ME 303 M.M. Yovanovich

Week 4

Lecture 1

Victoria Day. No Lecture.

Lecture 2

• Review dimensional and nondimensional forms of the one-dimensional Heat Equation (Diffusion Equation) in cartesian coordinates.

• In dimensional form $T = T(x, L, t, \alpha, T_i, T_0)$, 6 independent variables. The BCs and IC are nonhomogeneous.

• Introduce dimensionless variables: $\xi = x/L, \phi = (T(x,t) - T_i)/(T_0 - T_i)$ and $\tau = \alpha t/L^2 = t/(L^2/\alpha)$. Note that (L^2/α) is a characteristic time of the system.

• In nondimensional form: $\phi = \phi(\xi, \tau)$, 2 independent variables. One BC and the IC are now homogeneous.

• Dimensional heat transfer rate is based on Fourier's Law of Conduction: $Q = -kA\dfrac{\partial T}{\partial x}$ where Q is a function of position and time. The thermal conductivity of the rod is k, a constant, and A is the constant conduction area.

• Nondimensional form of heat transfer.

$$Q = -kA\frac{\partial T}{\partial x} = -kA\frac{T_0 - T_i}{L}\frac{\partial \phi}{\partial \xi}$$

Therefore

$$Q^\star = \frac{LQ}{kA(T_0 - T_i)} = -\frac{\partial \phi}{\partial \xi}$$

Lecture 3

• Fourier cosine and sine series.
• See Spiegel, Shaum's Outline Handbook of Mathematics: Section 23, pp. 131-135.

• See Spiegel's Text Book: pp. 382-395 with several examples.
• See ME 303 Web site for 3 page summary and Maple worksheets.
• Fourier coefficients: A_0, A_n, B_n
• Orthogonality Property of Cosines and Sines.

• Demonstrate that

$$\int_0^L \cos^2(m\pi \frac{x}{L})\, dx = \frac{L}{2}, \qquad m = 1, 2, 3, \ldots$$

• Odd and Even Functions on the interval $-L \leq x \leq L$.
• Odd functions: $x, x^3, \sin x, \sinh x$

• If $f(x)$ is *even*, then

$$\int_{-L}^{L} f(x)\, dx = 2 \int_0^L f(x)\, dx$$

• Even functions: $1, |x|, x^2, \cos x, \cosh x$
• If $f(x)$ is *odd*, then

$$\int_{-L}^{L} f(x)\, dx = 0$$

• Fourier Cosine and Sine Series for even and odd functions on the half-interval $0 \leq x \leq L$.

• Fourier Sine Series for "Saw Tooth", i.e. $f(x) = x$ on the interval $-L \leq x \leq L$. This is an odd function. Therefore the Fourier cofficients are:

$$A_0 = 0, \qquad A_n = 0, \quad n = 1, 2, 3, \ldots$$

and

$$B_n = \frac{2}{L} \int_0^L x \sin\left(n\pi \frac{x}{L}\right)\, dx$$

Integration by parts can be used. From Spiegel's Handbook, 14.340, p. 75 we have

$$\int x \sin(ax)\, dx = \frac{\sin(ax)}{a^2} - \frac{x\cos(ax)}{a}$$

But $a = n\pi/L$.
• Fourier coefficients are

$$B_n = -(-1)^n \left(\frac{2L}{n\pi}\right), \qquad n = 1, 2, 3, \ldots$$

Fourier sine series for the "Saw Tooth" profile is approximately

$$f(x) \approx \frac{2L}{\pi}\left[\sin\left(\frac{\pi x}{L}\right) - \frac{1}{2}\sin\left(\frac{2\pi x}{L}\right) + \frac{1}{3}\sin\left(\frac{3\pi x}{L}\right) - \cdots\right]$$

Note that the absolute value of the amplitude decreases with increasing values of n.
