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ABSTRACT

In this study an approximate analytical method, known as the Von Karman-Pohlhausen method, is used to investigate fluid flow and heat transfer from cylindrical pin fins. A fourth order velocity profile in the hydrodynamic boundary layer and a third order temperature profile in the thermal boundary layer are used to obtain a closed form solution for the fluid flow and heat transfer from a cylindrical pin fin. The momentum and energy equations in the integral form are used to obtain the solution. Both isothermal and isoflux boundary conditions are applied. The results for both boundary conditions are found to be in a good agreement with experimental/numerical data for a single circular cylinder. The effects of free stream turbulence and blockage are not considered in this study.

NOMENCLATURE

C_D	total drag coefficient
C_{Df}	friction drag coefficient
C_{Dp}	pressure drag coefficient
C_f	skin friction coefficient $\equiv 2\tau_w/\rho U_\infty^2$
C_p	pressure coefficient $\equiv 2\Delta p/\rho U_\infty^2$
c_p	specific heat of the fluid $[kJ/kgK]$
D	cylinder diameter $[m]$
k	thermal conductivity $[W/mK]$
$\bar{h}$	average heat transfer coefficient $[W/m^2K]$
Nu_D	average Nusselt number based on the diameter of the cylinder $\equiv \bar{h}\, D/k_f$
Pr	Prandtl number $\equiv \nu/\alpha$
p	pressure $[N/m^2]$
q	heat flux $[W/m^2]$
Re_D	Reynolds number based on the diameter of the cylinder $\equiv DU_\infty/\nu$
T	temperature $[^0C]$
x	distance along the curved surface of the circular cylinder measured from the forward stagnation point $[m]$
y	distance normal to and measured from the surface of the circular cylinder $[m]$
u	x - component of velocity in the boundary layer $[m/s]$
v	y - component of velocity in the boundary layer $[m/s]$
$U(x)$	potential flow velocity just outside the boundary layer $\equiv 2U_\infty \sin\left(\frac{2x}{D}\right)$ $[m/s]$

Subscripts

f	fluid or friction
∞	free stream conditions
p	pressure
r, θ	radial and transverse directions in the plane polar coordinates
s	separation
T	temperature
w	wall

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s	separation
T	temperature
w	wall

Greek Symbols

δ	hydrodynamic boundary-layer thickness $[m]$
δ_1	displacement thickness $[m]$
δ_2	momentum thickness $[m]$
δ_T	thermal boundary layer thickness $[m]$
λ	pressure gradient parameter
μ	absolute viscosity of the fluid $[kg/m\,s]$
ν	kinematic viscosity of the fluid $[m^2/s]$
ρ	density of the fluid $[kg/m^3]$
τ	shear stress $[N/m^2]$
θ	angle measured [radians]

INTRODUCTION

The main objective of this study is to investigate analytically the fluid flow and heat transfer from a cylindrical pin fin under isothermal and isoflux boundary conditions. In practice, a pin fin is rarely used as a single entity, but is instead, used in two-dimensional arrays. These arrays are found in many applications, such as cross flow heat exchangers, heat sinks, and electric heating elements in boilers. Cylindrical pin-fin heat sinks are used to cool electronic equipment, where heat loads are substantial and space is limited. In these heat sinks, where the arrays of pins are attached to a common base, the fluid flow over the arrays determines the manner in which the average heat transfer coefficient $\bar{h}$ varies with location in the arrays.

It has been observed experimentally by Žukauskas[1] that the flow characteristics and, consequently, the convective heat transfer for the first row of pins essentially resembles that of a single circular cylinder in cross flow. However, due to mutual interaction between the pins as well as the base plate, fluid flow and heat transfer characteristics are different in subsequent rows. If these interactions are neglected, then a single average heat transfer coefficient as well as drag coefficient can be assigned to each single pin which will be pertinent to an isolated cylinder. In this study a circular cylinder is considered in cross flow with air to investigate the fluid flow and heat transfer from a cylindrical pin fin for a wide range of parameters.

LITERATURE REVIEW

A review of existing literature reveals that most of the studies related to a single isolated cylinder are experimental or numerical. No analytical study exists to investigate the fluid flow and heat transfer together from a single isolated cylinder, that can be used for a wide range of Reynolds number as well as Prandtl number. Flow past a cylinder has been investigated experimentally/numerically by numerous authors. Žukauskas[1], Lamb[2], Roshko[3], Achenbach[4], and Schlichting[5] studied the influence of Reynolds number on separation point, skin friction, pressure distribution as well as the local velocity around the cylinder. Wieselsberger[6], and according to Schlichting[5], Flachsbart[7] and Roshko[3] investigated the influence of Reynolds number on the drag coefficients. Wieselsberger[6] performed extensive experimental work and showed that almost all the experimental points for the drag coefficient of circular cylinders of widely different diameters fall on a single curve. This curve is recognized as a standard curve to determine the drag coefficients of a circular cylinder. Churchill[8] reviewed many numerical studies for laminar flow around a cylinder. He compared different numerical methods by applying them to the calculation of friction, pressure and total drag coefficients of a circular cylinder in cross flow. He found that the most accurate results were those of Sucker and Brauer[9] among all the studies.

Heat transfer from a circular cylinder has been investigated also by many researchers. According to Žukauskas and Žiugžda[10], Kruzhilin[11], Frossling[12], and Eckert[13], as well as Drake et al.[14], Eckert and Soehngen[15], and recently, Refai Ahmed and Yovanovich[16] presented a number of calculation techniques, which involved analytical solutions of the boundary layer equations or of integral equations with the corresponding limiting conditions. Krall and Eckert[17], Cebeci and Smith[18], Lin et al.[19], and Chun and Boehm[20] obtained various finite difference solutions for low Reynolds numbers. Žukauskas and Žiugžda[10] presented a semi-analytical solution for the boundary-layer equations in the laminar, transitional, and turbulent parts of the boundary layer, taking into account the effects of free-stream turbulence, blockage factor, and Reynolds number on the heat transfer and fluid dynamics for a cylinder in cross flow. They found their results in good agreement with the results of Jones and Launder[21] and Karyakin and Sharov[22].

Giedt[23], Eckert and Soehngen[15], Quarmby and Fakhri[24], Refai Ahmed and Yovanovich [25] investigated experimentally the heat transfer from a circular cylinder under isothermal boundary condition. Krall and Eckert[17,26], and Sarma and Sukhatme[27] studied experimentally/numerically the local heat transfer from a horizontal cylinder to air under isoflux boundary condition. A summary of experimental/analytical correlations for the heat transfer from an isolated single cylinder under isothermal and isoflux boundary conditions is given in Table 1.

Table 1: Summary of Previous Experimental Correlations for Air (Pr = 0.71)

Authors	**Correlations/ Models**			**Conditions**
Churchill and Bernstein[33]	$Nu_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1+(0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$			Isothermal $Re_D\, Pr > 0.2$
	$Nu_D = C Re_D^n$			
	C	**n**	**$\mathbf{Re_D}$ Range**	**Boundary Condition**
Žukauskas[1]	0.6607 0.4493 0.2290 0.0669	0.4 0.5 0.6 0.7	$1 - 40$ $40 - 1 \times 10^3$ $1 \times 10^3 - 2 \times 10^5$ $2 \times 10^5 - 1 \times 10^6$	Isothermal
Morgan[34]	0.795 0.583 0.148 0.0208	0.384 0.471 0.633 0.814	$4 - 40$ $40 - 4 \times 10^3$ $4 \times 10^3 - 4 \times 10^4$ $4 \times 10^4 - 4 \times 10^5$	Isothermal
Hilpert[35]	0.891 0.821 0.615 0.174 0.0239	0.33 0.385 0.466 0.618 0.805	$1 - 4$ $4 - 40$ $40 - 4 \times 10^3$ $4 \times 10^3 - 4 \times 10^4$ $4 \times 10^4 - 4 \times 10^5$	Isothermal
Žukauskas and Žiugžda[10]	0.29	0.6	$1 \times 10^3 - 2 \times 10^5$	Isoflux
Sarma and Sukhatme[27]	0.62	0.505	$1200 - 4700$	Isoflux

It is obvious from the literature survey that all experimental/analytical correlations are applicable over a fixed range of conditions. Furthermore, no analytical study gives a closed form solution for the fluid flow and heat transfer from a circular cylinder for a wide range of Reynolds and Prandtl numbers. At most, they provide a solution at the front stagnation point or a solution of boundary layer equations for very low Reynolds numbers. In this study, a closed form solution is obtained for the drag coefficients and Nusselt number, which can be used for a wide range of parameters. For this purpose, the

Von Karman-Pohlhausen method is used, which was first introduced by Pohlhausen[28] at the suggestion of Von Karman[29] and then modified by Walz[30] and Holstein and Bohlen[31]. Schlichting[5] has explained and applied this method to the general problem of a two-dimensional boundary layer with pressure gradient. He obtained general solutions for the velocity profiles and the thermal boundary layers and compared them with the exact solution of a flat plate at zero incidence.

ANALYSIS

Consider a uniform flow of a Newtonian fluid past a fixed circular cylinder of diameter D, with vanishing circulation around it, as shown in Figure 1. The approaching velocity of the air is U_∞ and the ambient temperature of the air is assumed to be T_∞. The surface temperature of the wall is $T_w (< T_\infty)$ in the case of the isothermal cylinder and the heat flux is q for the isoflux boundary condition. The flow is assumed to be laminar, steady, and two dimensional. Using order-of-magnitude analysis, the reduced equations of continuity, momentum and energy in the plane polar coordinates for an incompressible fluid can be written as:

Continuity:

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} = 0 \tag{1}$$

θ - Momentum:

$$u_r\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} = -\frac{1}{r\rho}\frac{\partial p}{\partial \theta} + \nu\left\{\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r^2}\right\} \tag{2}$$

r - Momentum:

$$\frac{\partial p}{\partial r} = 0 \tag{3}$$

Energy:

$$u_r\frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} = \alpha\left\{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right\} \tag{4}$$

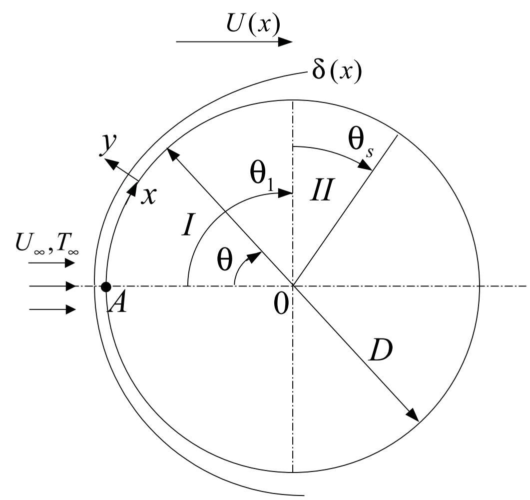

Fig. 1 Flow over a circular cylinder

These equations can be rewritten by adopting a curvilinear system of coordinates in which x denotes distance along the curved surface of the circular cylinder measured from the forward stagnation point A and y is the distance normal to and measured from the surface as shown in Fig. 1. In this system of coordinates, the velocity components u_θ and u_r are replaced by u and v in the local $x-$ and $y-$ directions whereas $rd\theta$ and dr are replaced by dx and dy respectively. The potential flow velocity just outside the boundary layer is denoted by $U(x)$. Therefore, the governing equations in this curvilinear system will be:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

x-Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{6}$$

y-Momentum:

$$\frac{dp}{dy} = 0 \tag{7}$$

Energy:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{8}$$

Bernoulli Equation:

$$-\frac{1}{\rho}\frac{dp}{dx} = U(x)\frac{dU(x)}{dx} \tag{9}$$

Hydrodynamic Boundary Conditions

At the cylinder surface, i.e., at $y = 0$:

$$u = 0 \qquad \text{and} \qquad \frac{\partial^2 u}{\partial y^2} = \frac{1}{\mu}\frac{\partial p}{\partial x} \tag{10}$$

At the edge of the boundary layer, i.e., at $y = \delta(x)$:

$$u = U(x), \qquad \frac{\partial u}{\partial y} = 0 \qquad \text{and} \qquad \frac{\partial^2 u}{\partial y^2} = 0 \quad (11)$$

Thermal Boundary Conditions

The boundary conditions for the uniform wall temperature (UWT) and uniform wall flux (UWF) are:

$$y = 0, \begin{cases} T = T_w & \text{for UWT} \\ \dfrac{\partial T}{\partial y} = -\dfrac{q}{k_f} & \text{for UWF} \end{cases} \quad (12)$$

$$y = 0, \qquad \frac{\partial^2 T}{\partial y^2} = 0 \quad (13)$$

$$y = \delta_T, \quad T = T_\infty \quad \text{and} \quad \frac{\partial T}{\partial y} = 0 \quad (14)$$

Velocity Distribution

Assuming a thin boundary layer around the cylinder, the velocity distribution in the boundary layer can be approximated by a fourth order polynomial as suggested by Pohlhausen[28]:

$$\frac{u}{U(x)} = (2\eta - 2\eta^3 + \eta^4) + \frac{\lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4) \quad (15)$$

where $0 \leq \eta = y/\delta(x) \leq 1$ and λ is the pressure gradient parameter, given by:

$$\lambda = \frac{\delta^2}{\nu} \frac{dU(x)}{dx} \quad (16)$$

With the help of velocity profiles, Schlichting[5] showed that the parameter λ is restricted to the range $-12 \leq \lambda \leq 12$.

Temperature Distribution

Assuming a thin thermal boundary layer around the cylinder, the temperature distribution in the thermal boundary layer can be approximated by a third order polynomial:

$$\frac{T - T_\infty}{T_w - T_\infty} = A + B\,\eta_T + C\,\eta_T^2 + D\,\eta_T^3 \quad (17)$$

where $\eta_T = y/\delta_T(x)$. Using the above mentioned thermal boundary conditions, the temperature distribution will be:

$$\frac{T - T_\infty}{T_w - T_\infty} = 1 - \frac{3}{2}\eta_T + \frac{1}{2}\eta_T^3 \quad (18)$$

for the isothermal boundary condition and:

$$T - T_\infty = \frac{2q\delta_T}{3k_f}\left(1 - \frac{3}{2}\eta_T + \frac{1}{2}\eta_T^3\right) \quad (19)$$

for the isoflux boundary condition.

Boundary-Layer Parameters

In dimensionless form, the momentum integral equation can be written as:

$$\frac{U\,\delta_2}{\nu}\frac{d\delta_2}{dx} + \left(2 + \frac{\delta_1}{\delta_2}\right)\frac{\delta_2^2}{\nu}\frac{dU}{dx} = \frac{\delta_2}{U}\left.\frac{\partial u}{\partial y}\right|_{y=0} \quad (20)$$

where

$$\delta_1 = \delta \int_0^1 \left[1 - \frac{u}{U(x)}\right] d\eta \quad (21)$$

and

$$\delta_2 = \delta \int_0^1 \frac{u}{U(x)}\left[1 - \frac{u}{U(x)}\right] d\eta \quad (22)$$

Using velocity distribution from Eq. (15), Eqs. (21) and (22) can be written as:

$$\delta_1 = \frac{\delta}{10}\left(3 - \frac{\lambda}{12}\right) \quad (23)$$

and

$$\delta_2 = \frac{\delta}{63}\left(\frac{37}{5} - \frac{\lambda}{15} - \frac{\lambda^2}{144}\right) \quad (24)$$

Assuming

$$Z = \frac{{\delta_2}^2}{\nu} \qquad \text{and} \qquad K = Z\,\frac{dU}{dx}$$

Equation (20) can be reduced to a non-linear differential equation of the first order for Z, which is given by:

$$\frac{dZ}{dx} = \frac{H(K)}{U} \quad (25)$$

where $H(K) = 2f_2(K) - 2K[2 + f_1(K)]$ is a universal function and is approximated by Walz[30] by a straight line:

$$H(K) = 0.47 - 6K \quad (26)$$

with

$$f_1(K) = \frac{63(3 - \lambda/12)}{10(37/5 - \lambda/15 - \lambda^2/144)} \quad (27)$$

$$f_2(K) = \frac{1}{63}\left(2 + \frac{\lambda}{6}\right)\left(\frac{37}{5} - \frac{\lambda}{15} - \frac{\lambda^2}{144}\right) \tag{28}$$

and

$$K = \frac{\lambda}{3963}\left(\frac{37}{5} - \frac{\lambda}{15} - \frac{\lambda^2}{144}\right)^2 \tag{29}$$

Solving Eq. (25) with Eq. (26), the local dimensionless momentum thickness can be written as:

$$\frac{\delta_2}{D} = \frac{0.3428}{\sqrt{Re_D}}\sqrt{\frac{1}{\sin^6\theta}\int_0^\theta \sin^5\zeta\, d\zeta} \tag{30}$$

This equation was solved numerically by using MAPLE 7, a symbolic mathematics software. From Eq. (16), the local dimensionless boundary layer thickness can be written as:

$$\frac{\delta}{D} = \sqrt{\frac{\lambda}{4\,Re_D\cos\theta}} \tag{31}$$

By solving Eqs. (24) and (31) and comparing the results with Eq. (30), the values of the pressure gradient parameter λ are obtained corresponding to each position along the cylinder surface. These values are positive in region I from $0 \le \theta \le \theta_1 = 90^o$ and negative in region II from $\theta_1 \le \theta \le \theta_s = 107.71^o$ (Fig.1). So the whole range of interest $0 \le \theta \le \theta_s$ can be divided into two regions and the λ values can be fitted separately by the least squares method into two polynomials, i.e., for region I:

$$\lambda_1 = 7.239\sum_{j=0}^{7} a_j\theta^j \tag{32}$$

and for region II:

$$\lambda_2 = 0.3259\sum_{j=0}^{10} b_j\theta^j \tag{33}$$

where a_j and b_j are the coefficients given in Table 2. These polynomials will be used to determine the drag and the local heat transfer coefficients in both regions.

Table 2: Coefficients Used in Eqs. (32) and (33)

j	0	1	2	3	4	5
a_j	1.000	-0.0053	-0.1344	-0.2998	0.6335	-0.7937
b_j	-2521.735	2834.998	-219.186	-262.703	109.967	-349.374

j	6	7	8	9	10
a_j	0.4583	-0.1123	-	-	-
b_j	131.419	-7.360	43.564	-21.629	1.564

Fluid Flow

The first parameter of interest is fluid friction which manifests itself in the form of the drag force F_D, where F_D is the sum of the skin friction drag D_f and pressure drag D_p. Skin friction drag is due to viscous shear forces produced at the cylinder surface predominantly in those regions where the boundary layer is attached. The component of shear force in the flow direction is given by

$$D_f = \int_A \tau_w \frac{D}{2}\sin\theta\, d\theta \tag{34}$$

where τ_w is the shear stress along the cylinder wall and it can be determined from Newton's law of viscosity:

$$\tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{35}$$

In dimensionless form, it can be written as:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho U_\infty^2} = \frac{4}{3}\frac{\lambda + 12}{\sqrt{Re_D}}\sin\theta\sqrt{\frac{\cos\theta}{\lambda}} \tag{36}$$

The friction drag coefficient can be defined as:

$$\begin{aligned} C_{Df} &= \int_0^\pi C_f\sin\theta\, d\theta \\ &= \int_0^{\theta_s} C_f\sin\theta\, d\theta + \int_{\theta_s}^\pi C_f\sin\theta\, d\theta \end{aligned} \tag{37}$$

Since no shear stress acts on the cylinder surface after boundary layer separation, therefore, the second integral will be zero and the friction drag coefficient can be written as:

$$\begin{aligned} C_{Df} &= \int_0^{\theta_s} C_f \sin\theta \, d\theta \\ &= \frac{4}{3\sqrt{Re_D}} \left\{ \int_0^{\theta_1} (\lambda_1 + 12)\sin^2\theta \sqrt{\frac{\cos\theta}{\lambda_1}}\, d\theta \right. \\ &\quad \left. + \int_{\theta_1}^{\theta_s} (\lambda_2 + 12)\sin^2\theta \sqrt{\frac{\cos\theta}{\lambda_2}}\, d\theta \right\} \\ &= \frac{5.784}{\sqrt{Re_D}} \end{aligned} \tag{38}$$

Pressure drag is due to the unbalanced pressures which exist between the relatively high pressures on the upstream surfaces and the lower pressures on the downstream surfaces. The component of pressure force in the flow direction is given by

$$D_p = \int_A p \, \frac{D}{2} \, \cos\theta \, d\theta \tag{39}$$

In dimensionless form, it can be written as:

$$C_{Dp} = \int_0^{\pi} C_p \, \cos\theta \, d\theta \tag{40}$$

where C_p is the pressure coefficient and can be defined as:

$$C_p = \frac{\Delta p}{\frac{1}{2}\rho U_\infty^2} \tag{41}$$

The pressure difference Δp can be obtained by integrating Eq. (2) w.r.t. θ. In dimensionless form, it can be written as:

$$\frac{\Delta p}{\frac{1}{2}\rho U_\infty^2} = 2(1 - \cos\theta) + \frac{8}{Re_D}(1 - \cos\theta) \tag{42}$$

So, the pressure drag coefficient for the cylinder up to the separation point will be:

$$\begin{aligned} C_{Dp} &= \int_0^{\theta_s} C_p \cos\theta d\theta \\ &= 1.152 + \frac{1.260}{Re_D} \end{aligned} \tag{43}$$

The total drag coefficient C_D can be written as the sum of both drag coefficients:

$$C_D = \frac{5.784}{\sqrt{Re_D}} + 1.152 + \frac{1.260}{Re_D} \tag{44}$$

Heat Transfer

The second parameter of interest in this study is the dimensionless average heat transfer coefficient, Nu_D for large Prandtl numbers. This parameter is determined by integrating Eq. (8) from the cylinder surface to the thermal boundary layer edge. Assuming the presence of a thin thermal boundary layer δ_T along the cylinder surface, the energy integral equation for the isothermal boundary condition can be written as:

$$\frac{d}{dx} \int_0^{\delta_T} (T - T_\infty) u \, dy = -\alpha \left. \frac{\partial T}{\partial y} \right|_{y=0} \tag{45}$$

Using velocity and temperature profiles, and assuming $\zeta = \delta_T/\delta < 1$, Eq. (45) can be simplified to:

$$\delta_T \frac{d}{dx} \left[U(x) \delta_T \zeta (\lambda + 12) \right] = 90\alpha \tag{46}$$

This equation can be rewritten separately for the above mentioned two regions, i.e.:

$$\delta_T \frac{d}{dx} \left[U(x) \delta_T \zeta (\lambda_1 + 12) \right] = 90\alpha \tag{47}$$

for the region I, and:

$$\delta_T \frac{d}{dx} \left[U(x) \delta_T \zeta (\lambda_2 + 12) \right] = 90\alpha \tag{48}$$

for the region II. Multiplying both equations by $U(x)\zeta$ and integrating separately in the two regions with respect to x, one can solve these two equations for the local thermal boundary layer thicknesses:

$$\frac{\delta_{T_1}}{D} = \frac{1}{Re_D^{1/2} Pr^{1/3}} \sqrt[3]{\frac{22.5 f_1(\theta)}{(\lambda_1 + 12)^2 \sin^2\theta} \sqrt{\frac{\lambda_1}{\cos\theta}}} \tag{49}$$

and

$$\frac{\delta_{T_2}}{D} = \frac{1}{Re_D^{1/2} Pr^{1/3}} \sqrt[3]{\frac{22.5 f_3(\theta)}{\sin^2\theta} \sqrt{\frac{\lambda_2}{\cos\theta}}} \tag{50}$$

where δ_{T_1} and δ_{T_2} are the thicknesses of the thermal boundary layers in the two respective regions and the functions $f_1(\theta)$ and $f_3(\theta)$ are given by:

$$f_1(\theta) = \int_0^{\theta} \sin\theta (\lambda_1 + 12) d\theta \tag{51}$$

and

$$f_3(\theta) = \frac{f_1(\theta)}{\lambda_1 + 12} + \frac{f_2(\theta)}{\lambda_2 + 12} \tag{52}$$

with

$$f_2(\theta) = \int_{\theta_1}^{\theta_s} \sin\theta(\lambda_2 + 12) d\theta \tag{53}$$

For the isothermal boundary condition, the local heat transfer coefficient can be defined as follows:

$$\begin{aligned} h(\theta) &= -\frac{k_f \left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{T_w - T_\infty} \\ &= \frac{3k_f}{2\delta_T} \end{aligned} \tag{54}$$

So, the local heat transfer coefficients for both the regions can be written as:

$$h_1(\theta) = \frac{3k_f}{2\delta_{T_1}} \qquad \text{and} \qquad h_2(\theta) = \frac{3k_f}{2\delta_{T_2}} \tag{55}$$

The average heat transfer coefficient can be defined as:

$$\begin{aligned} \bar{h} &= \frac{1}{\pi}\int_0^{\pi} h(\theta)\, d\theta \\ &= \frac{1}{\pi}\left\{\int_0^{\theta_s} h(\theta)\, d\theta + \int_{\theta_s}^{\pi} h(\theta)\, d\theta\right\} \end{aligned} \tag{56}$$

It has been observed experimentally by many researchers that, at low Reynolds numbers (up to $Re_D = 5000$, according to Žukauskas and Žiugžda[10]), there is no appreciable increase in the local heat transfer after separation point. However, at high Reynolds numbers, the local heat transfer increases from the separation point to the rear stagnation point but the effects of this increase on the average heat transfer are observed to be smaller and they could be reduced further if the free stream turbulence and the blockage effects were included in the analysis. Hence, the average heat transfer coefficient can be written as:

$$\begin{aligned} \bar{h} &= \frac{1}{\pi}\int_0^{\theta_s} h(\theta)\, d\theta \\ &= \frac{1}{\pi}\left\{\int_0^{\theta_1} h_1(\theta)\, d\theta + \int_{\theta_1}^{\theta_s} h_2(\theta)\, d\theta\right\} \end{aligned} \tag{57}$$

Using Eqs. (49) - (55), Eq. (57) can be solved for the average heat transfer coefficient which gives the average Nusselt number for an isothermal cylinder:

$$Nu_D|_{isothermal} = 0.5930\, Re_D^{1/2}\, Pr^{1/3} \tag{58}$$

For the isoflux boundary condition, the energy integral equation can be written as:

$$\frac{d}{dx}\int_0^{\delta_T} (T - T_\infty) u\, dy = \frac{q}{\rho\, c_p} \tag{59}$$

For constant heat flux and thermophysical properties, Eq. (59) can be simplified to:

$$\frac{d}{dx}\left[U(x)\delta_T^2\, \zeta(\lambda + 12)\right] = 90\frac{\nu}{Pr} \tag{60}$$

Rewritting Eq. (60) for the two regions in the same way as Eq. (46), one can obtain local thermal boundary layer thicknesses δ_{T_1} and δ_{T_2} under isoflux boundary condition. The local surface temperatures for the two regions can then be obtained from Eq. (19):

$$\Delta T_1(\theta) = \frac{2q\delta_{T_1}}{3k_f} \tag{61}$$

and

$$\Delta T_2(\theta) = \frac{2q\delta_{T_2}}{3k_f} \tag{62}$$

The local heat transfer coefficient can now be obtained from its definition as:

$$h_1(\theta) = \frac{q}{\Delta T_1(\theta)} \qquad \text{and} \qquad h_2(\theta) = \frac{q}{\Delta T_2(\theta)} \tag{63}$$

Following the same procedure for the average heat transfer coefficient as mentioned above, one can obtain the average Nusselt number for an isoflux cylinder as:

$$Nu_D|_{isoflux} = 0.6321\, Re_D^{1/2}\, Pr^{1/3} \tag{64}$$

This Nusselt number is 6% greater than the average Nusselt number for an isothermal cylinder. Combining the results for both thermal boundary conditions, we have

$$\frac{Nu_D}{Re_D^{1/2} Pr^{1/3}} = \begin{cases} 0.5930 & \text{for UWT} \\ 0.6321 & \text{for UWF} \end{cases} \tag{65}$$

RESULTS AND DISCUSSION

Flow Characteristics

The dimensionless local shear stress, $C_f\sqrt{Re_D}$, is plotted in Fig. 2. It shows that C_f is zero at the stagnation point and reaches a maximum at $\theta \approx 60^0$. The increase in shear stress is caused by the deformation of the velocity profiles in the boundary layer, a higher velocity gradient at the wall and a thicker boundary layer. In the region of decreasing C_f preceeding the separation point, the pressure gradient decreases further and finally C_f falls to zero at $\theta = 107.7^0$, where boundary-layer separation occurs. Beyond this point, C_f remains close to zero up to the rear stagnation point. These results are compared with the experimental results of Žukauskas and Žiugžda[10].

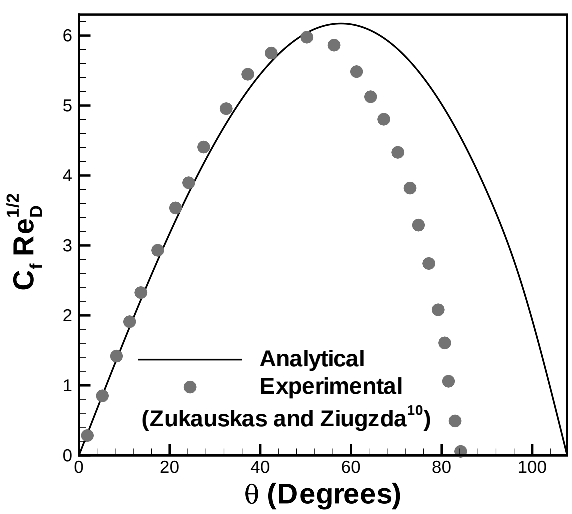

Fig. 2 Distribution of Shear Stress on a Circular Cylinder in Air

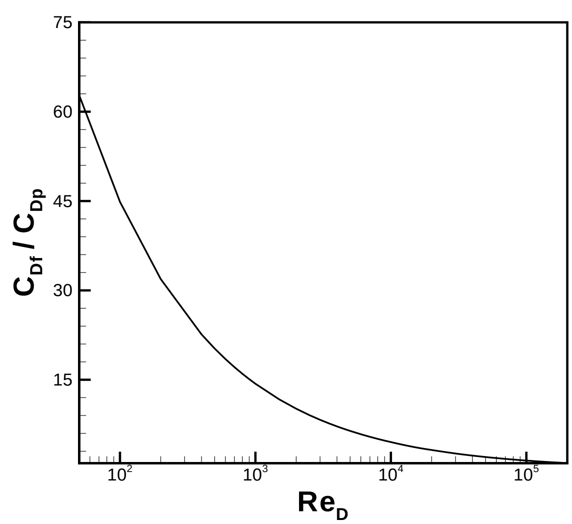

Fig. 3 Contribution of Friction Drag to Pressure Drag as a Function of Re_D

Equations (38) and (43) show the effects of Reynolds number on the friction and pressure drag. At low Reynolds number the pressure drag is insignificant and is proportional to $1/Re_D$, but at high Reynolds numbers it is independent of Re_D. The contribution of the friction drag to the pressure drag was found to be in the range 50 to 2% for Re_D from 30 to 10^4 by Goldstein[32], 3 to 1% for Re_D from 5×10^3 to 10^6 by Žukauskas and Žiugžda[10]. Figure 3 shows similar results of the contribution of the friction drag to the pressure drag obtained in the present analysis.

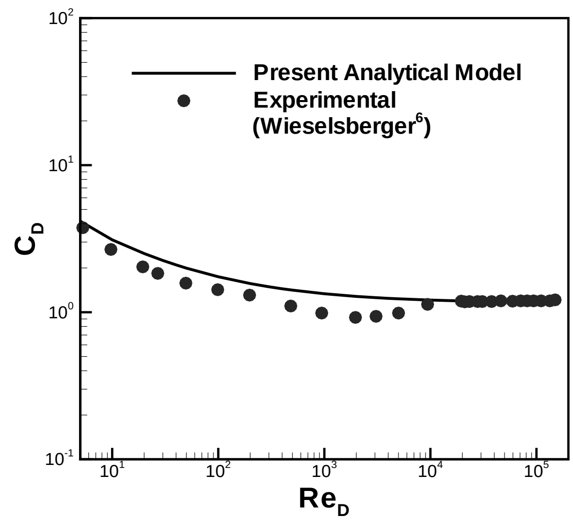

Fig. 4 Drag Coefficient as a function of Re_D for a Circular Cylinder

The variation of the total drag coefficient C_D with Re_D is illustrated in Fig. 4 for an infinite cylinder. The present results are compared with the experimental results of Wieselsberger [6]. It is clear that the present results are in good agreement except at $Re_D = 2 \times 10^3$, where a downward deviation (23.75%) in the experimental results was noticed. No physical explanation could be found in the literature for this deviation.

Heat Transfer Characteristics

The comparison of local Nusselt Numbers for the isothermal and isoflux boundary conditions is presented in Fig. 5. The isoflux boundary condition gives a higher heat transfer coefficient over the larger part of the circumference. On the front part of the cylinder (up to $\theta \approx 30^0$), there is no appreciable effect of boundary condition. Higher heat transfer coefficients have also been observed numerically by Krall and Eckert[17] and Chun and Boehm[20] and experimentally by Žukauskas and Žiugžda[10] with the isoflux boundary condition.

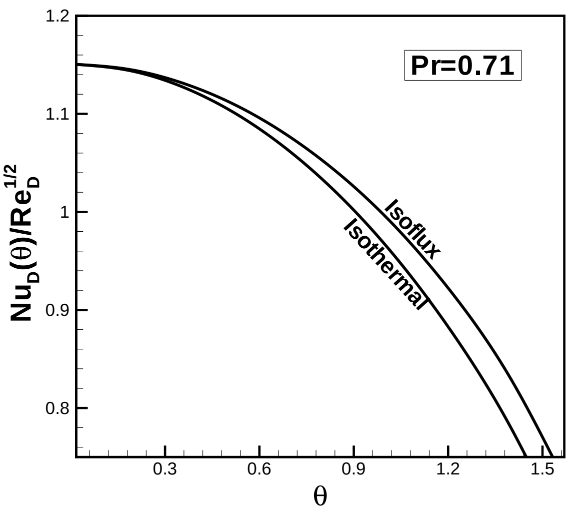

Fig. 5 Local Nusselt Numbers for different Boundary Conditions

The results of heat transfer from a single isothermal cylinder are shown in Fig. 6, where they are compared with the correlations of Churchill and Bernstein[33], Žukauskas[1], Morgan[34], and of Hilpert[35]. It shows that the previous correlations are

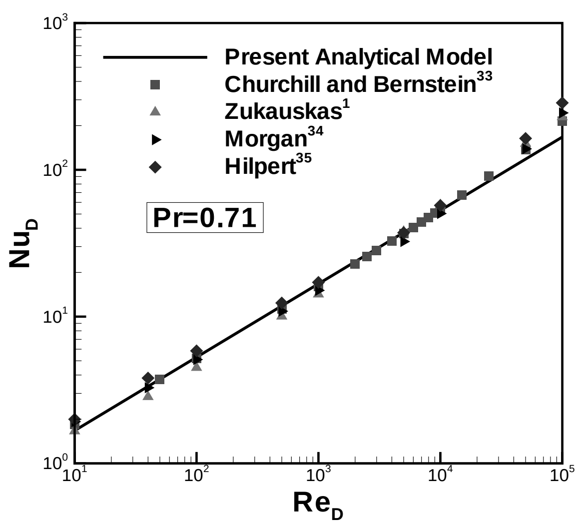

Fig. 6 Variation of Average Nusselt Number with Reynolds Number for Isothermal Boundary Condition

in very good agreement with the present correlation for $40 < Re_D < 4 \times 10^4$. Beyond this range, the discrepancy increases as the Reynolds number increases. This discrepancy could be the effect of free-stream turbulence or vortex shedding in actual experiments. It was demonstrated by Kestin[36], Smith and Kuethe[37], Dyban and Epick[38], and Kestin and Wood[39] that the heat transfer coefficient increases with turbulence intensity and that this effect is more intense when the Reynolds number is higher. In the present analysis these effects are not included, so the discrepancy can be observed clearly in Fig. 6 for higher Reynolds numbers.

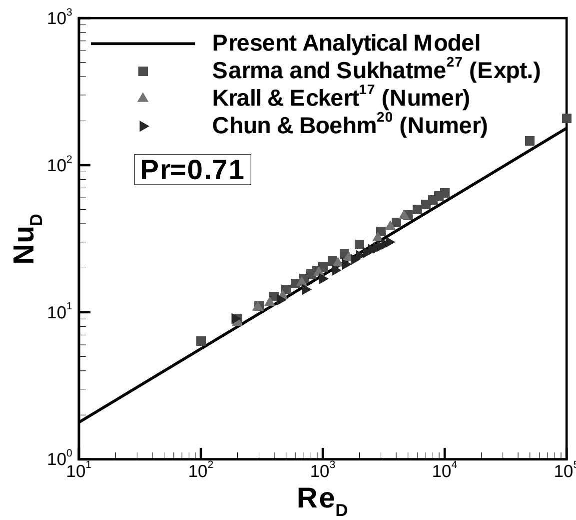

Fig. 7 Variation of Average Nusselt Number with Reynolds Number for Isoflux Boundary Condition

Average Nusselt numbers for the isoflux boundary condition are compared in Fig. 7 with the experimental/numerical results. The average Nu_D values are found to be in a good agreement with both numerical results of Krall and Eckert[17] and Chun and Boehm[20]. However, the experimental results of Sarma and Sukhatme[27] are found to be higher ($\approx 5\%$).

SUMMARY

The Von Karman-Pohlhausen method was used to investigate the fluid flow and heat transfer from a circular cylinder. Three correlations are obtained, Eq. (44) for total drag coefficient, Eq. (58) for heat transfer from an isothermal cylinder, and Eq. (64) for heat transfer from a cylinder under isoflux boundary condition.

The present results indicate good agreement with the experimental results for the full laminar range of Reynolds numbers in the absence of free stream turbulence and blockage effects. However, a downward deviation was noticed in the experimental drag curve at $Re_D = 2000$ and the effects of free stream turbulence were noticed in heat transfer results for high Reynolds numbers. These correlations can be used to determine the drag coefficient and the dimensionless heat transfer coefficient from a pin fin.

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