Comprehensive Review of Natural Convection in Horizontal Circular Annuli

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ABSTRACT

A review of the currently available experimental data, numerical results, and analytical models and correlations for natural convection in a horizontal circular annulus with isothermal boundary conditions is presented. These results have direct application to air-layer insulation for pipelines, nuclear reactor design, and should provide an effective starting point for the analysis of heat transfer in sealed electronic enclosures. Comparisons are made between the available data, models and correlations for a wide range of aspect rations, $1.2 \le D_o/D_i \le 57$, for both air and water-filled annuli. Through these comparisons specific recommendations are made concerning the accuracy and application of the data and correlations presented in each of the previous studies.

NOMENCLATURE

C_1, C_2	=	correlation coefficients, Eqs. (9, 11)
D_o, D_i	=	outer, inner cylinder diameters (m)
g	=	gravitational acceleration (m/s^2)
G	=	correlation coefficient, Eq. (20)
Gr	=	Grashof number, Eq. (2)
h	=	average heat transfer coefficient, (W/m^2K)
k	=	thermal conductivity (W/mK)
k_e	=	effective conductivity, Eq. (6) (W/mK)
Nu	=	Nusselt number, Eq. (7)
Pr	=	Prandtl number, $\equiv \nu/\alpha$
Q	=	heat flow rate per unit length (W/m)
Ra	=	Rayleigh number, Eq. (3)
S	=	conduction shape factor
T_o, T_i	=	outer, inner cylinder temperatures (K)

Greek Symbols

α	=	thermal diffusivity (m^2/s)
β	=	thermal expansion coefficient $(1/K)$
δ	=	gap spacing, Eq. (1) (m)
ϵ	=	eccentricity (m)
ν	=	kinematic viscosity (m^2/s)

Subscripts

$cond$	=	conductive limit
$conv$	=	convective limit
D_i	=	based on inner cylinder diameter
δ	=	based on gap spacing
i	=	inner cylinder
o	=	outer cylinder

INTRODUCTION

Natural convection heat transfer from a heated body to its surrounding enclosure is currently of interest to designers of microelectronic systems and cabinets. The rapid growth of wireless and cellular communications has lead to a more widespread use of "outside plant" applications, products designed to withstand harsh, outdoor environments. The

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sensitive electronics contained in these products require a sealed enclosure to protect them from moisture and contaminants in their surroundings, and cannot rely on a supply of fresh, ambient air to dissipate the heat produced during typical operation. Analytical models that can be used to predict natural convection from electronics packages and circuit boards to their surrounding enclosure are of interest to engineers for trade-off and "what-if" studies during preliminary product design.

Although the current literature does not contain analytical models for the specific problem of circuit boards in a sealed enclosure, a number of researchers have examined problems involving more simple geometries. One of these configurations, the horizontal circular annulus, has been studied extensively, and these references provide a wide range of experimental and numerical results, as well as analytically-based correlations and models. The information provided by these studies should provide an effective starting point for the analysis of the sealed electronic enclosure, and it also has direct application to air-layer insulation for pipelines (Gröber et al., 1961), underground electric transmission cables and nuclear reactor design (Kuehn and Goldstein, 1976a).

Because of the large time span over which these studies were performed, over 65 years, and the large number of correlations and data available, some confusion may arise concerning the proper choice of analytical model or correlation for any particular configuration. The accuracy and application of the available correlations and data are in many cases limited to particular ranges of the independent parameters, such as the fluid type, geometry, or the Rayleigh and Grashof numbers. This paper presents a comprehensive review and comparison of all models and data currently available for natural convection in the horizontal circular annulus. This review includes recommendations for which correlations are appropriate for any particular configuration, and discussion of the expected accuracy of the predicted values.

General heat transfer texts and handbooks typically present correlations from a single source, and do not compare the results with the available data or comment on application of these correlations beyond the specified range. No comprehensive review of this type is currently available in the literature.

BACKGROUND

The problem of interest, shown in Fig. 1, involves natural convection in an annular region with uniform temperature boundary conditions T_i and T_o on the inner and outer surfaces, respectively. In all previous studies, the geometry of the annulus is expressed in one of two ways. The first method is based on the aspect ratio, a non-dimensional quantity defined as D_o/D_i, with values for all configurations

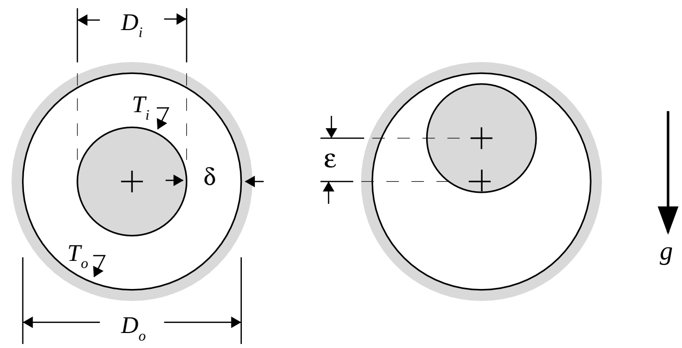

Figure 1: Schematic of Horizontal Circular Annulus Problem

limited to the range:

$$1 < \frac{D_o}{D_i} < \infty$$

The second method uses the gap spacing δ, defined by:

$$\delta = \frac{D_o - D_i}{2}\ , \qquad 0 < \delta < \infty \tag{1}$$

In all cases, the annulus length L is assumed to be large compared to the outer diameter, such that any axial heat flow can be neglected. Therefore the heat flow rate Q is the total per unit length in the axial direction, with units W/m.

Two of the models presented in this review include an eccentricity term, where ϵ is defined as the distance between the centers of the inner and outer cylinders as shown in Fig. 1. This dimensional quantity is limited to the range:

$$0 \le \epsilon < \frac{D_o - D_i}{2}$$

The remaining independent parameter used in many of the previous studies, particularly in air-filled annuli, is the Grashof number, defined as:

$$Gr_{\mathcal{L}} = \frac{g\beta\,(T_i - T_o)\,\mathcal{L}^3}{\nu^2} \tag{2}$$

where $\mathcal{L}$ represents a general characteristic length. The Rayleigh number is also widely used, especially for fluids with Pr larger than one, such as water or various oils:

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} \cdot Pr = \frac{g\beta\,(T_i - T_o)\,\mathcal{L}^3}{\nu\alpha} \tag{3}$$

Two different characteristic lengths were used in the previous work: the inner cylinder diameter D_i and the gap spacing δ. The gap spacing δ was first proposed as a characteristic length by Kraussold (1934), who noted that dependence of the solution on aspect ratio could be reduced and a single curve could be used to fit the results for all

D_o/D_i. However, for the limiting case of large aspect ratio, $D_o >> D_i$, where the solution is expected to approach that of a single, isothermal cylinder, a Rayleigh number based on the gap spacing approaches infinity. This limiting case is treated correctly when the diameter of the inner cylinder is used as the characteristic length, but the solution remains a strong function of the aspect ratio.

Average heat transfer rate in the annulus is non-dimensionalized using one of two methods: the effective conductivity ratio and the Nusselt number. The dimensionless effective conductivity ratio, first proposed by Beckmann (1931), is based on the formulation for dimensionless conduction shape factor in a circular annulus:

$$S = \frac{2\pi}{\ln(D_o/D_i)} \tag{4}$$

Through the use of an effective conductivity, k_e, the effects of convection on heat transfer in the annulus are included in this conduction expression:

$$Q = S\, k_e\,(T_i - T_o) \tag{5}$$

Solving for k_e and normalizing using the actual thermal conductivity of the medium yields:

$$\frac{k_e}{k} = \frac{Q}{k\,(T_i - T_o)} \frac{\ln(D_o/D_i)}{2\pi} \tag{6}$$

where $k_e/k \geq 1$ in all cases.

The second method for non-dimensionalizing the results uses the Nusselt number, defined as:

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k} = \frac{Q\mathcal{L}}{\pi D_i k\,(T_i - T_o)} \tag{7}$$

where both the inner cylinder diameter D_i and the gap spacing δ have been used as the characteristic dimension in previous studies.

The dimensionless effective conductivity ratio can be related to the Nusselt number based on inner diameter using the following relationship:

$$Nu_{D_i} = \frac{k_e}{k} \frac{2}{\ln(D_o/D_i)} \tag{8}$$

For the purposes of the proposed review, a single non-dimensionalization and length scale should be chosen to allow comparison of the available models and data. All results will be converted in terms of Nusselt and Grashof numbers, or Rayleigh numbers for fluids other than air. All non-dimensionalized values will use the inner cylinder diameter D_i as characteristic length, and the annulus geometry will be reported in terms of the aspect ratio D_o/D_i.

Figure 2 is a plot of typical results anticipated for this problem, and it clearly demonstrates three distinctive regions in the solution. The conductive limit, characterized

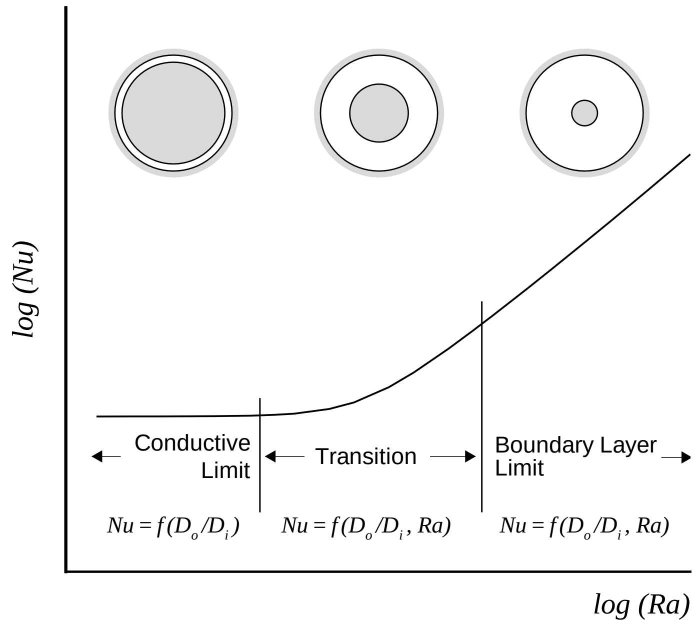

Figure 2: Schematic of Solution Behavior

by small Ra or D_o/D_i, is a strong function of the aspect ratio and is independent of the Rayleigh number. The boundary layer limit occurs at large Ra and large aspect ratios, where the difference in surface area between the inner and outer boundaries leads to the inner cylinder controlling the heat transfer in the annulus. It is anticipated that at this limit the solution will be a strong function of Ra, but a somewhat weaker function of the aspect ratio because of the relative unimportance of the outer boundary for large D_o/D_i.

In the intermediate region between the conduction-dominated and boundary layer regions, the solution is a strong function of both Ra and D_o/D_i as it moves smoothly from one limiting case to the other. The location of this transition on a log-log plot, such as that shown in Fig. 2, is strongly dependent on the aspect ratio and may vary over a few decades of Rayleigh number.

The following sections present a brief, chronological review of the previous studies with a description of the available data, models and correlations. Although some of these works include local temperature, velocity and heat flux distributions as a function of angular or radial position, these local effects are beyond the scope of this presentation and will not be considered here.

REVIEW OF PREVIOUS STUDIES

One of the first, well documented studies of heat transfer in horizontal annular enclosures was presented by Beckmann (1931). He performed experimental measurements

Table 1: Summary of Previous Experimental Data and Numerical Results

Authors	Test	Fluid	Ranges	Number of Aspect Ratios
Beckmann (1931)	experimental	H_2, Air, CO_2	$1.1875 \leq D_o/D_i \leq 8.1$ $3.4 \times 10^3 \leq Gr_{D_i} \leq 1.5 \times 10^7$	7
Voigt and Krischer (1932)	experimental	Air	$1.4 \leq D_o/D_i \leq 4.29$ $2.1 \times 10^3 \leq Gr_{D_i} \leq 7.9 \times 10^5$	4
Kraussold (1934)	experimental	Water, Oil - $Pr \approx 500$ Oil - $Pr \approx 5000$	$1.234 \leq D_o/D_i \leq 3.0$ $2 \times 10^5 \leq Ra_{D_i} \leq 1 \times 10^8$	3
Liu, Mueller and Landis (1961)	experimental	Air, Water, Oil - $Pr \approx 3000$	$1.157 \leq D_o/D_i \leq 7.52$ $4.9 \times 10^3 \leq Ra_{D_i} \leq 7.2 \times 10^8$	5
Crawford and Lemlich (1962)	numerical	Air	$2 \leq D_o/D_i \leq 57$ $1 \leq Gr_{D_i} \leq 1 \times 10^5$	3
Grigull and Hauf (1966)	experimental	Air	$1.3 \leq D_o/D_i \leq 6.3$ $5.7 \times 10^3 \leq Gr_{D_i} \leq 1.1 \times 10^6$	9
Lis (1966)	experimental	Air	$2.0 \leq D_o/D_i \leq 4.0$ $6.2 \times 10^5 \leq Gr_{D_i} \leq 4.6 \times 10^{10}$	3
Koshmarov and Ivanov (1973)	experimental	Air	$1.9 \leq D_o/D_i \leq 9.4$ $10^{-9} < Gr_{D_i} \leq 1.9 \times 10^5$	3
Kuehn and Goldstein (1976a)	experimental	Air, Water	$D_o/D_i = 2.6$ $4.1 \times 10^4 \leq Gr_{D_i} \leq 1.9 \times 10^6$	1
	numerical	Air	$2.6 \leq D_o/D_i \leq 17$ $20 \leq Gr_{D_i} \leq 1.4 \times 10^5$	6

for three different gases, air, H_2 and CO_2, for the ranges of aspect ratio and Gr_{D_i} listed in Table 1. An additional contribution by this author is his fundamental analysis of this problem, resulting in the definition of the dimensionless effective conductivity ratio k_e/k used in many subsequent works. He also proposed a simple correlation for his results, as presented in Table 2. Values for the correlation coefficient C_1 are determined as a function of the aspect ratio using a plot provided in the paper. Beckmann also includes a correlation for C_1 as a function of D_o/D_i, but its predictions vary significantly from those shown in the plot and lead to substantial errors in the results. The following updated correlation predicts coefficient values to within $\pm 1\%$ of those presented by the plot:

$$C_1 = \left(\ln\left[(D_o/D_i)^{0.207}\right]\right)^{1.22} \tag{9}$$

Subsequent publications have raised questions concerning the accuracy of Beckmann's measurements, particularly the air data for $D_o/D_i > 2.1$. Liu, Mueller and Landis (1961) attribute the large axial conduction errors along the tube walls to Beckmann's failure to employ guard heaters at the tube ends. The effects of this axial conduction would be most noticeable when the length of the tube becomes small relative to its diameter, as in his $D_o/D_i = 3.3, 5.8$ and 8.1 cases.

Voigt and Krischer (1932) presented a follow-up to Beckmann's work, performing experimental measurements for air at four different aspect ratios and smaller values of Gr_{D_i}, as listed in Table 1. No correlation for their data was provided.

Kraussold (1934) expanded the research of Beckmann (1931) to liquid-filled annuli, performing heat transfer measurements for water, transformer oil ($Pr \approx 500$) and machine oil ($Pr \approx 5000$). Because of the large differences in

Table 2: Summary of Previous Correlations and Models

Authors	Correlation	Ranges
Beckmann (1931)	$\frac{k_e}{k} = C_1 \, {Gr_{D_i}}^{0.258}$	$1 < D_o/D_i \leq 56, \qquad Pr \approx 1$
Kraussold (1934)	$\frac{k_e}{k} = 1$	$Ra_\delta < 6.3 \times 10^3$
	$\frac{k_e}{k} = 0.11 {Ra_\delta}^{0.29}$	$6.3 \times 10^3 \leq Ra_\delta \leq 1 \times 10^6$
	$\frac{k_e}{k} = 0.40 {Ra_\delta}^{0.20}$	$Ra_\delta > 1 \times 10^6$
Liu et al. (1961)	$\frac{k_e}{k} = 1$	$\left(\frac{Pr}{1.36 + Pr}\right) Ra_\delta < 1 \times 10^3$ $1.5 \leq (D_o/D_i) \leq 7.5$
	$\frac{k_e}{k} = 0.135 \left(\frac{Pr}{1.36 + Pr}\right)^{0.278} {Ra_\delta}^{0.278}$	$3.2 \times 10^3 \leq \left(\frac{Pr}{1.36 + Pr}\right) Ra_\delta < 1 \times 10^8$ $1.5 \leq (D_o/D_i) \leq 7.5$
Lis (1966)	$\log\left(\frac{k_e}{k}\right) = 0.0794 + 0.0625 \log(X) + 0.0154 \left[\log(X)\right]^2$ $X = Ra_{Di} \left(1 - \frac{1}{D_o/D_i}\right)^{6.5}$	$1 \times 10^4 \leq Ra_{D_i} \left(1 - \frac{1}{D_o/D_i}\right)^{6.5} \leq 1 \times 10^9$ $2 \leq D_o/D_i \leq 4, \qquad 0.645 \leq Pr \leq 1.32$
	$\frac{k_e}{k} = 0.389 \left[Ra_{D_i} \left(1 - \frac{1}{D_o/D_i}\right)\right]^{0.237}$	$4 \times 10^4 \leq Ra_{D_i} \left(1 - \frac{1}{D_o/D_i}\right)^{6.5} \leq 1 \times 10^8$
	$\frac{k_e}{k} = 0.087 \left[Ra_{D_i} \left(1 - \frac{1}{D_o/D_i}\right)\right]^{0.329}$	$1 \times 10^8 < Ra_{D_i} \left(1 - \frac{1}{D_o/D_i}\right)^{6.5} \leq 5 \times 10^{10}$
Grigull and Hauf (1966)	$Nu_\delta = \left[0.2 + 0.145(\delta/D_i)\right] {Gr_\delta}^{1/4} \exp(-0.02(\delta/D_i))$	$3 \times 10^4 \leq Gr_\delta \leq 7.2 \times 10^5$, $2.1 \leq D_o/D_i \leq 6.3, \qquad Pr = 0.7$
Itoh et al. [10]	$\frac{k_e}{k} = 0.2 \left[0.5 \sqrt{D_o/D_i} \ln(D_o/D_i)\right]^{3/4} Ra_{D_i}^{1/4}$	$Ra_{D_i} \left[0.5 \sqrt{D_o/D_i} \ln(D_o/D_i)\right]^3 \geq 7.1 \times 10^3$
Kuehn and Goldstein (1976b)	Eqs. (14 - 24)	
Raithby and Hollands (1975)	$Nu_{D_i} = \frac{2.425}{\pi \left[1 + (D_o/D_i)^{-3/5}\right]^{5/4}} \left(\frac{Pr}{0.861 + Pr}\right)^{1/4} {Ra_{D_i}}^{1/4}$	$Nu_{D_i} > \frac{2}{\ln(D_o/D_i)}$

the Prandtl number between these fluids, he presents his results in terms of Ra_{D_i}. The range of values tested is shown in Table 1. Kraussold (1934) also recognized the dimensionless effective conductivity ratio as a function of the Rayleigh number and aspect ratio:

$$\frac{k_e}{k} = F(Ra_{D_i}, D_o/D_i) \qquad (10)$$

and proposed that, when the gap thickness δ is used as the characteristic length, the variation of the effective conductivity ratio as a function of aspect ratio is minimized. Kraussold (1934) presented a piecewise correlation of his results as shown in Table 2, with three different formulations for particular ranges of Ra.

Senftleben and Gladisch (1948) presented experimental results for other gases, including propane, methane, butane, and several others. The results for all gases are well correlated by a single curve when the gap thickness is used as the characteristic length, as recommended by Kraussold (1934). These authors presented their measurements graphically for average Nusselt number as a function of Gr, but did not provide any correlations for their results. Because of their incomplete documentation of Pr and the obscure nature of the gases considered in this study, the results of Senftleben and Gladisch (1948) will not be included in the comparison that follows.

An experimental investigation of horizontal circular annuli is presented by Liu, Mueller and Landis (1961) for five different aspect ratios and three different fluids: air, water, and silicon oil ($Pr \approx 3000$). The ranges of D_o/D_i and Ra_{D_i} examined in these tests are summarized in Table 1. These authors proposed a correlation of their results using the gap width δ as the characteristic length, and stipulate that the effective conductivity ratio k_e/k should approach unity at low values of Ra_δ, where pure conduction effects dominate. In addition, the effects of Pr are included in the proposed correlation by the following correction term:

$$\left(\frac{Pr^2}{1.36 + Pr}\right)^{0.278}$$

derived from an integral analysis of laminar free convection from a vertical flat plate. The resulting correlation, presented in Table 2, predicts their experimental measurements to within $\pm 20\%$ for all cases.

Crawford and Lemlich (1962) reported the results of their numerical simulations of this problem for the range of values shown in Table 1. Most notable in their study are the results for $D_o/D_i = 57$, the largest aspect ratio data available in the literature. Although these authors compare their results against the models of Beckmann (1931) and Liu, Mueller and Landis (1961), they provided no additional analysis or correlation of their own.

Temperature fields and flow lines in air-filled horizontal circular annuli were examined by Grigull and Hauf (1966) using Mach-Zhender interferometry, and these results are integrated over the circumference of the inner cylinder to determine average heat transfer results. The ranges of D_o/D_i and Gr for these tests are described in Table 1. Grigull and Hauf (1966) were the first authors to identify three distinct regions in the solution as a function of the independent parameters: the pseudo-conductive region ($Gr_\delta < 2400$) where the solution is independent of Gr_δ; the transition region ($2400 \leq Gr_\delta \leq 30000$); and the fully developed convective limit ($Gr_\delta > 30000$). Finally, these authors presented a correlation of their experimental results for fully developed convective flow, as given in Table 2.

Lis (1966) presented an experimental study for natural convection in air-filled horizontal circular annuli containing helical and axial spacers. Part of this study involved simple annuli without any spacers, and the range of experimental measurements reported by Lis (1966) are summarized in Table 1. All tests performed for the cases without spacers fall within the previously defined boundary layer limit, and are effectively correlated by the author's proposed polynomial equation, as presented in Table 2. Lis (1966) also presented a piecewise correlation of the data by the two simplified equations given in Table 2. It is suggested that two correlations with different exponents on the Rayleigh number are required to correctly fit the data because the dependence of the results on Ra varies between laminar and turbulent flow, where the usually accepted values of the exponent are 0.25 and 0.33, respectively.

An alternative correlation of the available experimental data was presented by Itoh et al. (1970), where the characteristic length is based on the geometric mean of the inner and outer cylinder radii combined with the conduction shape factor for the annulus. This correlation, presented in Table 2, is compared with the previous experimental results of Beckmann (1931), Kraussold (1934) and Grigull and Hauf (1966) for a wide range of aspect ratios, Rayleigh numbers and fluid types.

Koshmarov and Ivanov (1973) presented experimental results for heat transfer through rarefied gases (air and argon) in circular annuli for three different aspect ratios, summarized in Table 1. Although rarefied gas effects are outside the scope of the present review, these authors included extensive data for the conduction, transition and boundary layer regions. No correlation of these results was presented by the authors.

One of the most widely used models for heat transfer between concentric cylinders was developed by Raithby and Hollands (1975), based on an analysis technique involving average conduction layer thicknesses within the fluid adjacent to the inner and outer boundaries. Using the effective conductivity ratio originally defined by Beckmann (1931), these authors proposed the following expression:

$$\frac{k_e}{k} = \frac{C_2}{2} \frac{\ln(D_o/D_i)}{\delta^{\,3/4} \left(1/D_i^{\,3/5} + 1/D_o^{\,3/5}\right)^{5/4}} Ra_\delta^{\,1/4} \tag{11}$$

where the coefficient $C_2 = 0.634$ is determined based on the available empirical data. The effects of variations in the Prandtl number are included in this model by the following formulation:

$$\frac{k_e}{k} = 0.386 \left(\frac{Pr}{0.861 + Pr}\right)^{1/4} \cdot \frac{\ln(D_o/D_i)}{\delta^{\,3/4} \left(1/D_i^{\,3/5} + 1/D_o^{\,3/5}\right)^{5/4}} Ra_\delta^{\,1/4} \tag{12}$$

The authors recommended that this equation be used for all cases where $k_e/k > 1$, and that $k_e/k = 1$ be assumed for cases where the use of this equation gives an effective conductivity ratio less than unity.

In a subsequent publication, Raithby and Hollands (1985) expanded their model to include eccentric annuli. Through the use of the eccentricity, ϵ, as defined in Fig. 1, the conduction limit for the effective conductivity of eccentric cylinders with eccentricity in the range $0 \le \epsilon < \delta$ is determined by:

$$\left(\frac{k_e}{k}\right)_{cond} = \frac{\ln(D_o/D_i)}{\cosh^{-1}\left[\left(D_o^{\,2} + D_i^{\,2} - 4\epsilon^2\right) / \left(2 D_o D_i\right)\right]} \tag{13}$$

For the transition and boundary layer regions, the expression from the previous study (Raithby and Hollands, 1975), Eq. (12) is recommended, but the authors warn that errors must be expected for cases with $\epsilon > 0$ when $k_e/k \approx 1$.

In his recently published heat transfer textbook, Bejan (1993) recast the Raithby and Hollands relation using the inner cylinder diameter as the characteristic length, and presented the results in terms of the Nusselt number Nu_{D_i}, as shown in Table 2.

In the first of two publications, Kuehn and Goldstein (1976a) presented the results of their experimental tests and numerical simulation of the horizontal annulus, as summarized in Table 1. Their experimental results included extensive documentation of temperature profiles and flow fields in the annular cavity, as well as average heat transfer results for air and water-filled annuli. Numerical predictions of temperature and velocity profiles were also presented, and average effective conductivities were reported for $Pr = 0.7$ for various aspect ratios and Rayleigh numbers. Simple correlations of the experimental results from a least-squares regression were also presented.

Kuehn and Goldstein (1976b) also developed a comprehensive model for heat transfer in concentric and eccentric circular annuli suitable for all values of the independent parameters D_o/D_i, Pr and Ra. The formulation of this model included terms for pure conduction for both concentric and eccentric geometries, as well as laminar and turbulent convection for the boundary layer limit. Results are expressed in terms of the Nusselt number based on D_i and the dimensionless effective conductivity ratio k_e/k.

The overall Nusselt number Nu_{D_i} were determined by a combination of the limiting cases discussed earlier, the pure conduction and boundary layer flow, using the Churchill and Usagi (1972) composite solution technique:

$$Nu_{D_i} = \left[Nu_{D_i\,cond}^{\;\;15} + Nu_{D_i\,conv}^{\;\;15}\right]^{1/15} \tag{14}$$

where $Nu_{D_i\,cond}$ and $Nu_{D_i\,conv}$ refer to expressions for the conductive and convective limits, respectively. The conductive limit in Eq. (14) is found by:

$$Nu_{D_i\,cond} = \frac{2}{\cosh^{-1}\left(\dfrac{D_i^{\,2} + D_o^{\,2} - 4\epsilon^{\,2}}{2\,D_i\,D_o}\right)} \tag{15}$$

where ϵ describes the eccentricity of the cylinders. For those cases involving concentric cylinders, $\epsilon = 0$, a simpler expression can be used:

$$Nu_{D_i\,cond} = \frac{2}{\ln\left(\dfrac{D_o}{D_i}\right)} \tag{16}$$

The convection term in Eq. (14) is determined by the following expression:

$$Nu_{D_i\,conv} = \frac{2}{\ln\left(\dfrac{1 + 2/Nu_i}{1 - 2/Nu_o}\right)} \tag{17}$$

The convection on the inner and outer surfaces, Nu_i and Nu_o, for the full range of Ra_{D_i} including both laminar and turbulent flow, are calculated by:

$$Nu_i = \left[\left\{0.518\left[1 + \left(\frac{0.559}{Pr}\right)^{3/5}\right]^{-5/12} Ra_{D_i}^{\;\;1/4}\right\}^{15} + \left\{0.1 Ra_{D_i}^{\;\;1/3}\right\}^{15}\right]^{1/15} \tag{18}$$

$$Nu_o = \left[\left\{\left[39.24 + \left(0.587 G\left(\frac{D_o}{D_i}\right)^{3/4} Ra_{D_i}^{1/4}\right)^{5/3}\right]^{3/5}\right\}^{15} + \left\{0.1\left(\frac{D_o}{D_i}\right) Ra_{D_i}^{\;\;1/3}\right\}^{15}\right]^{1/15} \tag{19}$$

where:

$$G = \left[\left(1 + \frac{0.6}{Pr^{\,0.7}}\right)^{-5} + \left(0.4 + 2.6\,Pr^{\,0.7}\right)^{-5}\right]^{-1/5} \qquad (20)$$

These components, Eqs. (18,19), can be simplified significantly if the terms describing the turbulent flow are removed:

$$Nu_i \;=\; 0.518\left[1 + \left(\frac{0.559}{Pr}\right)^{3/5}\right]^{-5/12} {Ra_{D_i}}^{1/4} \qquad (21)$$

$$Nu_o \;=\; \left[39.24 + \left(0.587\,G\left(\frac{D_o}{D_i}\right)^{3/4} {Ra_{D_i}}^{1/4}\right)^{5/3}\right]^{3/5} \qquad (22)$$

where the coefficient G is determined by Eq. (20). For the particular case of an air-filled annulus, assuming $Pr = 0.7$ further simplifies the model:

$$Nu_i \;=\; 0.3987\,{Ra_{D_i}}^{1/4} \qquad (23)$$

$$Nu_o \;=\; \left(39.24 + \left(\frac{D_o}{D_i}\right)^{5/4} {Ra_{D_i}}^{5/12}\right)^{3/5} \qquad (24)$$

Combining these two expressions and the conduction term, Eq. (16), into the expression for the overall Nusselt number, Eq. (14), results in a significant simplification of the model for air-filled, concentric cylinders. From Fig. 3 it can be seen that the simplified model is identical to the full model presented by Kuehn and Goldstein (1976b) for the conduction and transition regions, and significant differences occur only at large values of Ra_{D_i}.

COMPARISON OF DATA AND MODELS

Due to the large amount of data available for the horizontal circular annulus, a complete evaluation of all experimental data and numerical results is beyond the scope of this review. However, certain comparisons of data for the more commonly used fluids and aspect ratios with the available models and correlations are possible. The following section compares the available experimental data and numerical results with a number of different correlations and models for air and water-filled annuli for six different aspect ratios. All data and correlations are presented using dimensionless quantities based on the inner cylinder diameter. The independent variable for those cases involving air is the Grashof number, while the Rayleigh number is used in the presentation of the results for the water-filled annulus. All results are plotted in terms of the average Nusselt number Nu_{D_i}.

Figure 4 compares the experimental data of Beckmann (1931) and Liu, Mueller and Landis (1961) for air-filled annuli with aspect ratio approximately equal to 1.175 with

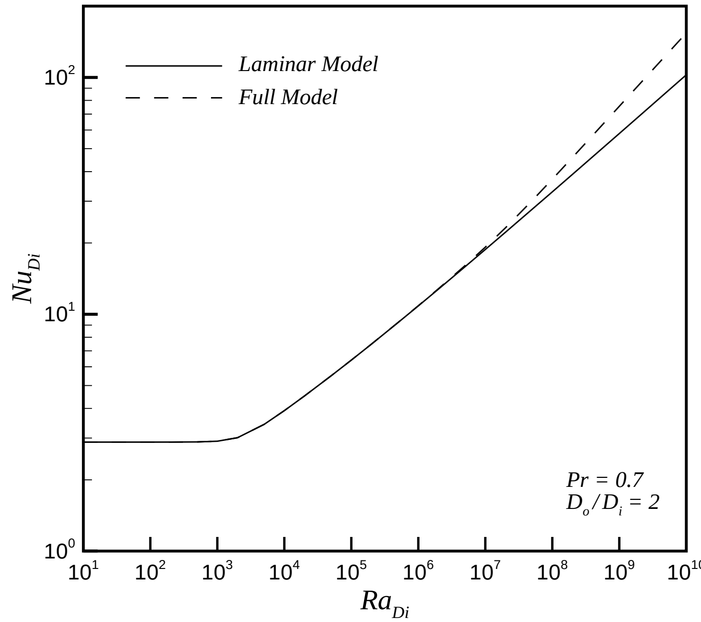

Figure 3: Comparison of Laminar and Full Kuehn and Goldstein (1976b) Models

the models of Kuehn and Goldstein (1976b) and Raithby and Hollands (1975). The actual values of aspect ratio for the Beckmann (1931) and Liu et al. (1961) data are 1.19 and 1.16, respectively. The difference in the conduction limit between these values and $D_o/D_i = 1.175$ used in the model is approximately 8%. Both data sets lie within the previously defined transition region between conduction and convection dominated heat transfer.

The data of Beckmann (1931) lies approximately 5% below the conduction limit predicted by the models, as expected for an annulus with a slightly larger diameter ratio. The data are essentially constant for all values of Gr_{D_i} tested, which is consistent with the definition of the conduction region. The data of Liu, Mueller and Landis (1961) exhibits some scatter and are up to 30% above the predictions of the models in some cases. Liu et al. (1961) acknowledged this scatter in their air data, and recommended that a greater emphasis should be placed on their silicon oil test results, where experimental uncertainty was better controlled.

The models of Raithby and Hollands (1975) and Kuehn and Goldstein (1976b) predict the majority of the data to within $\pm 10\%$, and both models predict transition at approximately the same value of Gr_{D_i}. For larger Grashof numbers in the boundary layer flow region, the Kuehn and Goldstein (1976b) predictions are larger than those of the Raithby and Hollands (1975) correlation, by approximately 9% at $Gr_{D_i} = 1 \times 10^7$ and 15% at $Gr_{D_i} = 1 \times 10^8$.

Data and model predictions for a water-filled annulus with aspect ratio $D_o/D_i = 1.5$ are compared in Fig. 5. The

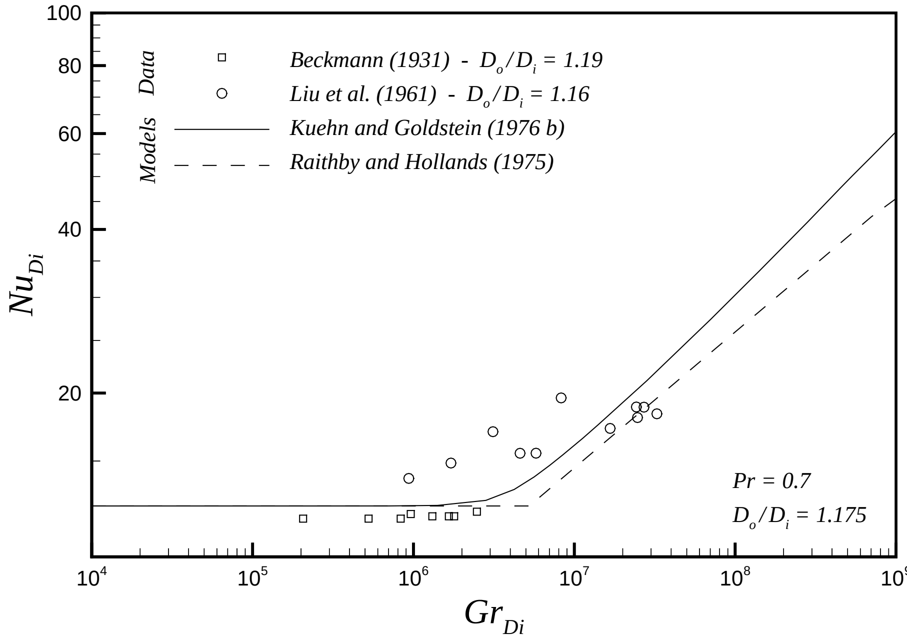

Figure 4: Comparison of Available Models, Correlations and Data for $Pr = 0.7$ and $D_o/D_i = 1.175$

measured values of Kraussold (1934) and Liu, Mueller and Landis (1961) are plotted along with the correlations of Kraussold (1934) and Liu et al. (1961), and the models of Kuehn and Goldstein (1976b) and Raithby and Hollands (1975). In this case, both data sets are well within the boundary layer flow region.

The Liu et al. (1961) data are in poor agreement with the other data and correlations in a manner similar to that described previously for their air data, and the authors' explanation of experimental uncertainty also applies in this case. The Kraussold (1934) data are in good agreement with his correlation, as well as the correlation of Liu et al. (1961) and the model of Kuehn and Goldstein (1976b). The Raithby and Hollands (1975) model falls above each of these for all but very large Ra_{D_i}.

Figure 6 compares data from a wide range of sources for air-filled annuli having aspect ratio $D_o/D_i \approx 2$ with the correlations of Grigull and Hauf (1966), Lis (1966), Itoh et al. (1970) and Beckmann (1931), and the model of Kuehn and Goldstein (1976b). The available data cover the full range of Gr_{D_i}, from the conduction limit data of Koshmarov and Ivanov (1973) to the Lis (1966) data for the laminar and turbulent boundary layer regime. However, the majority of the available data fall within the laminar boundary layer limit, $5 \times 10^4 \leq Gr_{D_i} \leq 10^7$.

There is good agreement between the data shown in Fig. 6, with variation between the data points of less than 10% in most cases. A few of the data points of Beckmann are significantly lower than the other data, due to experimental error or incorrect use of thermophysical properties when reducing the data. Also the first few data points of Grigull and Hauf (1966) for low Gr_{D_i} fall below the other values, and the trend of these data do not suggest that the results are approaching the conduction limit as Gr_{D_i} becomes smaller.

Within the laminar boundary layer region all correlations with the exception of Beckmann (1931) agree closely with each other. The turbulence term in the Kuehn and Goldstein (1976b) models does cause it to over-predict the data above $Gr_{D_i} = 10^8$, but the model seems to follow the trend of the Lis (1966) data in this region. In the transition region the predictions of the Kuehn and Goldstein (1976b) model fall below the data of Koshmarov and Ivanov (1973) by up to 30%, but closely fits the data of Voigt and Krischer (1932). The correlation of Beckmann (1931), using Eq. (9) for the coefficient C_1, has significant error for all Gr_{D_i} for this aspect ratio.

The plot shown in Fig. 7 compares the air data of Grigull and Hauf (1966) for $D_o/D_i = 3.9$ and Lis (1966) at $D_o/D_i = 4.0$ with the correlations of Beckmann (1931), Grigull and Hauf (1966), Lis (1966), Itoh et al. (1970) and the model of Raithby and Hollands (1975). All of the available data for this configuration are contained within the laminar boundary flow limit.

For low values of Grashof number, $5 \times 10^3 \leq Gr_{D_i} \leq 10^6$, there is excellent agreement between the data and the correlations of Grigull and Hauf (1966), Itoh et al. (1970) and Raithby and Hollands (1975), with an average difference of

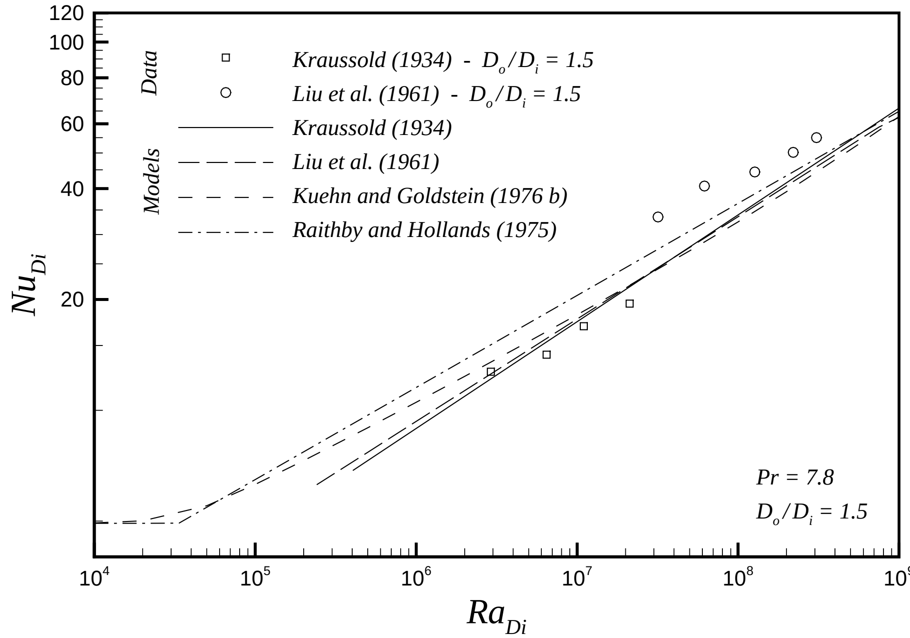

Figure 5: Comparison of Available Models, Correlations and Data for $Pr = 7.8$ and $D_o/D_i = 1.5$

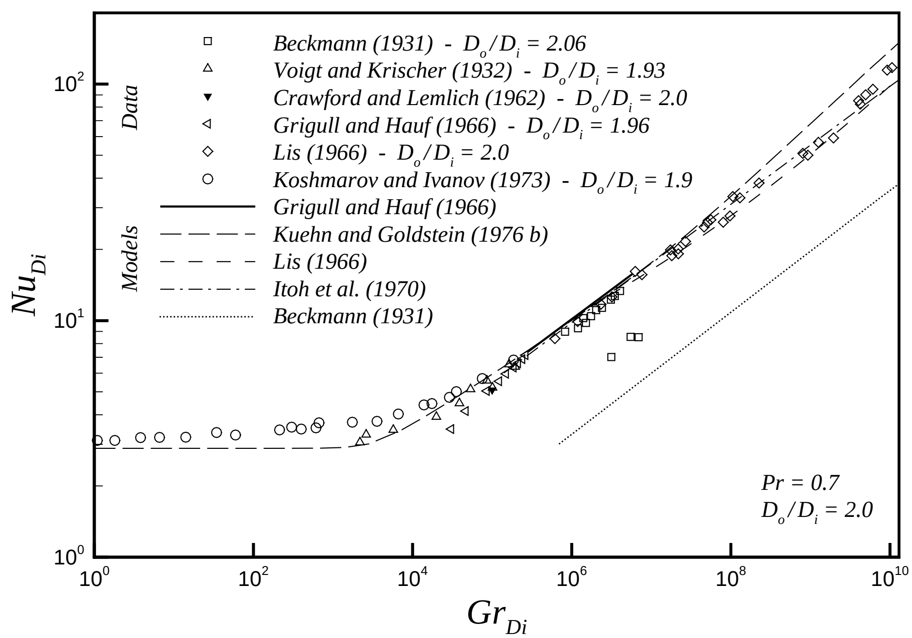

Figure 6: Comparison of Available Models, Correlations and Data for $Pr = 0.7$ and $D_o/D_i = 2.0$

less than 2% over this range. The correlation of Lis (1966) clearly underpredicts the data and the polynomial shape of the curve does not reflect the established physical behavior of the data. The correlation of Beckmann (1931) again underpredicts the data, but the difference is less than for the $D_o/D_i = 2$ case.

The final plots, shown in Fig. 8, compare data and models for two different aspect ratios. The first data set shown in Fig. 8 compares the data of Koshmarov and Ivanov (1973)

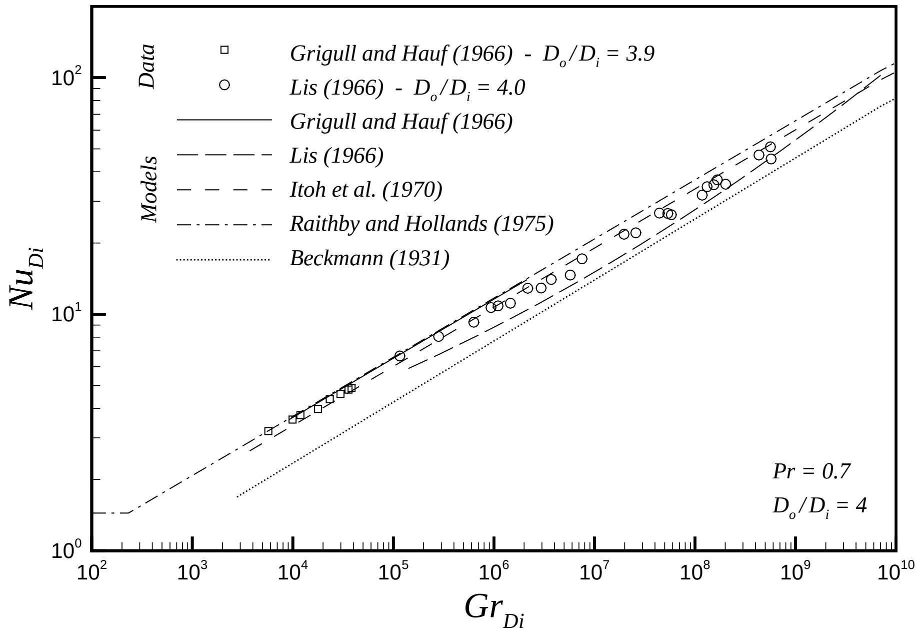

Figure 7: Comparison of Available Models, Correlations and Data for $Pr = 0.7$ and $D_o/D_i = 4.0$

for $D_o/D_i = 9.4$ with the models of Kuehn and Goldstein (1976b) and Raithby and Hollands (1975). This plot clearly demonstrates the excellent agreement between the Kuehn and Goldstein (1976b) model and the data over the entire transition region, while the piecewise approach of the Raithby and Hollands (1975) model can lead to an underprediction of the data by as much as 30%.

The second data set shown in Fig. 8 compares the numerical results of Crawford and Lemlich (1962) for an air-filled annulus of aspect ratio $D_o/D_i = 57$ with the models of Kuehn and Goldstein (1976b) and Raithby and Hollands (1975). Once again, the model of Kuehn and Goldstein (1976b) is in good agreement with the data, while the Raithby and Hollands piecewise correlation leads to an underprediction of up to 40% in the transition region.

SUMMARY AND CONCLUSIONS

A review of the available experimental data, numerical results, analytical models and correlations for heat transfer in a horizontal circular annulus has been presented. Comparisons between the data, correlations and models for several different configurations have been presented, and there is generally good agreement between the results of each of the studies, with the following exceptions. The use of the correlation of Beckmann (1931), even with the updated correlation for C_1 given in Eq. (9), should be avoided in all cases. Although the exponent on Gr_{D_i} appears correct, the dependence of this correlation on the aspect ratio is incorrect and leads to substantial errors for small values of D_o/D_i. Because of their large experimental uncertainty, the air and water data of Liu, Mueller and Landis (1961) should also be avoided in any subsequent correlation or validation studies.

Most of the correlations examined in this review provide acceptable accuracy, within $\pm 10\%$, for the laminar boundary layer region. However, these correlations should not be used beyond their recommended ranges. Some of these correlations, such as the polynomial fit of Lis (1966) or the exponential formulation of Grigull and Hauf (1966), may result in unpredictable behavior and substantial errors if used beyond their recommended limits of aspect ratio or Ra_{D_i}.

By their use of the Churchill and Usagi (1972) composite solution technique, Kuehn and Goldstein (1976b) presented the only model in the current study that attempts to fit data within the transition region. Caution is required when using piecewise correlations or models, such as Raithby and Hollands (1975), in the transition region, which may underpredict the data by as much as 40% in some cases.

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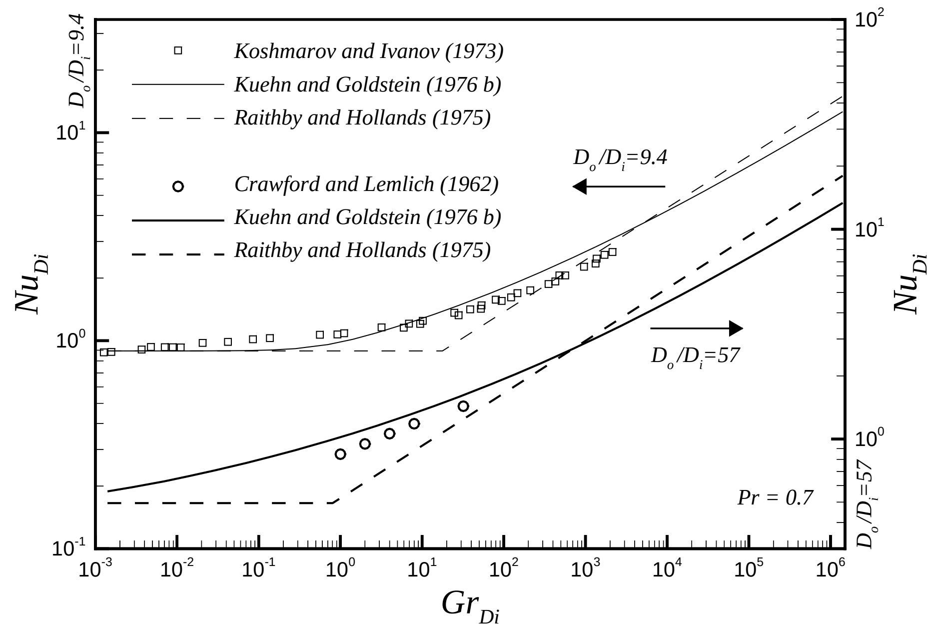

Figure 8: Comparison of Available Models, Correlations and Data for $Pr = 0.7$ and $D_o/D_i = 9.4, 57$

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