Optimization of Pin-Fin Heat Sinks Using Entropy Generation Minimization

W. A. Khan, J. R. Culham, and M. M. Yovanovich
Microelectronics Heat Transfer Laboratory
Department of Mechanical Engineering
University of Waterloo
Waterloo, Ontario, Canada N2L 3G1
Phone: (519)885-1211 Ex (5247)
Fax: (519)746-9141
Email: wkhan@mhtlab.uwaterloo.ca

ABSTRACT

In this study, an entropy generation minimization, EGM, technique is applied as a unique measure to study the thermodynamic losses caused by heat transfer and pressure drop in cylindrical pin-fin heat sinks. The use of EGM allows the combined effect of thermal resistance and pressure drop to be assessed through the simultaneous interaction with the heat sink. A general expression for the entropy generation rate is obtained by considering the whole heat sink as a control volume and applying the conservation equations for mass and energy with the entropy balance. Analytical/empirical correlations for heat transfer coefficients and friction factors are used in the optimization model, where the characteristic length is used as the diameter of the pin and reference velocity used in Reynolds number and pressure drop is based on the minimum free area available for the fluid flow. Both in-line and staggered arrangements are studied and their relative performance is compared on the basis of equal overall volume of heat sinks. It is shown that all relevant design parameters for pin-fin heat sinks, including geometric parameters, material properties and flow conditions can be simultaneously optimized.

KEY WORDS: Optimization, EGM, Heat Sinks, In-Line, Staggered, Heat Sink Resistance.

NOMENCLATURE

A	total base area $[m^2]$
A_{bp}	exposed area of base plate $[m^2]$
A_c	cross-section or contact area of a single pin $[m^2]$
A_{fin}	surface area of a single pin
A_{hs}	surface area of heat sink $[m^2]$
D	pin diameter $[m]$
f	friction factor
g, l	equality and inequality constraints
H	pin height $[m]$
h	heat transfer coefficient $[W/m^2K]$
j	number of imposed constraints
k	thermal conductivity $[W/mK]$
$\mathcal{L}$	Lagrangian function
L	length of heat sink in flow direction $[m]$
N	total number of pins in heat sink $\equiv N_T N_L$
n	number of design variables
N_L	number of rows in streamwise direction
N_T	number of rows in spanwise direction
Nu_D	Nusselt number based on pin diameter
Nu_L	Nusselt number based on the heat sink length in the flow direction
Q	total base heat flow rate $[W]$
R	resistance $[^0C/W]$
Re_D	Reynolds number $\equiv DU_a/\nu$
$\dot{S}_{gen}$	entropy generation rate $[W/K]$
$\mathcal{S}_D$	dimensionless diagonal pitch $\equiv S_D/D$
$\mathcal{S}_L$	dimensionless streamwise pitch $\equiv S_L/D$
$\mathcal{S}_T$	dimensionless spanwise pitch $\equiv S_T/D$
S_D	diagonal pitch $[m]$
S_L	pin spacing in streamwise direction $[m]$
S_T	pin spacing in spanwise direction $[m]$
T	absolute temperature $[K]$
t	thickness $[m]$
U_a	approach velocity $[m/s]$
U_{max}	maximum velocity in minimum flow area $[m/s]$
x_i	design variables

Subscripts

a	approach
amb	ambient
bp	base plate
c	contact
w	wall
f	fluid
fin	single fin

$fins$	all fins with exposed base plate area
hs	heat sink
m	bulk material

Greek Symbols

ΔP	pressure drop across heat sink $[N/m^2]$
γ	slenderness ratio $\equiv H/D$
λ, χ	Lagrangian multipliers
μ	absolute viscosity of fluid $[kg/m\,s]$
ν	kinematic viscosity of fluid $[m^2/s]$
ρ	fluid density $[kg/m^3]$

INTRODUCTION

The selection of an appropriate heat sink has become crucial to the overall performance of electronic packages as heat flux densities increase with product miniaturization. Applications utilizing arrays of pin-fins for cooling have increased significantly during the last few decades, especially in microelectronics. Cylindrical pin-fin heat sinks are commonly used in applications that exhibit severe space and acoustic restrictions and for applications that possess a substantial amount of heat. The effective cooling scheme for these heat sinks is forced convection where the pins are in cross-flow with air.

Following studies, related to optimization of pin-fin heat sink geometries, are found in the open literature.

Poulikakos and Bejan [1] established a theoretical framework to determine the optimum fin dimensions for minimum entropy generation in forced convection. They first developed an expression for the entropy generation rate for a general fin and then applied it to select the optimum dimensions of pin fins, rectangular plate fins, plate fins with trapezoidal cross section, and triangular plate fins with rectangular cross section. Their study seems to be inconclusive as to which geometry offers advantages over others.

Bejan and Morega [2] reported the optimal geometry of an array of fins that minimizes the thermal resistance between the substrate and the forced flow through the fins. They modeled pin-fin arrays as the Darcy-flow porous medium and expressed the local thermal conductance in dimensionless form.

Lin and Lee [3] performed a second law analysis on a pin-fin array under forced flow conditions. They evaluated optimal operational/design conditions for both the in-line and staggered fin alignments. They considered the heat transfer contributions from the base wall as well as from the fin surface and found the optimal Re_D values as 2068 for the in-line and 1974 for the staggered alignment. It is also noted that in the range where $Re_D < Re_{Dopt}$, the in-line array would generate more entropy than does the staggered arrays and that Re_{Dopt} increases with the decreasing slenderness ratio, whereas the corresponding entropy generation number decreases only slightly.

Jubran et al. [4] performed an experimental investigation on the effects of inter fin spacing, shroud clearance, and missing pins on the heat transfer from cylindrical pin fins arranged in staggered and in-line arrays. They found that the optimum inter fin spacing in both span wise and stream wise directions is $2.5D$ regardless of the type of array and shroud clearance used. They also found the effect of missing fin to be negligible for the in-line array but more significant for the staggered arrays. Later, Bejan [5] extended the previous work of Jubran et al. and proved the existence of an optimal spacing between the cylinders. He showed that this optimal spacing increases with the length of the bundle and decreases with the applied pressure difference and the Prandtl number.

Tahat et al. [6] studied the effects of varying the geometrical configurations of the pin-fins and found the optimal pin-fin separation in both streamwise as well as spanwise directions to achieve maximum heat transfer rate. They established a general empirical correlation for the average Nusselt number which can be used for both in-line and staggered arrangements. After 6 years Tahat et al. [7] repeated previous experiments for a wider range of Reynolds number, S_T/W, and S_L/L to give separate correlations for the in-line and staggered arrangements.

Azar and Mandrone [8] investigated the effect of pin-fin density on thermal performance of unshrouded pin-fin heat sinks. They found an optimal number of pin fins beyond which thermal resistance actually increased. They also found that thermal resistance was a function of the approach velocity and the governing flow pattern. Further, pin-fin heat sinks with a small number of pins had the best performance at low and moderate forced convection cooling.

Minakami and Iwasaki [9] conducted experiments to investigate the pressure loss characteristics and heat transfer performance of pin-fin heat sinks exposed to air flow in a cross-flow direction, varying the pin pitch as a parameter. They found that as the longitudinal pitch increased, the heat transfer coefficient increased and the pressure loss also increased. Further, as the transverse pitch decreased, the heat transfer coefficient increased, but the pressure loss increased drastically compared to the Nu_D.

Babus'Haq et al. [10] investigated experimentally the thermal performance of a shrouded vertical Duralumin pin-fin assembly in the in-line and staggered configurations . They found that under similar flow conditions and for an equal number of pin-fins, the staggered configuration yields a higher steady-state rate of heat transfer than the in-line configuration. They studied the effect of changing the thermal conductivity of the pin-fin material and found that the optimal separation between the pin-fins in the streamwise direction increased with the thermal conductivity of the pin-fin material, whereas the optimal separations in the spanwise direction remained invariant.

Jonsson and Palm [11] performed experiments to compare the thermal performance of the heat sinks with different fin designs including straight fins and pin fins with circular, quadratic and elliptical cross sections. They evaluated the thermal performance by comparing the thermal resistance of the heat sinks at equal average velocity and equal pressure drop. They recommended elliptical pin-fin heat sinks at high velocities and circular pin-fin heat sinks at mid-range velocities.

Stanescu et al. [12] performed an experimental, numerical and analytical study of the optimal spacing between cylinders in cross flow forced convection. They determined optimal cylinder-to-cylinder spacing by maximizing the overall thermal conductance between all the cylinders and the free stream. They found that the optimal spacing decreases as the Re_D increases, and as the flow length of the array L decreases.

Wirtz et al. [13] reported experimental results on the thermal performance of model pin-fin fan-sink assemblies. They used cylindrical, square, and diamond shape cross section pin-fins and found that cylindrical pin-fins give the best overall fan-sink performance. Furthermore, the overall heat sink thermal resistance decreases with an increase in either applied pressure rise or fan power and fin height. Zapach [14] verified experimentally a model for the optimization of pin-fin heat sinks. This model was based on Zukauskas [15] correlations of flow resistance and heat transfer from studies of tube bank heat exchangers. With some minor modification to the heat transfer correlation, he presented a model that can be used to optimize inter-pin spacing based on a constant fluid velocity or a fan curve.

Kondo et al. [16] presented a semi-empirical zonal approach for the design and optimization of pin-fin heat sinks cooled by impingement. They calculated the thermal resistance and pressure drop for an air-cooled heat sink and performed experiments and flow visualization to validate the model's predictions.

Following Kern and Kraus [17], Sonn and Bar-Cohen [18] and Iyengar and Bar-Cohen [19, 20] performed a least material optimization of cylindrical pin-fin, plate-fin, and triangular-fin array geometries by extending the use of least-material single fin analysis to multiple fin arrays. Bar-Cohen and Jelinek [21] developed guidelines and design equations for optimum plate-fin arrays.

It is obvious from the literature survey that all optimization studies, related to cylindrical pin-fin heat sinks, are limited to the optimization of one or two design variables and are applicable over a fixed range of conditions only. In this study, all relevant design parameters for pin-fin heat sinks, including geometric parameters, material properties and flow conditions are optimized simultaneously by minimizing entropy generation rate $\dot{S}_{gen}$ subject to manufacturing and design constraints.

ASSUMPTIONS

This study assumes the following design considerations:

1. No flow bypassing
2. Adiabatic fin tips
3. Isotropic material
4. Uniform approach velocity
5. Fully developed heat and fluid flow
6. Airflow normal to pin axis
7. Steady, laminar and 2-D flow
8. Incompressible fluid with constant properties
9. Uniform heat spreading over entire base plate

MODEL DEVELOPEMENT

The entropy generation associated with heat transfer and frictional effects serve as a direct measure of lost potential for work or in the case of a heat sink, the ability to transfer heat to surrounding cooling medium. A model that establishes a relationship between entropy generation and heat sink design parameters can be optimized in such a manner that all relevant design conditions combine to produce the best possible heat sink for the given constraints. Following Bejan [22] and applying the laws of conservation of mass and energy with the entropy balance for a fluid flowing across a heat sink (Fig. 1), one can obtain an expression for the entropy generation rate:

$$\dot{S}_{gen} = \left(\frac{Q}{T_{amb}}\right)^2 R_{hs} + \frac{\dot{m}\Delta P}{\rho T_{amb}} \qquad (1)$$

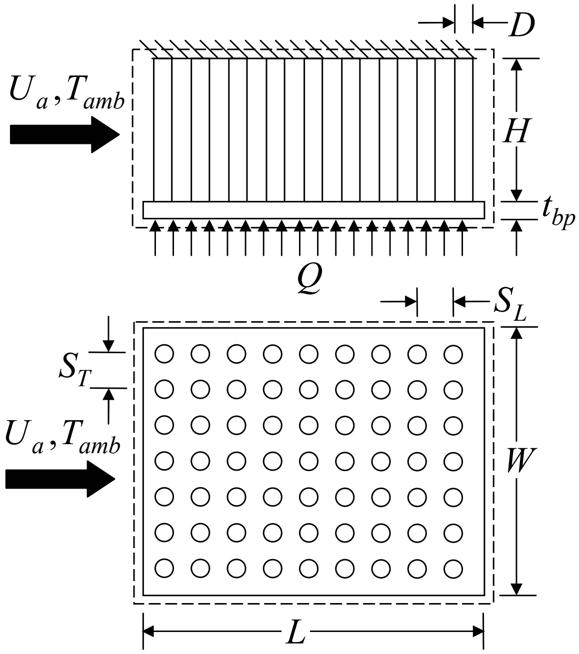

Fig. 1 control volume for a heat sink model

This expression shows that the entropy generation rate depends on the heat sink resistance and the pressure drop across the heat sink, provided that the heat load, mass flow rate and ambient conditions are specified. The lumped heat sink resistance is given by:

$$R_{hs} = R_m + R_{fins} \tag{2}$$

where R_m is the bulk material resistance, given by:

$$R_m = \frac{t_{bp}}{kA} \tag{3}$$

and R_{fins} is the overall resistance of the fins and the exposed base plate, which can be written as:

$$R_{fins} = \frac{1}{\dfrac{N}{R_c + R_{fin}} + \dfrac{1}{R_{bp}}} \tag{4}$$

where

$$R_c = \frac{1}{h_c A_c} \tag{5}$$

$$R_{fin} = \frac{1}{h_{fin} A_{fin} \eta_{fin}} \tag{6}$$

$$R_{bp} = \frac{1}{h_{bp} A_{bp}} \tag{7}$$

with

$$\eta_{fin} = \frac{\tanh(mH)}{mH} \tag{8}$$

$$m = \sqrt{\frac{4h_{fin}}{kD}} \tag{9}$$

Khan [23] has developed following analytical correlation for dimensionless heat transfer coefficient for the cylindrical fin in a fin array:

$$Nu_{Dfin} = \frac{h_{fin} D}{\nu} = C_1 Re_D^{1/2} Pr^{1/3} \tag{10}$$

where C_1 is a constant which depends upon the longitudinal and transverse pitches, arrangement of the pins, and thermal boundary conditions. For isothermal boundary condition, it is given by:

$$C_1 = \frac{[0.2 + \exp(-0.55\mathcal{S}_T)]\mathcal{S}_T^{\ 0.785}\mathcal{S}_L^{\ 0.212}}{(\mathcal{S}_T - 1)^{\ 0.5}} \tag{11}$$

for in-line arrangement, and

$$C_1 = \frac{0.61\,\mathcal{S}_T^{\ 0.591}\mathcal{S}_L^{\ 0.053}}{(\mathcal{S}_T - 1)^{\ 0.5}\,[1 - 2\exp(-1.09\,\mathcal{S}_T)]} \tag{12}$$

for staggered arrangement. The heat transfer coefficient for the base plate, h_{bp}, can be determined by considering it a finite plate. Khan [23] has developed following analytical correlation for dimensionless heat transfer coefficient for a finite plate:

$$Nu_L = \frac{h_{bp} L}{\nu} = 0.75 Re_L^{1/2} Pr^{1/3} \tag{13}$$

where L is the length of the base plate in the streamwise direction. The mass flow rate through the pins is given by:

$$\dot{m} = \rho U_a N_T \mathcal{S}_T H D \tag{14}$$

The pressure drop associated with flow across the pin fins is given by:

$$\Delta P = f \frac{\rho U_{max}^2}{2} N_L \tag{15}$$

where the friction factor f depends on the Reynolds number and the array geometry, and can be written as:

$$f = K_1\{0.233 + 45.78/(\mathcal{S}_T - 1)^{1.1} Re_D\} \tag{16}$$

for in-line arrangement, and

$$f = K_1\{378.6/\mathcal{S}_T^{13.1/\mathcal{S}_T}\}/Re_D^{0.68/\mathcal{S}_T^{1.29}} \tag{17}$$

for staggered arrangement, and K_1 is a correction factor depending upon the flow geometry and arrangement of the pins. It is given by:

$$K_1 = 1.009\left(\frac{\mathcal{S}_T - 1}{\mathcal{S}_L - 1}\right)^{1.09/Re_D^{0.0553}} \tag{18}$$

for in-line arrangement, and

$$K_1 = 1.175(\mathcal{S}_L/\mathcal{S}_T Re_D^{0.3124}) + 0.5Re_D^{0.0807} \tag{19}$$

for staggered arrangement. All the correlations for friction and correction factors are derived from graphs given in Zukauskas [15]. The velocity U_{max}, in Eq. (15), represents the maximum average velocity seen by the array as flow accelerates between pins, and is given by:

$$U_{max} = max\left\{\frac{\mathcal{S}_T}{\mathcal{S}_T - 1}U_a, \frac{\mathcal{S}_T}{\mathcal{S}_\mathcal{D} - 1}U_a\right\} \tag{20}$$

where $\mathcal{S}_\mathcal{D} = \sqrt{\mathcal{S}_L^2 + (\mathcal{S}_T/2)^2}$ is the dimensionless diagonal pitch. The calculation procedure for optimized design variables is best illustrated by the flow chart given in Fig. 2. This flow chart assumes that the heat load is known and the fluid properties are calculated at the ambient temperature. Within the temperature range encountered in typical microelectronic applications, the fluid properties can be considered constant without introducing significant error.

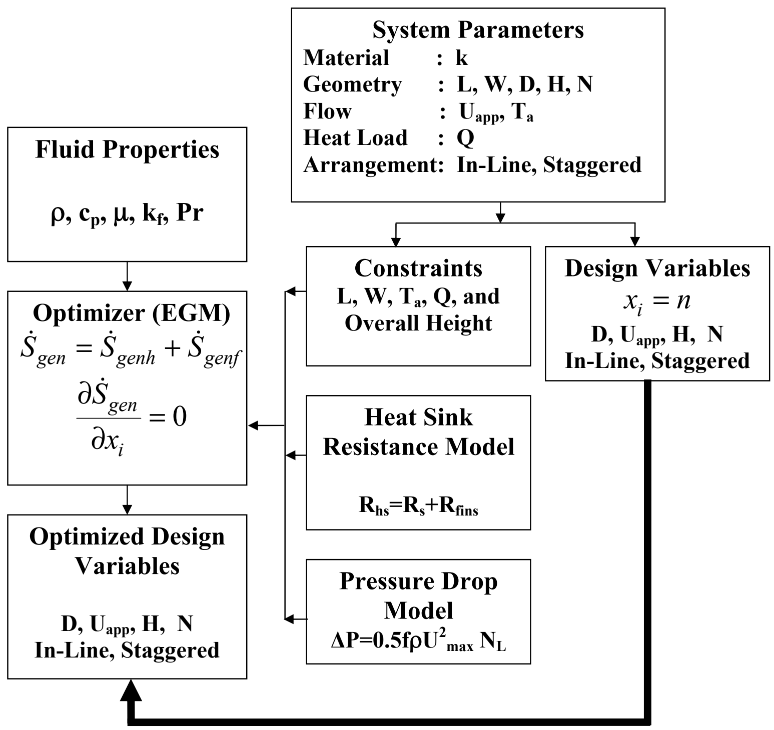

Figure 1: Flowchart for Calculation of Optimized design Variables

OPTIMIZATION MODEL

The problem considered in this study is to minimize the entropy generation rate, given by Eq. (1), for the optimal overall performance of the cylindrical pin-fin heat sink. If $f(\mathbf{x})$ represent the entropy generation rate that is to be minimized subject to equality constraints $g_j(x_1, x_2, .., x_n) = 0$ and inequality constraints $l_k(x_1, x_2, .., x_n) \geq 0$, then the complete mathematical formulation of the optimization problem may be written in the following form:

$$minimize \;\; f(\mathbf{x}) = \dot{S}_{gen}(\mathbf{x}) \tag{21}$$

subject to the equality constraints:

$$g_j(\mathbf{x}) = 0, \qquad j = 1, 2, ..., m \tag{22}$$

and inequality constraints

$$l_j(\mathbf{x}) \geq 0, \qquad j = m + 1, ..., n \tag{23}$$

where g_j and l_j are the imposed equality and inequality constraints and $\mathbf{x}$ denotes the vector of the design variables $(x_1, x_2, x_3, ..., x_n)^T$. The objective function can be redefined by using Lagrangian function as follows:

$$\mathcal{L}(\mathbf{x}, \lambda, \chi) = f(\mathbf{x}) + \sum_{j=1}^{m} \lambda_j g_j(\mathbf{x}) - \sum_{j=m+1}^{n} \chi_j l_j(\mathbf{x}) \quad (24)$$

where λ_j and χ_j are the Lagrange multipliers. The λ_j can be positive or negative but the χ_j must be ≥ 0. In addition to Kuhn-Tucker conditions, the other necessary condition for $\mathbf{x}^*$ to be a local minimum of the problem, under consideration, is that the Hessian matrix of $\mathcal{L}$ should be positive semidefinite, i.e.

$$\mathbf{v}^T \nabla^2 [(\mathbf{x}^*, \lambda^*, \chi^*)] \mathbf{v} \geq 0 \quad (25)$$

For a local minimum to be a global minimum, all the eigen-values of the Hessian matrix should be ≥ 0.

A system of non-linear equations is obtained, which can be solved using numerical methods such as a multivariable Newton-Raphson method. This method has been described in Stoecker [24] and applied by Culham and Muzychka [25] , and Culham et al. [26] to study the optimization of plate fin heat sinks. In this study, the same approach is used to optimize the overall performance of a cylindrical pin-fin heat sink in such a manner that all relevant design conditions combine to produce the best possible heat sink for the given constraints. The optimized results are then compared for in-line and staggered arrangements.

RESULTS AND DISCUSSION

Due to high heat loads and small space in many applications, optimization of the heat sinks is one of the most important issues to be taken into consideration in electronic equipment design. The main objectives of this study are to optimize and compare the overall performance of in-line and staggered pin-fin heat sinks.

For optimization, the objective is set to select the best heat sink to fit the 25.4×25.4 mm foot print but not to exceed a maximum overall height of 12 mm. The maximum height restriction is selected to represent a typical board pitch found in communications systems. It is also assumed that a total heat dissipation of 10 W is uniformly applied over the entire base plate which has a uniform thickness of 2 mm. The ambient temperature is fixed at $27\,^oC$ and the problem is solved for the two extreme thermal conductivities 25 and 400 (enhanced plastics and copper). Parametric variations include the pin diameter, D, approach velocity, U_a, and the total number of pins, N. With reference to this, three design options are examined by determining the heat sink geometry that leads to overall optimized performance where both heat transfer and viscous effects are considered.

In the first design, a fixed approach velocity is considered with the overall constraints of a maximum heat sink volume of $25.4 \times 25.4 \times 12$ mm. The problem is solved, in terms of pin diameter, for four extreme cases, i.e. low and high thermal conductivities with low and high pin densities. Results are summarized in Table 1, where the heat sink resistance, pressure drop and corresponding entropy generation rate are shown for each extreme case.

Table 1 Results For Optimization Of Single Parameter

k	N	Arrangement	Optimized Design Variables	Performance Parameters		
			D	R_{hs}	ΔP	$\dot{S}_{gen}$
$W/m.K$	$N_T \times N_L$		(mm)	$^oC/W$	N/m^2	W/K
25	5×5	In-Line	4.49	1.88	380.2	0.0024
	6×4	Staggered	5.41	1.26	249.2	0.00171
	10×10	In-Line	2.03	1.22	268.4	0.00159
	11×9	Staggered	2.17	0.95	229.0	0.0013
400	5×5	In-Line	4.47	1.47	351.3	0.00191
	6×4	Staggered	5.39	.905	229.9	0.0013
	10×10	In-Line	1.98	0.76	207.8	0.00102
	11×9	Staggered	2.12	0.53	181.4	0.00078

Table 2 Results For Optimization Of Two Parameters

k W/mK	N $N_T \times N_L$	Arrangement	Optimized Design Variables D (mm)	U (m/s)	R_{hs} $^oC/W$	ΔP N/m^2	$\dot{S}_{gen}$ W/K
25	5×5	In-Line	2.00	4.93	3.05	48.375	0.003
	6×4	Staggered	4.00	2.54	1.969	47.488	0.002
	10×10	In-Line	1.00	3.38	2.185	58.03	0.002
	11×9	Staggered	1.00	1.73	1.691	68.371	0.002
400	5×5	In-Line	2.00	4.91	2.32	40.017	0.002
	6×4	Staggered	4.00	2.66	2.446	51.424	0.002
	10×10	In-Line	0.90	3.20	1.308	40.97	0.001
	11×9	Staggered	1.00	1.56	1.068	53.763	0.001

Table 3 Results For Optimization Of Three Parameters

k W/mK	Arrangement	Optimized Design Variables D (mm)	U (m/s)	$N_T \times N_L$	R_{hs} $^oC/W$	ΔP N/m^2	$\dot{S}_{gen}$ W/K
25	In-Line	2.00	4.94	5×5	3.059	48.34	0.0030
	Staggered	3.00	2.49	6×4	2.296	55.21	0.0020
400	In-Line	0.80	3.04	11×11	1.233	41.45	0.0010
	Staggered	0.10	0.35	85×83	0.191	47.42	0.0002

It is clear that although a high thermal conductivity heat sink with high pin density is superior to all other cases, but a low thermal conductivity heat sink with high pin density still results in acceptable performance in terms of entropy generation rate. In both cases in-line arrangement is the best.

In the second design, both pin diameter and approach velocity are kept as free variables. The results are presented for all the four extreme cases in Table 2. Once again, the high thermal conductivity heat sink with high pin density leads the other cases. Finally, the problem is re-analyzed with three free variables D, U_a, and N. The results of this design are shown in Table 3. The heat sink resistance and pressure drop, for both arrangements, decrease with the thermal conductivity of the material and the pin density. But the heat sink resistance is higher for in-line arrangement whereas the pressure drop is lower under the same conditions.

Figure 3 shows the entropy generation rate for in-line arrangement for high thermal conductivity of the material and high pin density. It can be seen that the higher approach velocity gives the least optimum pin diameter. Figure 4 shows the same trend for staggered arrangement under the same conditions.

The comparison of entropy generation rate, for both arrangements, is shown in Fig. 5 under the same flow conditions and for the same material. It can be inferred that the staggered arrangement gives the higher entropy generation rate for the whole range of pin diameters.

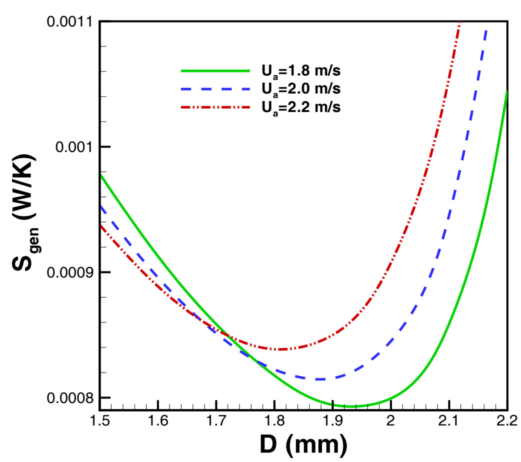

Fig. 3 Entropy generation Rate For In-Line Arrangement

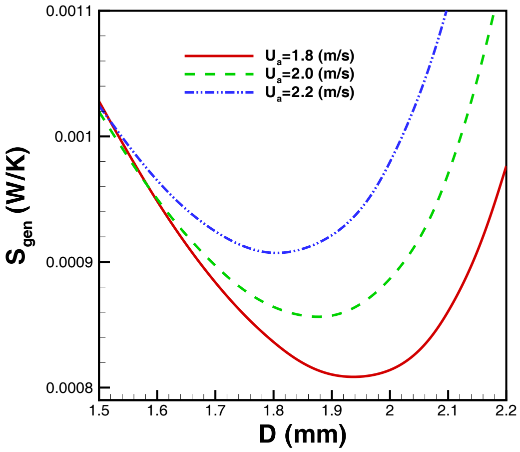

Fig. 4 Entropy generation Rate For Staggered Arrangement

The comparison of entropy generation rate, for both arrangements, is shown in Fig. 5 under the same flow conditions and for the same material. It can be inferred that the staggered arrangement gives the higher entropy generation rate for the whole range of pin diameters. The total entropy generation rate includes the contributions due to heat transfer and viscous friction. As the approach velocity is increased, the contribution due to heat transfer decreases and that of viscous friction, increases for both arrangements. This contribution is shown in Fig. 6 for the staggered arrangement. There is an optimum diameter for each approach velocity, pin density, thermal conductivity of the material, and arrangement of pins.

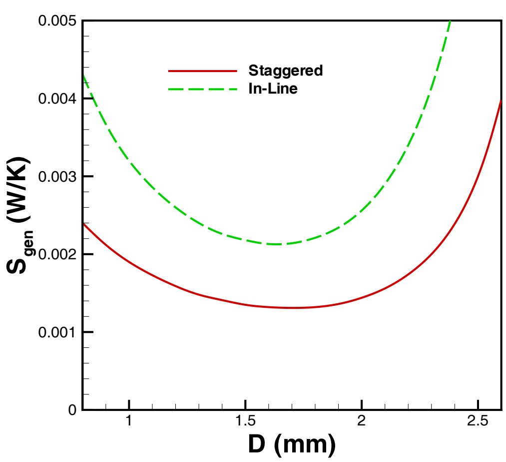

Fig. 5 Comparison of Entropy generation Rate for In-Line and Staggered Arrangement

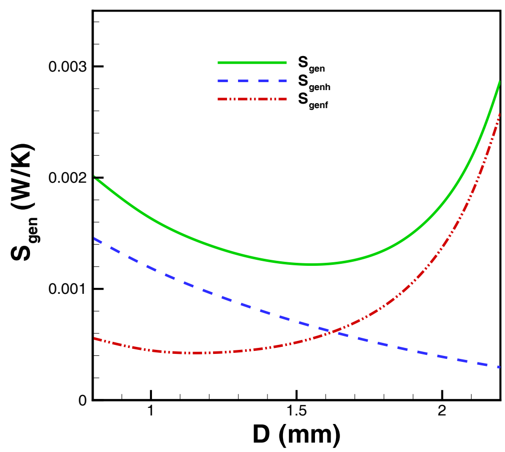

Fig. 6 Contribution of Entropy generation Rate Due To Heat And Fluid Friction For Staggered Arrangement

CONCLUSION

A scientific procedure is presented for determining optimum heat sink conditions given the simultaneous consideration of both heat transfer and viscous dissipation. The effects of approach velocity, pin density and heat sink thermal conductivity are examined with respect to its role in influencing optimum design conditions and the overall performance of the heat sink. It is demonstrated that the entropy generation rate is higher for staggered arrangement in all cases and it decreases with the increase in thermal conductivity of the material. While the conventional heat sinks with high pin density are superior to other heat sinks, a low conductivity heat sink also provides a viable alternative to those heat sinks.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support of Natural Sciences and Engineering Research Council of Canada and the Center for Microelectronics Assembly and Packaging.

REFERENCES

[1] Poulikakos, A. and Bejan, A., "Fin Geometry for Minimum Entropy Generation in Forced Convection," ASME Journal of Heat Transfer, Vol. 104, pp. 616-623, 1982.

[2] Bejan, A. and Morega, A. M., "Optimal Arrays of Pin Fins and Plate Fins in Laminar Forced Convection," ASME Journal of Heat Transfer, Vol. 115, pp. 75-81, 1993.

[3] Lin, W. W. and Lee, D. J., "Second-Law Analysis on a Pin Fin Array Under Crossflow," Int. J. Heat Mass Transfer, Vol. 40, No. 8, pp. 1937-1945, 1997.

[4] Jubran, B. A., Hamdan, M. A., and Abdullah, R. M., "Enhanced Heat Transfer, Missing Pin, and Optimization for Cylindrical Pin Fin Arrays," ASME Journal of Heat Transfer, Vol. 115, pp. 576-583, 1993.

[5] Bejan, A., "The Optimal Spacing for Cylinders in Crossflow Forced Convection," ASME Journal of Heat Transfer, Vol. 117, pp. 767-770, 1995.

[6] Tahat, M. A., Babus'Haq, R. F., and Probert, S. D., "Forced Steady-State Convections from Pin Fin Arrays," Applied Energy, Vol. 48, pp. 335-351, 1994.

[7] Tahat, M. A., Kodah, Z. H., Jarrah, B. A. and Probert, S. D., "Heat Transfer from Pin-Fin Arrays Experiencing Forced Convection," Applied Energy, Vol. 67, pp. 419-442, 2000.

[8] Azar, K. and Mandrone, C. D., "Effect of Pin Fin Density of the Thermal Performance of Unshrouded Pin Fin Heat Sinks," ASME Journal of Electronic Packaging, Vol. 116, pp. 306-309, 1994.

[9] Minakami, K. and Iwasaki, H., "Heat-Transfer Characteristics of Pin-Fins with In-Line Arrangement," Heat Transfer - Japanese Research, Vol. 23, No. 3, pp. 213-228, 1994.

[10] Babus'Haq, R. F., Akintunde, K. and Probert, S. D., "Thermal Performance of a Pin-Fin Assembly," Int. J. of Heat and Fluid Flow, Vol. 16, No. 1, pp. 50-55, 1995.

[11] Jonsson, H. and Palm, B., "Experimental Comparison of Different Heat Sink Designs for Cooling of Electronics," ASME, HTD-Vol. 329, National Heat Transfer conference, Vol. 7, pp. 50-55, 1996.

[12] Stanescu, G., Fowler, A. J. and Bejan, A., "The Optimal Spacing of Cylinders in Free-Stream Crossflow Forced Convection," Int. J. Heat Mass Transfer, Vol. 39, No. 2, pp. 311-317, 1996.

[13] Wirtz, R. A., Sohal, R., and Wang, H., "Thermal Performance of Pin-Fin Fan-Sink Assemblies," J. of Electronic Packaging, Vol. 119, March, pp. 26-31, 1997.

[14] Zapach, T., Newhouse, T., Taylor, J., and Thomasing, P., "Experimental Verification of a Model for the Optimization of Pin Fin Heat Sinks," The Seventh Inter Society Conference on Thermal Phenomena, Las Vegas, Nevada, USA, May 23 - 26, Vol. 1, pp. 63-69, 2000.

[15] Zukauskas, A., "Heat Transfer from Tubes in Crossflow," Advances in Heat Transfer, Vol. 8, pp. 93-160, 1972.

[16] Kondo, Y., Matsushima, H. and Komatsu, T., "Optimization of Pin-Fin Heat Sinks for Impingement Cooling of Electronic Packages," J. of Electronic Packaging, Vol. 122, September, pp. 240-246, 2000. [17] Kern, D. Q. and Kraus, A. D., "Extended Surface Heat Transfer," McGraw-Hill, New York, 1972. [18] Sonn, A. and Bar-Cohen, A., "Optimum Cylindrical Pin-Fin," Journal of Heat Transfer, Vol. 103, pp. 814-815, 1981. [19] Iyengar, M. and Bar-Cohen, A., "Least-Material Optimization of Vertical Pin-Fin, Plate-Fin, and Triangular-Fin Heat Sinks in Natural Convective Heat Transfer," Proceedings of the Intersociety