Modeling Contact between Rigid Sphere and Elastic Layer Bonded to Rigid Substrate

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***Abstract*—An approximate mechanical model is developed for predicting the radius of contact between a sphere and a layered substrate. The complex solution of Chen and Engel is reduced to the simple root finding procedure for the unknown contact radius. Numerical data from the model of Chen and Engel are obtained for several combinations of layer material. It is shown that with the proper selection of dimensionless parameters the numerical results fall on a single curve that is easily correlated. Radius predictions show good agreement with experimental measurements.**

***Index Terms*—Contact radius, elastic deformation, elastomer layers, thermal constriction resistance.**

NOMENCLATURE

a	Contact radius (m).
a_{L}	Contact radius corresponding to layer bound (m).
a_{S}	Contact radius corresponding to substrate bound (m).
a^*	Dimensionless contact radius.
CEM	Chen and Engel model.
d	Approach or penetration depth (m).
E	Elastic modulus (Pa).
F	Normal load (N).
k	Thermal conductivity (W/mK).
R	Constriction resistance (K/W).
R^*	Dimensionless constriction resistance.
r, z	Local polar coordinates.
t	Layer thickness (m).

Greek Symbols

α	Ratio of bounding radii $\equiv a_{\mathrm{L}}/a_{\mathrm{S}}$.
κ	Conductivity ratio ($\equiv k_1/k_2$).
ν	Poisson's ratio.
ρ	Radius of sphere.
τ	Relative layer thickness ($\equiv t/a$).
ϕ	Stress function.
ψ	Constriction parameter.
∇^2	Laplacian operator.

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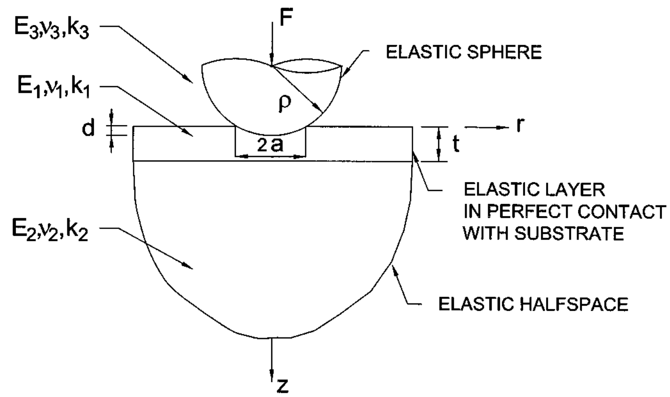

Fig. 1. Elastic contact of a sphere with a layered halfspace.

Superscripts

*	Dimensionless or reduced.

Subscripts

L, S	Layer, substrate.
$1, 2, 3$	Layer, substrate, sphere.

I. INTRODUCTION

THE resistance to heat flow due to thermal spreading or constriction at a joint formed between contacts is an important consideration in the development of high speed electronic equipment. Critical interfaces formed between electronic packages or silicon flip chips and heat sinks necessitate the use of soft, compliant interface materials to fill air gaps associated with nonconforming wavy surfaces. Without the use of an interface material, the overall thermal resistance between heat sources and the surrounding air will rise significantly, resulting in an increase in the operating temperature of integrated circuits and a subsequent decrease in component reliability.

As shown in Fig. 1, typically a joint consists of a deformable spherical body that penetrates into a thin, elastic layer, assumed to be in perfect contact with an elastic substrate of large extent. The temperature drop across the contact is related to the power dissipation and the constriction resistance which depends on the thermal conductivities of the three components, their elastic properties (Young's modulus, Poisson's ratio), the radius of curvature of the spherical body, the layer thickness and the applied mechanical load. The constriction resistance can be reduced significantly through the proper selection of a layer which has a high thermal conductivity, low rigidity, and a thickness which is sufficiently large to cause the constriction on the layer-substrate side of the joint to occur primarily within the layer.

Since the radius of the contact area is much smaller than the radius of the sphere and the layered-substrate dimensions, the contact is modeled both thermally and mechanically as a circular contact connecting two halfspaces: the sphere on one side and the layered-substrate on the other.

The total constriction resistance of the sphere and the layered substrate is equal to the sum of two constriction resistances which are in series, the resistance of the contact area on the sphere and the resistance of the contact area on the layered substrate. The total resistance is given by

$$R = \frac{1}{4\,ak_3} + \frac{\psi(\tau,\kappa)}{4\,ak_2} \tag{1}$$

where k_2 and k_3 are the thermal conductivities of the substrate and the sphere, respectively. The thermal constriction parameter $\psi(\tau,\kappa)$ is obtained from the solution of the Laplace equation within the layer and the substrate. The dimensionless constriction resistance is defined with respect to the unknown contact radius a and the substrate thermal conductivity k_2

$$R^* = Rak_2 \tag{2}$$

which gives

$$R^* = \frac{1}{4}\left[\frac{k_2}{k_3} + \psi(\tau,\kappa)\right]. \tag{3}$$

The constriction parameter $\psi(\tau,\kappa)$ for an isothermal contact area is given by Dryden [4], where the relative layer thickness τ and conductivity ratio κ are defined as

$$\tau = \frac{t}{a} \quad \text{and} \quad \kappa = \frac{k_1}{k_2}.$$

The thermal conductivity ratio is constant for a given layer-substrate combination. The relative layer thickness varies with the mechanical load, geometry and physical properties. The evaluation of $\psi(\tau,\kappa)$ is not possible without first solving the mechanical portion of the contact resistance problem for the contact radius a, which depends on the relative layer thickness along with other dimensionless physical parameters. The mechanical problem is a complex mathematical problem in axisymmetric elasticity governed by the following differential equation:

$$\nabla^2\nabla^2\phi = 0 \tag{4}$$

where ϕ is the stress function and ∇^2 is the Laplacian operator in circular coordinates. The boundary conditions are of the mixed type, with the surface deflection prescribed within the contact area and the normal stress prescribed outside. For layered bodies it is not possible to obtain a closed form solution of (4). Instead, an iterative procedure must be used based on an initial estimate of the contact radius that is then updated until the calculated normal load equals the given load within some relative error criterion.

The first objective of this work is to review the existing mechanical models for predicting the radius of contact for the problem of interest and to select the most general solution. A second objective is to compute the contact radius for a wide range of the physical properties, layer thickness, sphere radius, and mechanical load by means of the most general solution. And finally, a third objective is to find appropriate dimensionless dependent and independent parameters which best characterize the mechanical problem. By means of a proper selection of dimensionless parameters the numerical results are expected to fall on a single curve which can be correlated in a relatively simple manner.

II. Literature Review

The penetration of an elastic layer by a frictionless indenter gives rise to two classes of problems referred to as complete and incomplete contacts. The radius of the contact region, a, is given in the complete contact problem, such as in the case of the flat-ended indenter. In the incomplete contact problem, the radius of the contact region is not known a priori but rather is a function of contact conditions, as in the case of a sphere/flat contact.

The mechanical problem of interest is the contact between a smooth sphere and a layer which is bonded to a substrate as shown in Fig. 1. Vorovich and Ustinov [13] were the first to obtain a solution for the problem of a rigid sphere contacting an elastic layer on a rigid substrate. They determined the radius of contact in terms of asymptotic expansions in powers of the dimensionless layer thickness τ. Their results were only applicable for thick layers where $\tau \geq 1.5$. Keer [5] extended the analysis of Vorovich and Ustinov by introducing an elastic indenter. He corrected their asymptotic expansion by introducing the ratio of the elastic properties into the solution.

While most researchers agree that the integral transform method appears to be the best tool for analyzing contact problems, Chen [1] identified a major shortcoming of this method. He found that integral formulations for contact problems in layered media were slow to converge at the surface, near the edge of the contact zone, especially for relatively thin layers. To circumvent this problem Chen and Engel [2] introduced a different approach, called the general approximate method (GAM), for the axisymmetric mixed boundary value problems in elasticity. The approach taken by Chen and Engel [2] was to replace the exact boundary conditions by approximate boundary conditions, such that the mixed conditions were reduced to the boundary condition of the second kind, which was then solved exactly. In their paper, Chen and Engel [2], compared their results with those of Dhaliwal [3] for a flat-ended punch and concluded that their approach was more accurate. Furthermore, they were able to investigate layers of all thicknesses. Also in their work the layer, substrate and indenter were assumed to be elastic.

McCormick [10] developed a numerical method to compute the pressure distribution and penetration depth in a generalized elliptical contact between layered elastic solids based on a discretized representation of the unknown pressure distribution. The procedure was applied to the contact between an indenter and layered half space and numerical solutions were presented for various material combinations. McCormick also demonstrated a favorable comparison of his numerical results with

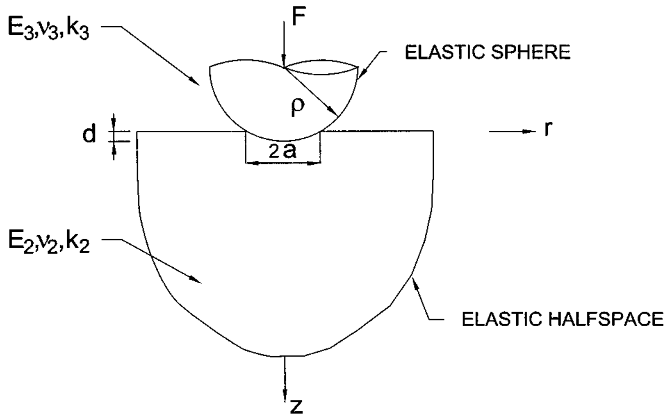

Fig. 2. Elastic contact of a sphere with a halfspace.

the Hertz solution (Fig. 2) for limiting cases as well as with the numerical results of Chen and Engel [2]. These solutions were presented graphically in dimensionless form for the values of contact radius $(a/a_{\rm H})$, penetration depth $(d/d_{\rm H})$ and contact load (F) over the range of a/t and E_1/E_2 with $E_1/E_3 = 0$. The values for contact radius and penetration depth determined by this method agreed within one percent of the Chen and Engel [2] results.

Matthewson [9] investigated the bonded interface formed between a rigid sphere and a soft, thin, elastic layer. The elastic modulus of the layer was assumed to be small compared with the moduli of the indenter and the substrate. The thickness of the layer was assumed to be small compared with the contact radius, which was small, compared with the characteristic linear dimension of the indenter. Friction between the indenter and the layer was ignored. Using all of these assumptions, he obtained differential equations which described the deformation of the layer. He found solutions for indentation of the sphere (or paraboloid) explicitly for general values of Poisson's ratio of the layer. Experiments were done using spherical and conical indenters and the results of his analyses were in good agreement with observed experimental results. He also compared his results to the numerical results of McCormick [10] and found good agreement only for $\tau \leq 0.5$.

Jaffar [6] developed a numerical method for solving axisymmetric contact problems for elastic layers bonded to a rigid substrate (bonded layer) or layers resting on a rigid substrate (unbounded layer), indented by a frictionless indenter. Rigid spherical and circular flat-ended indenters were considered in the range $0.05 \leq \tau \leq \infty$, for Poisson's ratio in the range $0 \leq \nu \leq 0.5$. He assumed that both the pressure and surface displacement over the contact region had an expansion in terms of modified Legendre polynomials. The method was tested by comparing the numerical results with the exact solution for the Hertzian problem $(\tau \rightarrow \infty)$ and very good agreement was obtained. Further computations were performed for $\tau < \infty$ and results were presented for pressure distributions, total load and penetration depth within the contact region and surface displacement outside the region. Results for $\tau < \infty$ when $\nu = 0.5$, were compared with the numerical results of McCormick [10] and the reported agreement was very good.

Stevanović and Yovanovich [12] developed a procedure for reducing the complex, computationally intensive solution of Chen and Engel [2], to a simple closed-form solution for the unknown contact radius. The method of Chen and Engel was chosen because it assumed that the sphere, substrate and layer were elastic and their model was also applicable over the full range of the layer thicknesses ie., $0 \leq \tau \leq \infty$.

The numerical data obtained from Chen and Engel [2] were presented graphically in dimensionless form for contact radius versus layer thickness. Selecting the proper dimensionless parameters

$$a^* = \frac{a - a_{\rm S}}{a_{\rm L} - a_{\rm S}} \quad \text{for contact radius}$$

$$\tau^* = \left(\frac{t}{a}\sqrt{\alpha}\right)^{1/3} \quad \text{for dimensionless layer thickness}$$

the numerical results fall on a single curve, which is easily correlated. The resulting correlation equation requires a simple, numerical root-finding method for computing the contact radius. Further investigation of this problem has led to the development of a closed-form solution for the unknown contact radius. The solution is presented in following form:

$$a = a_{\rm S} + (a_{\rm L} - a_{\rm S})\left(1 - \exp\left(-\pi^{1/4}\left(\frac{t\sqrt{\alpha}}{a_0}\right)^{\pi/4}\right)\right)$$

where a_0 is defined as

$$a_0 = a_{\rm S} + (a_{\rm L} - a_{\rm S})\left(1 - \exp\left(-\pi^{1/4}\left(\frac{2\,t\sqrt{\alpha}}{a_{\rm S} + a_{\rm L}}\right)^{\pi/4}\right)\right)$$

and $a_{\rm L}$ and $a_{\rm S}$ are the contact radii corresponding to the layer $(\tau \rightarrow \infty)$ and substrate $(\tau \rightarrow 0)$ bounds, respectively. The physical parameter α is defined as the ratio of the bounding radii

$$\alpha = \frac{a_{\rm L}}{a_{\rm S}}.$$

It is important to notice that for any common combination of metallic materials, the value of α is in the range $1 \leq \alpha \leq 2$. This model gives good agreement with the numerical values of the Chen and Engel model $\alpha \leq 2.5$. The maximum difference between Chen and Engel and the present model is 1.5% for $\alpha = 2.5$. For values of $\alpha > 2.5$ the proposed model does not have good agreement with the numerical data of Chen and Engel. To find a radius of contact for cases where $\alpha > 2.5$ it is necessary to develop a new model.

Table I provides a summary of the solution methods reviewed, the range of applicability and the elastic properties of the sphere, layer and substrate.

III. Mechanical Model Development

A mechanical model is developed for contact between a metallic sphere and an elastomeric layer bonded to a metallic substrate. The model can be used for layer-substrate material combinations with any value of Poisson's ratio where the bounding radii ratio, $\alpha > 2.5$ and the Young's modulus of elasticity of the substrate is more than 40 times greater than that of

TABLE I
REVIEW OF MECHANICAL MODELS

Authors	Solution Method	Range	Sphere	Substrate	Layer
Vorovich and Ustinov (1959)	Analytical	$\tau > 1.5$	rigid	rigid	elastic
Chen and Engel (1972)	Analytical	$0 < \tau < \infty$	elastic	elastic	elastic
McCormick (1978)	Numerical	$0 < \tau < \infty$	rigid	elastic	elastic
Matthewson (1981)	Numerical	$\tau \leq 0.5$	rigid	rigid	elastic
Jaffar (1987)	Numerical	$0.05 < \tau < \infty$	rigid	rigid	elastic
Stevanović and Yovanovich (1999)	Approximate	$0 < \tau < \infty$	elastic	elastic	elastic

the layer. It is assumed that when E_2 and E_3 are much greater than E_1 (40 times or more) the substrate and the sphere can be modeled as a rigid material compared to the layer. Given this assumption, the bonding radius corresponding to the substrate bond is equal to zero ($a_S = 0$) and the ratio of bounding radii α becomes infinitely large. For the new model, the contact radius will depend on a smaller subset of the physical parameters and will not contain a_S and α. Therefore

$$a = f(F, \rho, t, E_1, \nu_1).$$

Numerical data from the model of Chen and Engel are obtained for several layer material combinations. Values of E_1 vary between 0.05 MPa and 500 MPa. A MATLAB 5 computer code [8] based on the model of Chen and Engel requires that the value of E_2 and E_3 be set at some finite number. These values of E_2 and E_3 were set to 207 GPa to correspond to the value of stainless steel. During the computation all physical parameters: load, radius of sphere, layer thickness are varied. Since the model of Chen and Engel does not converge for very thin layers, i.e., $\tau < 0.01$, computed values are obtained for the range of $0.02 \leq \tau \leq 3$. For very thick layers, the radius of the contact, a, must approach a value of a_L. Calculated data from Chen and Engel are presented graphically in dimensionless form in Fig. 3.

Selecting a dimensionless radius as $a^* = a/a_L$ and dimensionless layer thickness as $\tau = t/a$, all numerical results fall on a single curve. This curve is correlated with a simple correlation equation

$$\frac{a}{a_L} = 1 - c_3 \exp\left(c_1 \tau^{c_2}\right) \quad (5)$$

where correlation coefficients are $c_1 = -1.73$, $c_2 = 0.734$ and $c_3 = 1.04$.

The maximum difference between the correlation and the Chen and Engel model is approximately 1.9% for the value of $\tau = 0.02$ with an RMS difference of 0.9.

The unknown radius of contact, a, appears on both sides of the correlation equation, which requires a simple numerical root-finding method for computing the contact radius. An iterative technique based on the Newton-Raphson method can be used

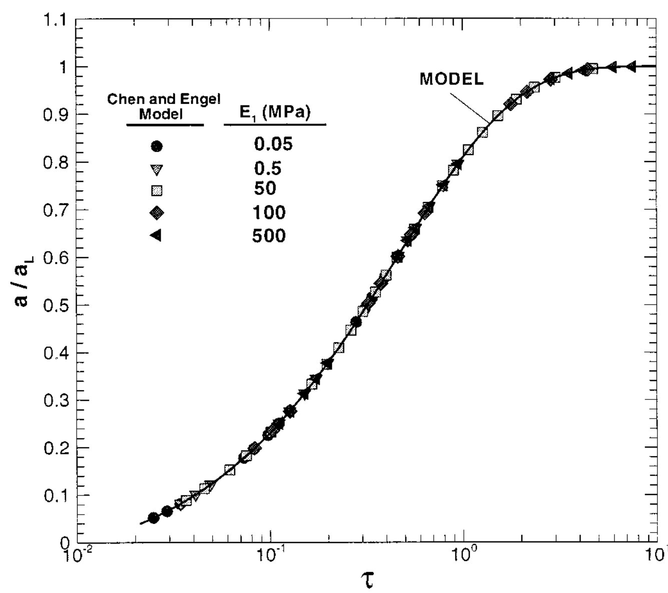

Fig. 3. Dimensionless contact radius versus τ.

to obtain the root of the equation. The solution procedure is described in the Appendix. An alternate approach for finding the root is the use of a computer algebra system, such as Maple [7].

IV. COMPARISON OF PRESENT MODEL TO OTHERS

The above proposed model is compared with that of Matthewson [9] and Vorovich and Ustinov [13]. These models can not be used for a wide range of τ. Matthewson [9] stated that his model gives good agreement for $\tau \leq 0.5$ while the model of Vorovich and Ustinov [13] is good only for $\tau > 1.5$. Since τ cannot go to zero because $\alpha = \infty$, the smallest value of τ is chosen to be $\tau = 0.02$. Fig. 4 shows comparisons between the proposed model and the other two models.

As can be seen from Fig. 4 the proposed model has good agreement with the model of Matthewson [9] for thin layers, $\tau < 0.2$. For $\tau > 0.2$ the agreement is poor because his model

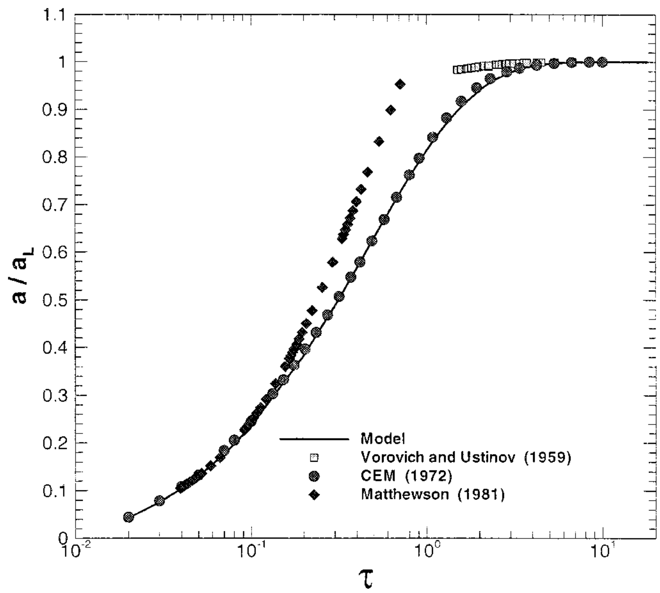

Fig. 4. Comparison of present model with other models.

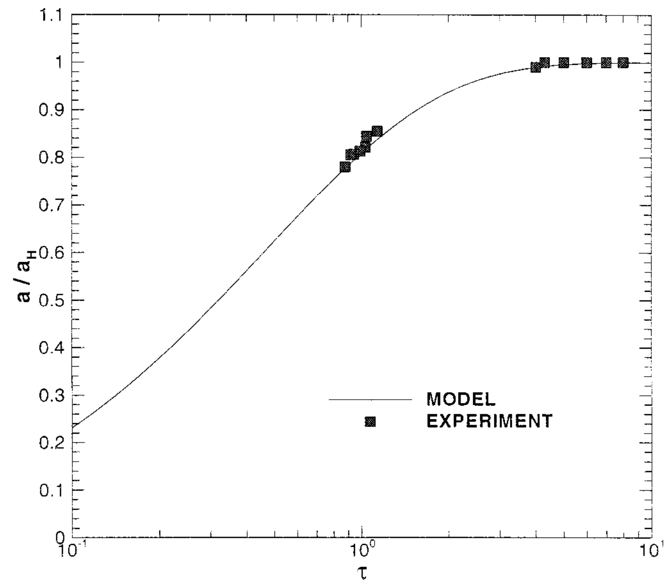

Fig. 5. Comparison of present model with experimental data.

is not able to approach unity for large τ due to the approximation made in his analysis. For the range of $\tau < 0.1$ the maximum error of the correlation with the data of Matthewson is approximately 3.0% at $\tau = 0.1$. The proposed model is not compared with the model of Jaffar [6] because he reported his numerical results in graphical form.

The proposed model shows good agreement with the model of Vorovich and Ustinov [13] for the thick layers, $\tau > 3.0$. For $\tau < 3.0$ the model is not in good agreement with the model of Vorovich and Ustinov [13] which is expected because their model is applicable only for thick layers.

V. Comparison of Experimental Data with Proposed Model

To verify the proposed model, contact radius measurements obtained by the authors are compared to the model predictions. The silicone rubber with the following properties from Stevanović [11], were used for the layer:

$$E_1 = 3.05 \text{ MPa}, \quad \nu_1 = 0.5$$

Tests were performed for two different layer thicknesses (7.0 mm and 43 mm), using two indenters ($\rho = 14.0$ mm and $\rho = 26.1$ mm) and for the load range 40–200 N. Experiments were done using an Instron, which can be used to measure the penetration depth d of the spherical indenter. Knowing the penetration depth d, the radius of the contact can be calculated using the simple geometrical relationship:

$$a = \sqrt{2\rho d} \tag{6}$$

Fig. 5 compares the present model and the experimental data for a range of τ. The maximum error of the correlation with the experimental data is 3.0%. These preliminary results verify the accuracy of the model for intermediate layer thicknesses.

VI. Summary

An approximate model for computing the contact radius of a sphere in contact with an elastic layer on a substrate is presented in the form of an equation which requires a simple root-finding procedure for the unknown radius of contact. The model is applicable for any layer-substrate material combination where $\alpha \geq 2.5$. The sphere and substrate are assumed to be rigid compared to the layer. Due to this assumption the model cannot predict the contact radius where the layer thickness is equal to zero, but the model can be used for the very thin relative layer thickness, $\tau \geq 0.01$. For $\alpha < 2.5$, the previously developed model is recommended.

A comparison to other models shows good agreement for the thin layers, Matthewson [9] and for the thick layers, Vorovich and Ustinov [13].

Experiments with silicone rubber layers were used to verify the accuracy of the proposed model in the ranges: $0.8 \leq \tau \leq 1.2$ and $\tau > 4$. The experimental results are in a good agreement with model predictions within 3%.

Future work is recommended to compare the proposed model with other models in the intermediate layer thickness, to experimentally verify the model for the thin layers and to obtain a closed-form solution for the unknown contact radius.

Appendix

The results of the application of the Chen and Engel model for the contact of a rigid sphere and an elastic layer bonded to a rigid substrate are correlated by the

$$\frac{a}{a_L} = 1 - c_3 \exp\left[c_1 \left(\frac{t}{a}\right)^{c_2}\right] \tag{7}$$

where

- a unknown contact radius;
- a_L contact radius corresponding to an infinitely thick layer;

t layer thickness.

The correlation coefficients are $c_1 = -1.73$, $c_2 = 0.734$, $c_3 = 1.04$.

The maximum contact radius is obtained from the Hertz relation

$$a_{\rm L} = \left[\frac{3}{4}\frac{(1-\nu_1^2)}{E_1}F\rho\right]^{1/3} \tag{8}$$

where

E_1, ν_1 Young's modulus and Poisson's ratio of the layer;
F load;
ρ radius of curvature of the sphere.

Since the unknown a appears on both sides of the equation, an iterative method must be used to calculate its root. The Newton–Raphson method can be used to obtain the root which is found from the following equation for $n \geq 0$:

$$a_{n+1} = a_n - \frac{[a_n - a_{\rm L}(1 - c_3 \exp[c_1(t/a_n)^{c_2}])]}{[1 - (a_{\rm L}/a_n)c_1c_2c_3(t/a_n)^{c_2}\exp[c_1(t/a_n)^{c_2}]]}. \tag{9}$$

Substituting the values for the correlation coefficients gives the equation for $n \geq 0$, [see (10)], shown at the bottom of the page.

This equation is valid for any combination of values for $a_{\rm L}$ and t.

The first guess for the iterative procedure can be based on the maximum contact radius $a_0 = a_{\rm L}$. Fewer than six iterations are required for convergence to eight digit accuracy.

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Mr. Culham is a member of ASME and the Professional Engineers Association of Ontario.

$$a_{n+1} = a_n - \frac{\left[a_n - a_{\rm L}\left(1 - 1.04\exp\left[-1.73(t/a_n)^{0.734}\right]\right)\right]}{\left[1 + 1.321(a_{\rm L}/a_n)(t/a_n)^{0.734}\exp\left[-1.73(t/a_n)^{0.734}\right]\right]} \tag{10}$$