AIAA 01-0366

APPLICATION OF THERMAL SPREADING RESISTANCE IN COMPOUND AND ORTHOTROPIC SYSTEMS

Y.S. Muzychka
Faculty of Engineering and Applied Science
Memorial University of Newfoundland
St. John's, NF, Canada, A1B 3X5

and

M.M. Yovanovich and J.R. Culham
Department of Mechanical Engineering
University of Waterloo
Waterloo, ON, Canada, N2L 3G1

39th AIAA Aerospace Sciences Meeting & Exhibit
8-11 January 2001 / Reno, NV

APPLICATION OF THERMAL SPREADING RESISTANCE IN COMPOUND AND ORTHOTROPIC SYSTEMS

Y.S. Muzychka*

Faculty of Engineering and Applied Science
Memorial University of Newfoundland
St. John's, NF, Canada, A1B 3X5

M.M. Yovanovich† and J.R. Culham‡

Department of Mechanical Engineering
University of Waterloo
Waterloo, ON, Canada, N2L 3G1

ABSTRACT

A review of thermal spreading resistance in compound and orthotropic systems is presented. Transformation of the governing equations and boundary conditions for orthotropic systems is discussed. Relationships between the solutions for isotropic and orthotropic systems are developed. Solutions for spreading resistance are presented in both cylindrical and cartesian systems.

NOMENCLATURE

a, b	=	radial dimensions, m
a, b, c, d	=	linear dimensions, m
A_b	=	baseplate area, m^2
A_s	=	heat source area, m^2
A_m, A_n, A_{mn}	=	Fourier coefficients
B_n		
Bi	=	Biot number, $h\mathcal{L}/k$
h	=	contact conductance or film coefficient, $W/m^2 \cdot K$
k	=	thermal conductivity, $W/m{\cdot}K$
k_{eff}	=	effective conductivity, W/mK
$\mathcal{L}$	=	arbitrary length scale, m
m, n	=	indices for summations
Q	=	heat flow rate, W, $\equiv qA_s$
q	=	heat flux, W/m^2
R	=	thermal resistance, K/W
R_{1D}	=	one-dimensional resistance, K/W
R_s	=	spreading resistance, K/W
R_T	=	total resistance, K/W
t, t_1, t_2	=	total and layer thicknesses, m
t_{eff}	=	effective thickness, m
T	=	temperature, K
$\overline{T}_s$	=	mean source temperature, K
T_f	=	sink temperature, K
Xc, Yc	=	heat source centroid, m

Greek Symbols

α	=	equation parameter, $\equiv \frac{1-\kappa}{1+\kappa}$
$\beta_{m,n}$	=	eigenvalues, $\equiv \sqrt{\lambda_m^2 + \delta_n^2}$
δ_n	=	eigenvalues, $(n\pi/b, m\pi/c)$
ϵ	=	relative contact size, $\equiv a/b$
θ	=	temperature excess, $\equiv T - T_f$, K
$\overline{\theta}$	=	mean temperature excess, $\equiv \overline{T} - T_f$, K
κ	=	relative conductivity, k_2/k_1
λ_m	=	eigenvalues, $(m\pi/a, n\pi/d)$
ϕ, φ	=	spreading resistance functions
ψ	=	spreading parameter, $4kaR_s$
ϱ	=	equation parameter, $\equiv = \frac{\zeta + h/k_2}{\zeta - h/k_2}$
τ	=	relative thickness, $\equiv t/\mathcal{L}$
ζ	=	dummy variable, m^{-1}
ξ	=	transform variable, $\equiv z/\sqrt{k_{tp}/k_{ip}}$

Subscripts

i	=	index denoting layers 1 and 2
ip	=	in plane
tp	=	through plane
r	=	r-plane
x	=	x-plane
xy	=	xy-plane
y	=	y-plane
z	=	z-plane

INTRODUCTION

Thermal spreading resistance theory finds widespread application in electronics cooling, both at the board and chip level and in heat sink applications. It also arises

*Assistant Professor

†Distinguished Professor Emeritus, Fellow AIAA

‡Associate Professor

in the thermal analysis of bolted joints and other mechanical connections resulting in discrete points of contact. Recently, a comprehensive review of the theory and application of thermal spreading resistances was undertaken by one of the authors[1]. Since this review, a number of new solutions and applications of spreading resistance theory have been addressed. These include, but are not limited to, prediction of thermal resistance of electronic devices known as Ball Grid Arrays (BGA)[2], the effect of heat source eccentricity[3], the effect of heat spreaders in compound systems[4−7], and the effect of orthotropic properties[8,9].

This paper presents a general review of thermal spreading resistance theory in compound and orthotropic systems. Presently, only a few analyses have been undertaken for orthotropic systems[8,9]. These solutions have only been presented for the circular disk and rectangular strip. Solutions for the thermal spreading resistance in compound disks and rectangular flux channels will be reviewed. It will be shown that with the appropriate transformation, these solutions may be applied to orthotropic systems with little effort. These new solutions may then be applied to a number of orthotropic systems, such as printed circuit boards.

THERMAL SPREADING RESISTANCE

Thermal spreading resistance arises in multi-dimensional applications where heat enters a domain through a finite area. The total thermal resistance of the system may be defined as

$$R_T = \frac{\overline{T}_s - T_f}{Q} = \frac{\overline{\theta}_s}{Q} \tag{1}$$

In applications involving spreading resistance, the total thermal resistance is composed of two terms: a uniform flow or one-dimensional resistance and a spreading or multi-dimensional resistance which vanishes as the source area approaches the substrate area. These two components are combined as follows:

$$R_T = R_{1D} + R_s \tag{2}$$

Thermal spreading resistance analysis requires the solution of Laplace's equation

$$\nabla \cdot (k \nabla T) = 0 \tag{3}$$

in more than one dimension. In most applications the following boundary conditions are applied:

$$\frac{\partial T_i}{\partial n} = 0 \qquad n = x, y, r \tag{4}$$

along the edges and center of the disk or channel, and

$$\begin{aligned} \frac{\partial T_1}{\partial z} &= 0, \qquad A > A_s \\ \frac{\partial T_1}{\partial z} &= -\frac{q}{k_z}, \quad A < A_s \end{aligned} \tag{5}$$

on the top surface where A_s is the area of the heat source, and

$$\frac{\partial T_N}{\partial z} + \frac{h}{k_z}(T_N - T_f) = 0 \tag{6}$$

on the bottom surface. In compound systems, Laplace's equation must be written for each layer in the system, and continuity of temperature and heat flux at the material interface is required, yielding two additional boundary conditions:

$$\begin{aligned} T_i &= T_{i+1} \\ k_i \frac{\partial T_i}{\partial z} &= k_{i+1} \frac{\partial T_{i+1}}{\partial z} \end{aligned} \tag{7}$$

Due the nature of the solution procedure, the total thermal resistance may be analyzed as two problems. One is steady one-dimensional conduction which yields the uniform flow component of the thermal reistance, while the other is a multi-dimensional conduction analysis using Fourier Series or Integral Transform methods to solve an eigenvalue problem. This paper is mainly concerned with the solution to the thermal spreading resistance component in systems with one or two layers.

COMPOUND SYSTEMS

An overview of thermal spreading resistance in cartesian and cylindrical systems is given for compound flux tubes, refer to Figs. 1 and 2. Solutions for many special cases involving spreading resistance in disks, flux tubes, and half spaces are reported in Yovanovich[1]. Since the publication of the review[1], new solutions for spreading resistance in rectangular flux channels have been obtained for many special cases. These and other similar solutions in cylindrical coordinates are presented below.

Cylindrical Systems

Thermal spreading resistance solutions in isotropic and compound disks, refer to Fig. 1, flux channels and half spaces are presented in Yovanovich et al.[5,6]. A general solution for the compound disk was first obtained by Yovanovich et al.[6]. The general solution[6] for $\psi = 4k_1 a R_s$ is

$$\psi = \frac{8}{\pi\epsilon} \sum_{n=1}^{\infty} A_n(n,\epsilon) B_n(n,\tau,\tau_1) \frac{J_1(\delta_n \epsilon)}{\delta_n \epsilon} \tag{8}$$

where

$$A_n = -\frac{2\epsilon J_1(\delta_n \epsilon)}{\delta_n^2 J_0^2(\delta_n)} \tag{9}$$

and

$$B_n = \frac{\phi_n \tanh(\delta_n) - \varphi_n}{1 - \phi_n)} \tag{10}$$

The functions ϕ_n and φ_n are defined as follows:

$$\phi_n = \frac{\kappa - 1}{\kappa} \cosh(\delta_n \tau_1) \left[\cosh(\delta_n \tau_1) - \varphi_n \sinh(\delta_n \tau_1)\right] \tag{11}$$

and

$$\varphi_n = \frac{\delta_n + Bi \tanh(\delta_n \tau)}{\delta_n \tanh(\delta_n \tau) + Bi} \tag{12}$$

The eigenvalues δ_n are solutions to $J_1(\delta_n) = 0$ and $Bi = hb/k_2$.

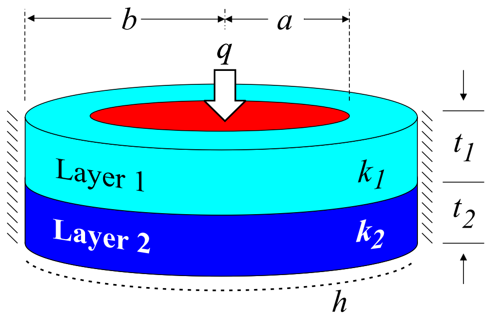

Fig. 1 - Compound disk with circular heat source.

Rectangular Systems

Thermal spreading resistance in rectangular systems has recently been examined by the authors[3,4]. In Yovanovich et al.[4], the authors obtained a solution for a compound rectangular flux tube having a central heat source, refer to Fig. 2. This general solution also simplifies for many cases of semi-infinite flux channels and half space solutions. More recently, the authors[3] developed a solution for a single eccentric heat source on compound and isotropic flux channels. The results of Muzychka et al.[3] were also extended to systems having multiple arbitrarily placed heat sources.

The spreading resistance of Yovanovich et al.[4] is obtained from the following general expression according to the notation in Fig. 2:

$$R_s = \frac{1}{2\,a^2\,c\,d\,k_1} \sum_{m=1}^{\infty} \frac{\sin^2(a\delta_m)}{\delta_m^3} \cdot \varphi(\delta_m) + \frac{1}{2\,b^2\,c\,d\,k_1} \sum_{n=1}^{\infty} \frac{\sin^2(b\lambda_n)}{\lambda_n^3} \cdot \varphi(\lambda_n) + \frac{1}{a^2\,b^2\,c\,d\,k_1} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{\sin^2(a\delta_m)\sin^2(b\lambda_n)}{\delta_m^2\,\lambda_n^2\,\beta_{m,n}} \cdot \varphi(\beta_{m,n}) \tag{13}$$

where

$$\varphi(\zeta) = \frac{\left(\alpha e^{4\zeta t_1} + e^{2\zeta t_1}\right) + \varrho\left(e^{2\zeta(2t_1+t_2)} + \alpha e^{2\zeta(t_1+t_2)}\right)}{\left(\alpha e^{4\zeta t_1} - e^{2\zeta t_1}\right) + \varrho\left(e^{2\zeta(2t_1+t_2)} - \alpha e^{2\zeta(t_1+t_2)}\right)} \tag{14}$$

and

$$\varrho = \frac{\zeta + h/k_2}{\zeta - h/k_2} \quad \text{and} \quad \alpha = \frac{1-\kappa}{1+\kappa}$$

with $\kappa = k_2/k_1$. The eigenvalues for these solutions are: $\delta_m = m\pi/c$, $\lambda_n = n\pi/d$ and $\beta_{m,n} = \sqrt{\delta_m^2 + \lambda_n^2}$.

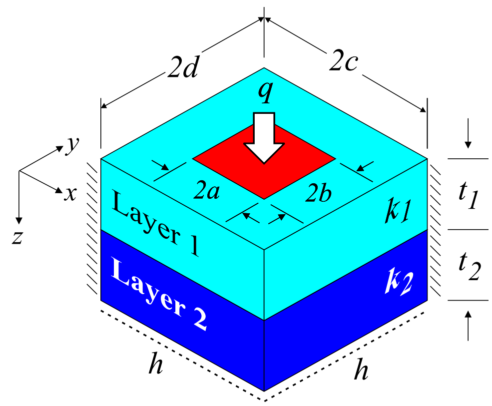

Fig. 2 - Compound flux channel with rectangular heat source.

Source Eccentricity

The general solution for the mean temperature excess of a single eccentric heat source was obtained by Muzychka et al.[3]. The thermal spreading resistance for a source using the notation of Fig. 3, is

$$R_s = 2\sum_{m=1}^{\infty} A_m \frac{\cos(\lambda_m X_c)\sin(\frac{1}{2}\lambda_m c)}{\lambda_m c} \quad 2\sum_{n=1}^{\infty} A_n \frac{\cos(\delta_n Y_c)\sin(\frac{1}{2}\delta_n d)}{\delta_n d} + 4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn} \frac{\cos(\delta_n Y_c)\sin(\frac{1}{2}\delta_n d)\cos(\lambda_m X_c)\sin(\frac{1}{2}\lambda_m c)}{\lambda_m c \delta_n d} \tag{15}$$

where

$$A_m = \frac{2\left[\sin\left(\frac{(2X_c+c)}{2}\lambda_m\right) - \sin\left(\frac{(2X_c-c)}{2}\lambda_m\right)\right]}{a\,b\,c\,k_1\,\lambda_m^2 \phi(\lambda_m)} \tag{16}$$

and

$$A_n = \frac{2\left[\sin\left(\frac{(2Y_c+d)}{2}\delta_n\right) - \sin\left(\frac{(2Y_c-d)}{2}\delta_n\right)\right]}{a\,b\,d\,k_1\,\delta_n^2\phi(\delta_n)} \tag{17}$$

and

$$A_{mn} = \frac{16\cos(\lambda_m X_c)\sin(\frac{1}{2}\lambda_m c)\cos(\delta_n Y_c)\sin(\frac{1}{2}\delta_n d)}{a\,b\,c\,d\,k_1\,\beta_{m,n}\lambda_m\delta_n\phi(\beta_{m,n})} \tag{18}$$

where

$$\phi(\zeta) = \frac{\zeta\sinh(\zeta t_1) + h/k_1\cosh(\zeta t_1)}{\zeta\cosh(\zeta t_1) + h/k_1\sinh(\zeta t_1)} \tag{19}$$

for an isotropic system, where ζ is replaced by the eigenvalues: $\lambda_m = m\pi/a$, $\delta_n = n\pi/b$, or $\beta_{m,n} = \sqrt{\lambda_m^2 + \delta_n^2}$, accordingly.

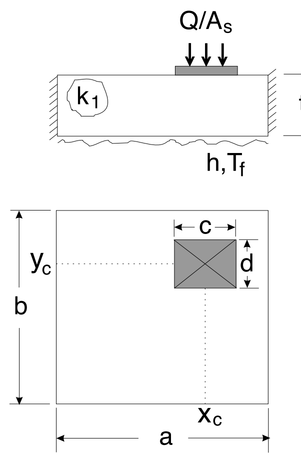

Fig. 3 - Eccentric Heat Source.

For a compound system

$$\phi(\zeta) = \frac{\left(\alpha e^{4\zeta t_1} - e^{2\zeta t_1}\right) + \varrho\left(e^{2\zeta(2t_1+t_2)} - \alpha e^{2\zeta(t_1+t_2)}\right)}{\left(\alpha e^{4\zeta t_1} + e^{2\zeta t_1}\right) + \varrho\left(e^{2\zeta(2t_1+t_2)} + \alpha e^{2\zeta(t_1+t_2)}\right)} \tag{20}$$

where

$$\varrho = \frac{\zeta + h/k_2}{\zeta - h/k_2} \quad \text{and} \quad \alpha = \frac{1-\kappa}{1+\kappa}$$

with $\kappa = k_2/k_1$, and ζ is replaced by λ_m, δ_n, or β_{mn}, accordingly.

The solution above may be used to calculate the total thermal resistance for a source located at any point on a compound or isotropic rectangular flux channel. Details of the local surface temperature distribution may be computed from additional expressions presented in Muzychka et al.[3], for single or multiple heat sources.

ORTHOTROPIC SYSTEMS

Laplace's equation, Eq. (3), may be expanded for an orthotropic system and written as

$$k_x\frac{\partial^2 T}{\partial x^2} + k_y\frac{\partial^2 T}{\partial y^2} + k_z\frac{\partial^2 T}{\partial z^2} = 0 \tag{21}$$

in cartesian co-ordinates, or

$$k_r\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) + k_z\frac{\partial^2 T}{\partial z^2} = 0 \tag{22}$$

in cylindrical co-ordinates.

The above equations may be transformed such that the governing equation and boundary conditions are reduced to those for an equivalent isotropic system. The solution for the orthotropic system is then easily obtained from this equivalent system. A transformation[10] for composite orthotropic materials was proposed for thermal spreading resistance of a circular heat source on a half-space. This transformation will be applied to isotropic spreading resistance solutions for finite circular and rectangular disks. Further discussion on the transformation of orthotropic systems to isotropic systems may be found in Carslaw and Jaeger[11] and Ozisik[12].

Results will be presented for two cases: the orthotropic disk, i.e., $k_r \neq k_z$ and the orthotropic channel where the in plane and through plane conductivities are different, i.e $k_x = k_y \neq k_z$. In addition, several other special cases which naturally arise from the general solutions, such as the half-space and semi-infinite flux tube, will also be discussed.

Circular Systems

In a cylindrical orthotropic system, refer to Fig. 4, Laplaces's equation, Eq. (22) may be transformed using $\xi = z/\beta$, where $\beta = \sqrt{k_z/k_r}$, and $\theta = T - T_f$, to yield,

$$\frac{\partial^2\theta}{\partial r^2} + \frac{1}{r}\frac{\partial\theta}{\partial r} + \frac{\partial^2\theta}{\partial\xi^2} = 0 \tag{23}$$

which is subjected to the following transformed boundary conditions:

$$\begin{aligned}
r = 0, b, \quad & \frac{\partial\theta}{\partial r} = 0 \\
\xi = 0, \quad & \frac{\partial\theta}{\partial\xi} = -\frac{q}{k_{eff}}, \quad 0 \le r < a \\
& \frac{\partial\theta}{\partial\xi} = 0, \quad a < r \le b \\
\xi = \frac{t}{\beta}, \quad & \frac{\partial\theta}{\partial\xi} = -\frac{h}{k_{eff}}\theta
\end{aligned} \tag{24}$$

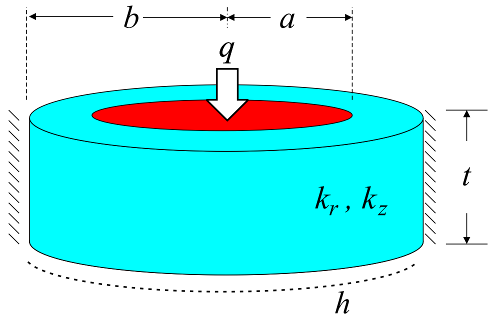

Fig. 4 - Finite Circular Orthotropic Disk

Equations (23, 24) are now in the same form as that for an isotropic disk, except an effective thermal conductivity $k_{eff} = \sqrt{k_r k_z}$ now replaces the isotropic thermal conductivity, and $t_{eff} = t/\beta$ now replaces the thickness. The solution for this case is[1]

$$\psi = 4 k_{eff} a R_s = \frac{16}{\pi\epsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\delta_n \epsilon)}{\delta_n^3 J_0^2(\delta_n)\phi_n} \tag{25}$$

where

$$\phi_n = \frac{\delta_n \tanh(\delta_n \tau_{eff}) + Bi}{\delta_n + Bi \tanh(\delta_n \tau_{eff})} \tag{26}$$

The dimensionless thickness and Biot number now become: $\tau_{eff} = t_{eff}/b$ and $Bi = hb/k_{eff}$, where the effective conductivity is $k_{eff} = \sqrt{k_r k_z}$, effective thickness is $t_{eff} = t/\beta$, and $\epsilon = a/b$. The eigenvalues δ_n are obtained from $J_1(\delta_n) = 0$.

Rectangular Systems

In a rectangular orthotropic system, refer to Fig. 5, with $k_x = k_y = k_{xy}$, Laplaces's equation, Eq. (21), may also be transformed using $\xi = z/\beta$, where $\beta = \sqrt{k_z/k_{xy}}$, and $\theta = T - T_f$, to yield,

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} + \frac{\partial^2\theta}{\partial \xi^2} = 0 \tag{27}$$

which is subjected to the following transformed boundary conditions:

$$\begin{aligned}
x = 0, c, \quad & \frac{\partial\theta}{\partial x} = 0 \\
y = 0, d, \quad & \frac{\partial\theta}{\partial y} = 0 \\
\xi = 0, \quad & \frac{\partial\theta}{\partial \xi} = -\frac{q}{k_{eff}}, \quad 0 \le x < a, \; 0 \le y < b \\
& \frac{\partial\theta}{\partial \xi} = 0, \quad a < x \le c, \; b < y \le d \\
\xi = \frac{t}{\beta}, \quad & \frac{\partial\theta}{\partial \xi} = -\frac{h}{k_{eff}}\theta
\end{aligned} \tag{28}$$

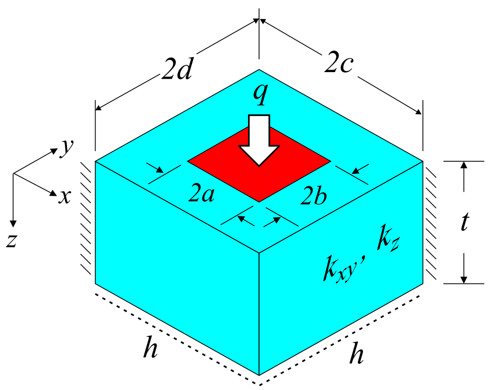

Fig. 5 - Finite Rectangular Orthotropic Flux Tube

Equations (27, 28) are now in the same form as those for an isotropic flux tube, except an effective thermal conductivity $k_{eff} = \sqrt{k_{xy} k_z}$ now replaces the isotropic thermal conductivity. The solution for this case is found in[4]

$$\begin{aligned}
R_s = & \frac{1}{2a^2 c\, d\, k_{eff}} \sum_{m=1}^{\infty} \frac{\sin^2(a\delta_m)}{\delta_m^3} \cdot \varphi(\delta_m) \\
& + \frac{1}{2b^2 c\, d\, k_{eff}} \sum_{n=1}^{\infty} \frac{\sin^2(b\lambda_n)}{\lambda_n^3} \cdot \varphi(\lambda_n) \\
& + \frac{1}{a^2 b^2 c\, d\, k_{eff}} \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \frac{\sin^2(a\delta_m)\sin^2(b\lambda_n)}{\delta_m^2 \lambda_n^2 \beta_{m,n}} \cdot \varphi(\beta_{m,n})
\end{aligned} \tag{29}$$

where

$$\varphi(\zeta) = \frac{(e^{2\zeta t_{eff}} + 1)\zeta - (1 - e^{2\zeta t_{eff}})h/k_{eff}}{(e^{2\zeta t_{eff}} - 1)\zeta + (1 + e^{2\zeta t_{eff}})h/k_{eff}} \tag{30}$$

where ζ is a dummy variable denoting the respective eigenvalues. The eigenvalues for these solutions are: $\delta_m = m\pi/c$, $\lambda_n = n\pi/d$ and $\beta_{m,n} = \sqrt{\delta_m^2 + \lambda_n^2}$.

Series-Parallel Models for k

In orthotropic systems such as printed circuit boards, series and parallel models are often used to define k_r and k_z. These are defined as[8]:

$$k_r, k_{xy} = \frac{\sum_{i=1}^{N} k_i t_i}{t} \tag{31}$$

and

$$k_z = \frac{t}{\sum_{i=1}^{N} \frac{t_i}{k_i}} \tag{32}$$

In general, thermal spreading resistance in multilayered systems is a strong function of the size and distribution of conducting layers. However, if the source size is considerably larger than the thickness of individual layers[10], the above relations for determining effective series and parallel conductivities may be applied.

Special Cases

The general solutions given by Eqs. (25) and (29) contain a number of special limits. These include: semi-infinite flux tubes and half-spaces. In addition, the effect of flux distribution over a circular heat source may also be considered[6]. These and other special cases are discussed in Yovanovich[1], Muzychka et al.[3], and Yovanovich et al.[4,5]. Finally, the effect of source eccentricity for a rectangular orthotropic disk may be considered using the solution of Muzychka et al.[3] and applying the appropriate transformations to the solution presented earlier for the compound isotropic disk.

SUMMARY AND CONCLUSIONS

A review of thermal spreading resistance solutions for compound and orthotropic systems was presented. New solutions for orthotropic systems were obtained using simple transformations on isotropic spreading resistance results. A number of special cases involving orthotropic systems were discussed. It was shown that orthotropic spreading resistance solutions can be obtained by applying the following rules:

$$k \rightarrow k_{eff} = \sqrt{k_{ip} k_{tp}} \tag{33}$$

where, k_{ip} and k_{tp} represent the in-plane and through-plane thermal conductivity, and

$$t \rightarrow t_{eff} = \frac{t}{\beta} \tag{34}$$

where $\beta = \sqrt{k_{tp}/k_{ip}}$, and

$$Bi \rightarrow \frac{h\mathcal{L}}{k_{eff}} \tag{35}$$

Finally, simple relationships for computing the in-plane and through-plane thermal conductivity for composite systems were presented.

ACKNOWLEDGMENTS

The authors would like to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada. The authors also acknowledge the assistance of Peter Teertstra for preparation of figures.

REFERENCES

[1] Yovanovich, M.M., "Conduction and Thermal Contact Resistances (Conductances)," *Chapter 3 in Handbook of Heat Transfer*, eds. W.M. Rohsenow, J.P. Hartnett, and Y.I. Cho, 1998, McGraw-Hill, New York.

[2] Ying, T.M. and Toh, K.C., "A Constriction Resistance Model in Thermal Analysis of Solder Ball Joints in Ball Grid Array Packages," Proceedings of the 1999 International Mechanical Engineering Congress and Exposition, Nashville, 1999, HTD-Vol. 364-1, pp. 29-36.

[3] Muzychka, Y.S., Culham, J.R., and Yovanovich, M.M., "Thermal Spreading Resistance of Eccentric Heat Sources on Rectangular Flux Channels," Presented at the *2000 International Mechanical Engineering Congress and Exposition*, Orlando, FL., November 5-10, 2000.

[4] Yovanovich, M.M., Muzychka, Y.S., Culham, J.R., "Spreading Resistance of Isoflux Rectangles and Strips on Compound Flux Channels," *Journal of Thermophysics and Heat Transfer*, Vol. 13, 1999, pp. 495-500.

[5] Yovanovich, M.M., Culham, J.R., and Teertstra, P.M., "Analytical Modeling of Spreading Resistance in Flux Tubes, Half Spaces, and Compound Disks," *IEEE Transactions on Components, Packaging, and Manufacturing Technology - Part A*, Vol. 21, 1998, pp. 168-176.

[6] Yovanovich, M.M., Tien, C.H., Schneider, G.E., "General Solution of Constriction Resistance within a Compound Disk," *Progress in Astronautics and Aeronautics: Heat Transfer, Thermal Control, and Heat Pipes*, MIT Press, Cambridge, MA, 1980, pp. 47-62.

[7] Muzychka, Y.S., Stevanovic, M., and Yovanovich, M.M., "Thermal Spreading Resistances in Compound Annular Sectors," AIAA 00-0875, 38th Aerospace Sciences Meeting and Exhibit, Reno, NV, 2000.

[8] Lam, T.T. and Fischer, W.D., "Thermal Reistance in Rectangular Orthotropic Heat Spreaders," ASME Advances in Electronic Packaging, Vol. 1, pp. 891-898, 1999.

[9] Ying, T.M. and Toh, K.C., "A Heat Spreading Resistance Model for Anisotropic Thermal Conductivity Materials in Electronic Packaging," *Proceedings of the 2000*

Inter-Society Conference on Thermal Phenomena.

[10] Yovanovich, M.M., "On the Temperature Distribution and Constriction Resistance in Layered Media," *Journal of Composite Materials*, Vol. 4, 1970, pp. 567-570.

[11] Carslaw, H.S. and Jaeger, J.C., *Conduction of Heat in Solids*, Oxford University Press, 1959, Oxford,UK.

[12] Ozisik, N., *Boundary Value Problems of Heat Conduction*, 1968, Dover Publications, New York.