

Week 12

Lecture 1

- Final Exam will be open book. Discussed this with Prof. Wilson.

- Entropy and the Second Law of Thermodynamics (SLOT)
- Read Chapter 6: Sections 1 through 5.
- See ECE 309 Web site for notes on Entropy.
- $S [J/K]$, extensive property
- $s = S/M [J/(kg \cdot K)]$, intensive property
- Perfect crystal (pure substance) at $0 K$, molecules are motionless and perfectly arranged; entropy is zero, i.e., $S = 0$
- Discuss $T - s$ diagram
- For mixture:

$$s = (1 - x)s_f + xs_g$$

- See example in text.
- Isolated system (system plus surroundings); (boundary closed to energy transfer in the form of heat and work):

$$(\Delta S)_{\text{isolated system}} = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surroundings}} \geq 0$$

- Reversible process $\Delta S = 0$ and irreversible process $\Delta S > 0$
- Production \mathcal{P}

$$\text{CE} \quad \mathcal{P}_M = M_2 - M_1 = 0$$

$$\text{FLOT} \quad \mathcal{P}_E = E_2 - E_1 = 0$$

$$\text{SLOT} \quad \mathcal{P}_S = S_2 - S_1 \geq 0$$

- Chapter 7 Consequences of the Second Law

- $S = S(U, V)$ and $s = s(u, v)$
- Thermodynamic Definition of Temperature

$$T = \frac{1}{(\partial S / \partial U)_V}$$

- Thermodynamic Definition of Pressure

$$P = \left(\frac{\partial S}{\partial V} \right)_U$$

- Definitions of T and P in terms of $s = s(u, v)$

$$\frac{1}{T} = \left(\frac{\partial s}{\partial u} \right)_v, \quad \frac{P}{T} = \left(\frac{\partial s}{\partial v} \right)_u$$

Lecture 2

- Gibbs equation for SCS based on $s = s(u, v)$

$$ds = \left(\frac{\partial s}{\partial u} \right)_v du + \left(\frac{\partial s}{\partial v} \right)_u dv$$

or

$$ds = \frac{du}{T} + \frac{P}{T} dv \quad \text{or} \quad T ds = du + P dv$$

- Macroscopic Evaluation of Entropy

$$s_2 - s_1 = \int_1^2 \frac{du}{T} + \int_1^2 \frac{P}{T} dv$$

- See example in Text for entropy change from saturated liquid to saturated vapor at constant pressure and temperature.
- Entropy change of solid or liquid. If SCS is incompressible, $dv = 0$, then

$$ds = \frac{du}{T}$$

If dv is small (liquid) or negligible (solid), then

$$ds \approx \frac{du}{T} \approx c \frac{dT}{T}$$

where c is the specific heat capacity. Assuming that $c = \text{constant}$:

$$s_2 - s_1 \approx c \ln \frac{T_2}{T_1}$$

- Example: One kilogram of water is heated from $T_1 = 20^\circ\text{C} + 273 = 293\text{K}$ to $T_2 = 90^\circ\text{C} + 273 = 363\text{K}$. Calculate the change in the entropy assuming constant specific heat capacity. Compare the result against the steam table value.

• Solution:

$$s_2 - s_1 \approx c \ln \frac{T_2}{T_1} \left[\frac{kJ}{kg \cdot K} \right]$$

But

$$\int_1^2 du = c_v \int_1^2 dT \quad \text{and} \quad u_2 - u_1 = c_v(T_2 - T_1)$$

Therefore

$$c_v = \frac{u_2 - u_1}{T_2 - T_1}$$

From Table B.1a, page 622 we find

$$\text{At } T_1 = 293 \text{ K}, \quad u_2 = 83.9 \left[\frac{kJ}{kg \cdot K} \right]$$

$$\text{At } T_2 = 363 \text{ K}, \quad u_1 = 376.8 \left[\frac{kJ}{kg \cdot K} \right]$$

This gives

$$c_v = \frac{376.8 - 83.9}{363 - 293} = 4.184 \left[\frac{kJ}{kg \cdot K} \right]$$

and

$$s_2 - s_1 = 4.184 \ln \left(\frac{363}{293} \right) = 0.8964 \left[\frac{kJ}{kg \cdot K} \right]$$

and a better estimate is

$$s_2 - s_1 = 4.184 \ln \left(\frac{363.2}{293.2} \right) = 0.8958 \left[\frac{kJ}{kg \cdot K} \right]$$

From steam tables:

$$s_2 - s_1 = s_{90^\circ C} - s_{20^\circ C} = 1.1927 - 0.2965 = 0.8962 \left[\frac{kJ}{kg \cdot K} \right]$$

The agreement is good. Water (liquid) can be approximated as an incompressible substance.

• Chapter 8:

- 8.2 The Gibbs Equation
- 8.3 Equation of State for Perfect Gas
- 8.4 Other $P - v - T$ Equations for Gases

- Entropy Change for Ideal Gas
- Gibbs Equation:

$$T ds = du + P dv$$

For ideal gas

$$du = c_v dT \quad \text{and} \quad \frac{P}{T} = \frac{R}{v}$$

Now

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

and the change in entropy is

$$s_2 - s_1 = \int_1^2 c_v \frac{dT}{T} + R \ln \frac{v_2}{v_1} \quad \text{and} \quad c_v = c_v(T)$$

Assuming that $c_v = \text{constant}$, gives

$$s_2 - s_1 \approx c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Similarly

$$T ds = dh - v dP$$

For ideal gas

$$dh = c_p dT \quad \text{and} \quad \frac{v}{T} = \frac{R}{P}$$

Now

$$ds = c_p \frac{dT}{T} - R \frac{dP}{P}$$

and the change in entropy is

$$s_2 - s_1 = \int_1^2 c_p \frac{dT}{T} - R \ln \frac{P_2}{P_1} \quad \text{and} \quad c_p = c_p(T)$$

Assuming that $c_p = \text{constant}$, gives

$$s_2 - s_1 \approx c_p \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Lecture 3

- 15 minutes at end of lecture to do Project 2 Part 2, and to hand in Part 1.

- Relationship between specific heat capacities

$$c_p - c_v = R$$

- Approximate Polynomial Fit to Experimental Data

$$c_p = a + bT + cT^2$$

where a, b, c are correlation coefficients fitted to data. Average deviation is $\pm 1\%$ and maximum deviation is $\pm 2\%$. See Table B.16 for values for several gases.

- Definition

$$\phi(T) = \int_{T_0}^T \frac{c_p(T)}{T} dT, \quad T_0 = \text{some reference temperature}$$

Entropy can now be expressed as

$$s_2 - s_1 = \phi_2 - \phi_1 - R \ln \frac{P_2}{P_1}$$

See Table B.9 for properties of air. Note that $T_0 = 0 \text{ R}$.

- Section 7.7 Two Idealized Systems

– Thermal Energy Reservoir (TER) is a reservoir for disorganized energy. It has a fixed volume $dV = 0$, and its temperature is T_{TER} for all time. The TER is a non-isolated system.

- Gibbs Equation for TER

$$T_{TER} dS = dU \quad \text{and} \quad PdV = 0$$

- FLOT for TER

$$dU = dQ \quad \text{and} \quad dS = \frac{dQ}{T_{TER}}$$

where Q enters the TER.