac{1}{z - ijS_T} = \frac{\pi}{S_T} \coth\left(\frac{\pi z}{S_T}\right) \tag{34}$$

the complex potential for in-line bank can be written as

$$w(z) = U_{\mathrm{app}} z + \frac{\mu}{2S_T} \sum_{k=-\infty}^{\infty} \coth \frac{\pi}{S_T}(z - kS_L) = U_{\mathrm{app}} z + \frac{\mu}{2S_T} T(z) \tag{35}$$

where

$$T(z) = \sum_{k=-\infty}^{\infty} \coth \frac{\mu}{2S_T}(z - kS_L) \tag{36}$$

Using complex variable theory, it can be shown that

$$T(z) = (S_T/2S_L)\cot(\pi z/S_L) \tag{37}$$

Therefore, the complex potential will be

$$W(z) = U_{\mathrm{app}} z + (\mu/4S_L)\cot(\pi z/S_L) \tag{38}$$

which gives the complex velocity $W'(z)$ as follows:

$$W'(z) = U_{\mathrm{app}} - \frac{\mu}{4S_L} \frac{1}{\sin^2(\pi/S_L)} \left(\frac{\pi}{S_L}\right) \tag{39}$$

At the surface of the tube, $W'(R) = 0$, $\Rightarrow \mu/4S_L = (U_{\mathrm{app}} S_L/\pi)\sin^2(\pi R/S_L)$ Therefore the required potential flow function for in-line tubes will be

$$W(z) = \phi + i\psi = U_{\mathrm{app}}\{z + C\cot(\pi z/S_L)\} \tag{40}$$

where ϕ and ψ are the potential and stream functions and C is a constant, given by

$$C = (S_L/\pi)\sin^2(\pi R/S_L) \tag{41}$$

It is interesting to note here that the potential flow field has no dependence on the transverse spacing S_T for the infinite number of rows. The stream function ψ in polar coordinates (r, θ) can be obtained from Eq. (40) as

$$\psi = U_{\mathrm{app}}\left\{r\sin\theta - C\frac{\sinh(C_1 r\sin\theta)}{\cosh(C_1 r\sin\theta) - \cos(C_1 r\cos\theta)}\right\} \tag{42}$$

where $C_1 = 2\pi/S_L$ is a constant. The radial and transverse components of velocity at the surface of the tube can be written as

$$u_r = -(1/r)(\partial\psi/\partial\theta)|_{r=R} \quad \text{and} \quad u_\theta = \partial\psi/\partial r|_{r=R} \tag{43}$$

which gives

$$u_r = 0 \quad \text{and} \quad u_\theta = U_{\mathrm{app}} f(\theta) \tag{44}$$

where

$$f(\theta) = \sin\theta - 2\sin^2\left(\frac{\pi}{2a}\right)\left\{\frac{\cosh[(\pi/a)\sin\theta]\sin\theta}{\cosh[(\pi/a)\sin\theta] - \cos[(\pi/a)\cos\theta]} + \sinh\left(\frac{\pi}{a}\sin\theta\right)\frac{\sinh[(\pi/a)\sin\theta]\sin\theta + \cos\theta\sin[(\pi/a)\cos\theta]}{[\cosh((\pi/a)\sin\theta) - \cos((\pi/a)\cos\theta)]^2}\right\} \tag{45}$$

The resultant potential flow velocity will be

$$U = U_{\mathrm{app}} f(\theta) \tag{46}$$

II. Cylindrical Tubes in Staggered Arrangement

Following Suh et al. [30], the complex potential for in-line arrays, subjected to uniform flow, can be written as

$$W(z) = \phi + i\psi = U_{\mathrm{app}}\left\{z + C\left[\cot\left(\frac{\pi z}{2S_L}\right) + \cot\left(\frac{\pi[z - (S_L + iS_T)]}{2S_L}\right)\right]\right\} \tag{47}$$

where ϕ and ψ are the potential and stream functions and C is a constant, given by

$$C = (2S_L/\pi)\sin^2(\pi R/2S_L) \tag{48}$$

The stream function ψ in polar coordinates (r, θ) can be obtained from Eq. (47) as

$$\psi = U_{\mathrm{app}}\left\{r\sin\theta - \frac{2S_L}{\pi}\sin^2\left(\frac{\pi R}{2S_L}\right)\left[\frac{\sinh((\pi r\sin\theta)/S_L)}{\cosh((\pi r\sin\theta)/S_L) - \cos((\pi r\sin\theta)/S_L)}\right] - \left[\frac{2S_L}{\pi}\sin^2\left(\frac{\pi R}{2S_L}\right)\frac{\sinh((\pi[r\sin\theta - S_T])/S_L)}{\cosh((\pi[r\sin\theta - S_T])/S_L) - \cos((\pi[r\cos\theta - S_L])/S_L)}\right]\right\} \tag{49}$$

The radial and transverse components of velocity at the surface of the tubes can be obtained by using Eq. (43) and can be written like Eq. (44), where

$$f(\theta) = \sin\theta - 2\sin^2\left(\frac{\pi}{4a}\right)\left\{\frac{\cosh[(\pi\sin\theta)/2a]\sin\theta}{\cosh[(\pi\sin\theta)/2a] - \cos[(\pi\cos\theta)/2a]} - \sinh\left(\frac{\pi\sin\theta}{2a}\right)\frac{\sinh[(\pi\sin\theta)/2a]\sin\theta + \sin(\frac{\pi\cos\theta}{2a})\cos\theta}{[\cosh[(\pi\sin\theta)/2a] - \cos[(\pi\cos\theta)/2a]]^2} + \frac{\cosh(\pi(\sin\theta - 2b)/2a)\sin\theta}{\cosh(\pi(\sin\theta - 2b)/2a) - \cos(\pi(\cos\theta - 2a)/2a)} - \sinh\left(\pi\frac{\sin\theta - 2b}{2a}\right)\frac{\sinh(\pi(\sin\theta - 2b)/2a)\sin\theta + \sin(\pi(\cos\theta - 2a)/2a)\cos\theta}{[\cosh(\pi(\sin\theta - 2b)/2a) - \cos(\pi(\cos\theta - 2a)/2a)]^2}\right\} \tag{50}$$

The resultant potential flow velocity will be:

$$U = U_{\mathrm{app}} f(\theta) \tag{51}$$

Acknowledgments

The authors gratefully acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada and the Centre for Microelectronics Assembly and Packaging.

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