Chapter 12: Forced Convection

Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. Convection is classified as *natural (or free*) and *forced* convection depending on how the fluid motion is initiated. In natural convection, any fluid motion is caused by natural means such as the buoyancy effect, i.e. the rise of warmer fluid and fall the cooler fluid. Whereas in forced convection, the fluid is forced to flow over a surface or in a tube by external means such as a pump or fan.

Mechanism of Forced Convection

Convection heat transfer is complicated since it involves fluid motion as well as heat conduction. The fluid motion enhances heat transfer (the higher the velocity the higher the heat transfer rate).

The rate of convection heat transfer is expressed by Newton's law of cooling:

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (W/m^2)$$
$$\dot{Q}_{conv} = hA(T_s - T_\infty) \qquad (W)$$

The convective heat transfer coefficient *h* strongly depends on the fluid properties and *roughness* of the solid surface, and the type of the fluid flow (*laminar* or *turbulent*).

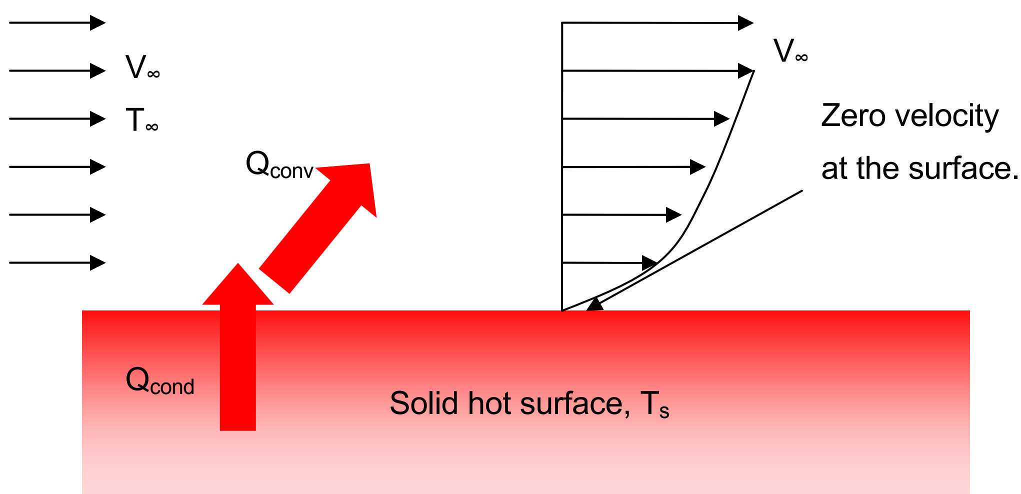


Fig. 12-1: Forced convection.

It is assumed that the velocity of the fluid is zero at the wall, this assumption is called *no-slip* condition. As a result, the heat transfer from the surface solid surface to the fluid layer adjacent to the surface is by pure conduction, since the fluid is motionless. Thus,

$$\left. \begin{array}{l} \dot{q}_{conv} = \dot{q}_{cond} = -k_{fluid} \left. \dfrac{\partial T}{\partial y} \right|_{y=0} \\ \dot{q}_{conv} = h(T_s - T_\infty) \end{array} \right\} \rightarrow h = \frac{-k_{fluid} \left. \dfrac{\partial T}{\partial y} \right|_{y=0}}{T_s - T_\infty} \qquad (W/m^2.K)$$

The convection heat transfer coefficient, in general, varies along the flow direction. The mean or average convection heat transfer coefficient for a surface is determined by (properly) averaging the local heat transfer coefficient over the entire surface.

Velocity Boundary Layer

Consider the flow of a fluid over a flat plate, the velocity and the temperature of the fluid approaching the plate is uniform at V_∞ and T_∞. The fluid can be considered as adjacent layers on top of each others.

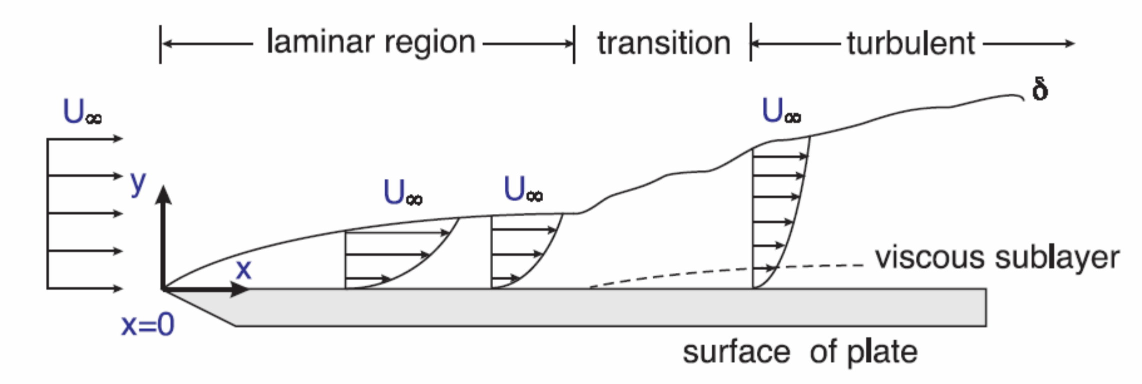


Fig. 12-2: Velocity boundary layer.

Assuming no-slip condition at the wall, the velocity of the fluid layer at the wall is zero. The motionless layer slows down the particles of the neighboring fluid layers as a result of friction between the two adjacent layers. The presence of the plate is felt up to some distance from the plate beyond which the fluid velocity V_∞ remains unchanged. This region is called *velocity boundary layer*.

Boundary layer region is the region where the viscous effects and the velocity changes are significant and the *inviscid region* is the region in which the frictional effects are negligible and the velocity remains essentially constant.

The friction between two adjacent layers between two layers acts similar to a drag force (friction force). The drag force per unit area is called the shear stress:

$$\tau_s = \mu \left. \frac{\partial V}{\partial y} \right|_{y=0} \qquad (N/m^2)$$

where μ is the dynamic viscosity of the fluid kg/m.s or N.s/m^2 .

Viscosity is a measure of fluid resistance to flow, and is a strong function of temperature.

The surface shear stress can also be determined from :

$$\tau_s = C_f \frac{\rho V_\infty}{2} \qquad \left(N/m^2\right)$$

where C_f is the friction coefficient or the drag coefficient which is determined experimentally in most cases.

The drag force is calculated from:

$$F_D = C_f A \frac{\rho V_\infty}{2} \qquad \left(N\right)$$

The flow in boundary layer starts as *smooth* and *streamlined* which is called *laminar flow*. At some distance from the leading edge, the flow turns *chaotic*, which is called *turbulent* and it is characterized by *velocity fluctuations* and highly disordered motion.

The transition from laminar to turbulent flow occurs over some region which is called *transition region*.

The velocity profile in the laminar region is approximately parabolic, and becomes flatter in turbulent flow.

The turbulent region can be considered of three regions: *laminar sublayer* (where viscous effects are dominant), *buffer layer* (where both laminar and turbulent effects exist), and *turbulent layer*.

The *intense mixing* of the fluid in turbulent flow enhances heat and momentum transfer between fluid particles, which in turn increases the friction force and the convection heat transfer coefficient.

Non-dimensional Groups

In convection, it is a common practice to non-dimensionalize the governing equations and combine the variables which group together into dimensionless numbers (groups).

Nusselt number: non-dimensional heat transfer coefficient

$$Nu = \frac{h\delta}{k} = \frac{\dot{q}_{conv}}{\dot{q}_{cond}}$$

where δ is the characteristic length, and is D for the tube and L for the flat plate. Nusselt number represents the enhancement of heat transfer through a fluid as a result of convection relative to conduction across the same fluid layer.

Reynolds number: ratio of inertia forces to viscous forces in the fluid

$$\mathrm{Re} = \frac{\text{inertia forces}}{\text{viscous forces}} = \frac{\rho V \delta}{\mu} = \frac{V\delta}{\nu}$$

At large Re numbers, the inertia forces, which are proportional to the density and the velocity of the fluid, are large relative to the viscous forces; thus the viscous

.

forces cannot prevent the random and rapid fluctuations of the fluid (turbulent regime).

The Reynolds number at which the flow becomes turbulent is called the critical *Reynolds number*. For flat plate the critical Re is experimentally determined to be approximately Re critical = 5 x10^5.

Prandtl number: is a measure of relative thickness of the velocity and thermal boundary layer

$$\Pr = \frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k}$$

where fluid properties are:

mass density : ρ, (kg/m^3)

specific heat capacity : C_p (J/kg · K)

dynamic viscosity : μ, (N · s/m^2)

kinematic viscosity : ν, μ / ρ (m^2/s)

thermal conductivity : k, (W/m· K)

thermal diffusivity : α, k/(ρ · C_p) (m^2/s)

Thermal Boundary Layer

Similar to velocity boundary layer, a thermal boundary layer develops when a fluid at specific temperature flows over a surface which is at different temperature.

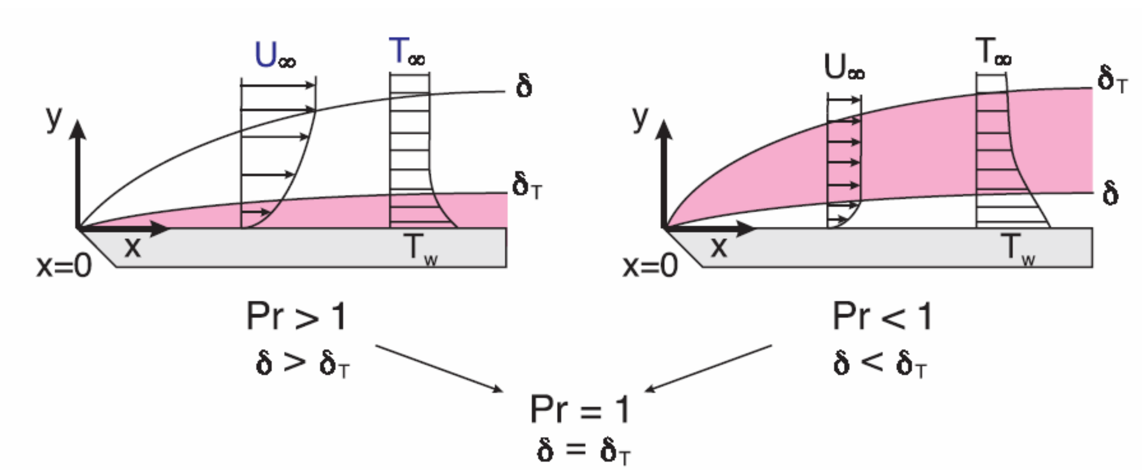


Fig. 12-3: Thermal boundary layer.

The thickness of the thermal boundary layer δ_t is defined as the distance at which:

$$\frac{T - T_s}{T_\infty - T_s} = 0.99$$

The relative thickness of the velocity and the thermal boundary layers is described by the Prandtl number.

For low Prandtl number fluids, i.e. liquid metals, heat diffuses much faster than momentum flow (remember Pr = ν/α<<1) and the velocity boundary layer is fully contained within the thermal boundary layer. On the other hand, for high Prandtl

.

number fluids, i.e. oils, heat diffuses much slower than the momentum and the thermal boundary layer is contained with in the velocity boundary layer.

Flow Over Flat Plate

The friction and heat transfer coefficient for a flat plate can be determined by solving the conservation of mass, momentum, and energy equations (either approximately or numerically). They can also be measured experimentally. It is found that the Nusselt number can be expressed as:

$$Nu = \frac{hL}{k} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

where C, m, and n are constants and L is the length of the flat plate. The properties of the fluid are usually evaluated at the *film temperature* defined as:

$$T_f = \frac{T_s + T_\infty}{2}$$

Laminar Flow

The local friction coefficient and the Nusselt number at the location x for laminar flow over a flat plate are

$$Nu_x = \frac{hx}{k} = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \quad \mathrm{Pr} \geq 0.6$$

$$C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}$$

where x is the distant from the leading edge of the plate and $\mathrm{Re}_x = \rho V_\infty x / \mu$.

The *averaged* friction coefficient and the Nusselt number over the entire isothermal plate for laminar regime are:

$$Nu = \frac{hL}{k} = 0.664\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3} \quad \mathrm{Pr} \geq 0.6$$

$$C_f = \frac{1.328}{\mathrm{Re}_L^{1/2}}$$

Taking the critical Reynolds number to be 5 x10^5, the length of the plate x_{cr} over which the flow is laminar can be determined from

$$\mathrm{Re}_{cr} = 5\times10^5 = \frac{V_\infty x_{cr}}{\nu}$$

Turbulent Flow

The local friction coefficient and the Nusselt number at location x for turbulent flow over a flat isothermal plate are:

$$Nu_x = \frac{hx}{k} = 0.0296\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60 \qquad 5\times10^5 \le \mathrm{Re}_x \le 10^7$$

$$C_{f,x} = \frac{0.0592}{\mathrm{Re}_x^{1/5}} \qquad 5\times10^5 \le \mathrm{Re}_x \le 10^7$$

The averaged friction coefficient and Nusselt number over the isothermal plate in turbulent region are:

$$Nu = \frac{hL}{k} = 0.037\,\mathrm{Re}_x^{4/5}\,\mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60 \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

Combined Laminar and Turbulent Flow

If the plate is sufficiently long for the flow to become turbulent (and not long enough to disregard the laminar flow region), we should use the average values for friction coefficient and the Nusselt number.

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,Laminar}\,dx + \int_{x_{cr}}^{L} C_{f,x,Turbulent}\,dx \right)$$

$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,Laminar}\,dx + \int_{x_{cr}}^{L} h_{,x,Turbulent}\,dx \right)$$

where the critical Reynolds number is assumed to be $5\mathrm{x}10^5$. After performing the integrals and simplifications, one obtains:

$$Nu = \frac{hL}{k} = \left(0.037\,\mathrm{Re}_x^{4/5} - 871\right)\mathrm{Pr}^{1/3} \quad 0.6 \le \mathrm{Pr} \le 60 \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5\times10^5 \le \mathrm{Re}_L \le 10^7$$

The above relationships have been obtained for the case of *isothermal* surfaces, but could also be used approximately for the case of non-isothermal surfaces. In such cases assume the surface temperature be constant at some average value.

For isoflux (uniform heat flux) plates, the local Nusselt number for laminar and turbulent flow can be found from:

$$Nu_x = \frac{hx}{k} = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \quad \text{Laminar (isoflux plate)}$$

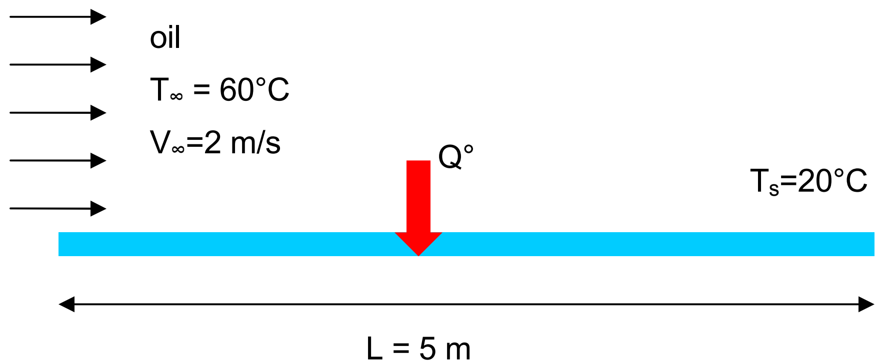
$$Nu_x = \frac{hx}{k} = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \quad \text{Turbulent (isoflux plate)}$$

Note the isoflux relationships give values that are 36% higher for laminar and 4% for turbulent flows relative to isothermal plate case.

.

Example 12-1

Engine oil at 60°C flows over a 5 m long flat plate whose temperature is 20°C with a velocity of 2 m/s. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



We assume the critical Reynolds number is 5×10^5. The properties of the oil at the film temperature are:

$$T_f = \frac{T_s + T_\infty}{2} = 40^\circ C$$
$$\rho = 876\ kg/m^3$$
$$k = 0.144\ W/(m.K)$$
$$\Pr = 2870$$
$$\nu = 242\times10^{-6}\ m^2/s$$

The Re number for the plate is:

$$Re_L = V_\infty L / \nu = 4.13\times10^4$$

which is less than the critical Re. Thus we have laminar flow. The friction coefficient and the drag force can be found from:

$$C_f = 1.328\,\mathrm{Re}_L^{-0.5} = 0.00653$$

$$F_D = C_f A \frac{\rho V_\infty^2}{2} = 0.00653\times\left(5\times1m^2\right)\frac{\left(876kg/m^3\right)\left(2m/s\right)^2}{2} = 57.2N$$

The Nusselt number is determined from:

$$Nu = \frac{hL}{k} = 0664\,\mathrm{Re}_L^{0.5}\,\Pr^{1/3} = 1918$$

Then,

$$h = 55.2\frac{W}{m^2K}$$

$$\dot{Q} = hA\left(T_\infty - T_s\right) = 11040W$$

.

Flow across Cylinders and Spheres

The characteristic length for a circular tube or sphere is the external diameter, D, and the Reynolds number is defined:

$$\mathrm{Re} = \frac{\rho V_\infty D}{\mu}$$

The critical Re for the flow across spheres or tubes is 2×10^5. The approaching the cylinder (a sphere) will branch out and encircle the body, forming a boundary layer.

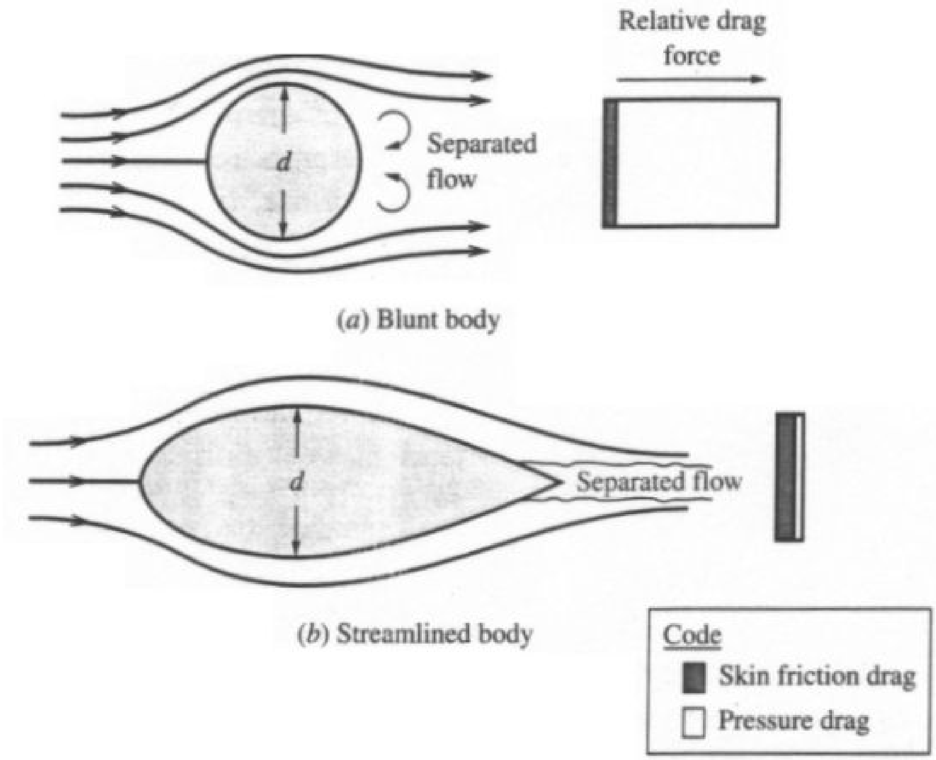


Fig. 12-4: Typical flow patterns over sphere and streamlined body and drag forces.

At low Re (Re<4) numbers the fluid completely wraps around the body. At higher Re numbers, the fluid is too fast to remain attached to the surface as it approaches the top of the cylinder. Thus, the boundary layer detaches from the surface, forming a wake behind the body. This point is called the *separation point*.

To reduce the drag coefficient, *streamlined bodies* are more suitable, e.g. airplanes are built to resemble birds and submarine to resemble fish, Fig. 12-4.

In flow past cylinder or spheres, flow separation occurs around 80° for laminar flow and 140° for turbulent flow.

$$F_D = C_D A_N \frac{\rho V_\infty^2}{2} \left(N\right) \qquad A_N \text{ : frontal area}$$

where *frontal area* of a cylinder is $A_N = L.D$, and for a sphere is $A_N = \pi D^2 / 4$.

.

The *drag force* acting on a body is caused by two effects: the *friction drag* (due to the shear stress at the surface) and the *pressure drag* which is due to pressure differential between the front and rear side of the body.

As a result of transition to turbulent flow, which moves the separation point further to the rear of the body, a large reduction in the drag coefficient occurs. As a result, the surface of golf balls is intentionally roughened to induce turbulent at a lower Re number, see Fig. 12-5.

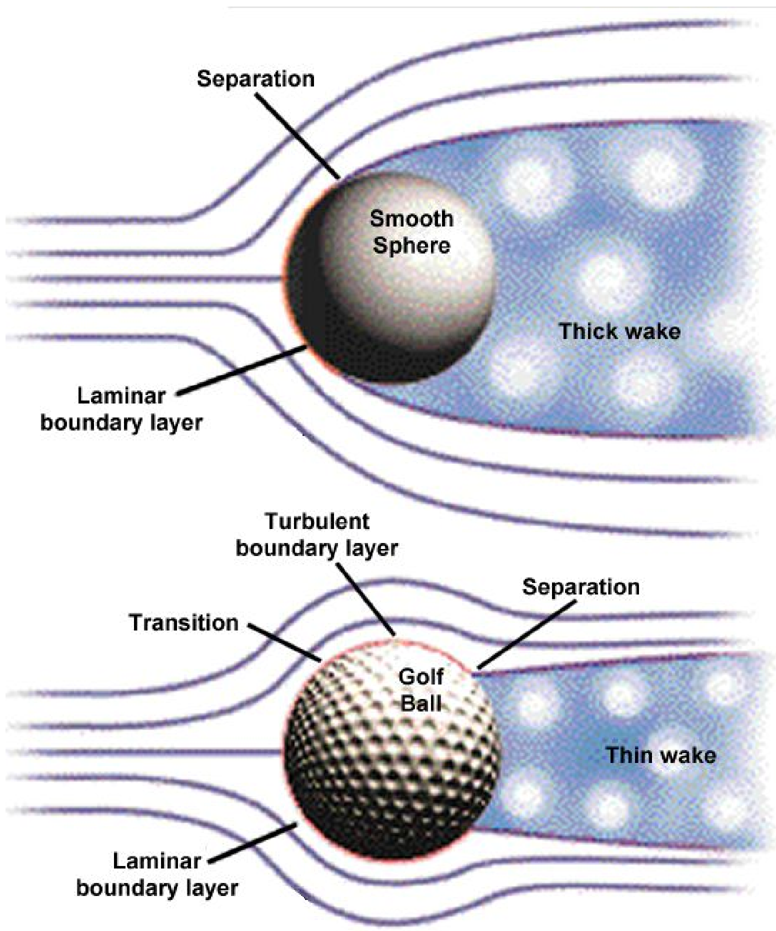


Fig. 12-5: Roughened golf ball reduces C_D.

The average heat transfer coefficient for cross-flow over a cylinder can be found from the correlation presented by Churchill and Bernstein:

$$Nu_{Cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4\,\mathrm{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

where fluid properties are evaluated at the film temperature $T_f = (T_s + T_\infty) / 2$.

For flow over a sphere, Whitaker recommended the following:

$$Nu_{Sph} = hD/k = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\mu_\infty / \mu_s\right)^{1/4}$$

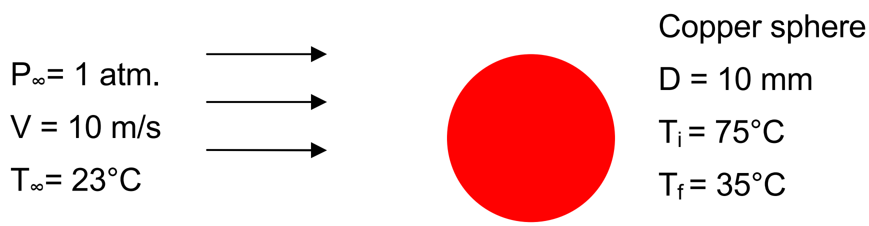
.

which is valid for 3.5 < Re < 80,000 and 0.7 < Pr < 380. The fluid properties are evaluated at the free-stream temperature T_∞, except for μ_s which is evaluated at surface temperature.

The average Nusselt number for flow across circular and non-circular cylinders can be found from Table 12-3 Cengel book.

Example 12-2

The decorative plastic film on a copper sphere of 10-mm diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an air stream at 1 atm. and 23°C having a velocity of 10 m/s, estimate how long it will take to cool the sphere to 35°C.



Assumptions:

1. Negligible thermal resistance and capacitance for the plastic layer.
2. Spatially isothermal sphere.
3. Negligible Radiation.

Copper at 328 K	Air at 296 K
ρ = 8933 kg / m^3 k = 399 W / m.K C_p = 387 J / kg.K	μ_∞ = 181.6 x 10-7 N.s / m^2 v = 15.36 x 10-6 m^2 / s k = 0.0258 W / m.K Pr = 0.709 μ_s = 197.8 x 10-7 N.s / m^2

The time required to complete the cooling process may be obtained from the results for a lumped capacitance.

$$t = \frac{\rho V C_P}{hA} \ln \frac{T_i - T_\infty}{T_f - T_\infty} = \frac{\rho C_p D}{6h} \ln \frac{T_i - T_\infty}{T_f - T_\infty}$$

Whitaker relationship can be used to find h for the flow over sphere:

$$Nu_{Sph} = hD/k = 2 + \left[0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right]\mathrm{Pr}^{0.4}\left(\mu_\infty / \mu_s\right)^{1/4}$$

where Re = ρVD / μ = 6510.

Hence,

.

$$Nu_{Sph} = hD/k = 2 + \left[0.4(6510)^{1/2} + 0.06(6510)^{2/3}\right](0.709)^{0.4}\left(\frac{181.6\times10^{-7}}{197.8\times10^{-7}}\right)^{1/4} = 47.4$$

$$h = Nu\frac{k}{D} = 122\, W/m^2K$$

The required time for cooling is then

$$t = \frac{\left(8933kg/m^3\right)\left(387J/kg.K\right)\left(0.01m\right)}{6\times122W/m^2.K}\ln\frac{75-23}{35-23} = 69.2\text{ sec}$$

.

Chapter 14: Natural Convection

In natural convection, the fluid motion occurs by natural means such as buoyancy. Since the fluid velocity associated with natural convection is relatively low, the heat transfer coefficient encountered in natural convection is also low.

Mechanisms of Natural Convection

Consider a hot object exposed to cold air. The temperature of the outside of the object will drop (as a result of heat transfer with cold air), and the temperature of adjacent air to the object will rise. Consequently, the object is surrounded with a thin layer of warmer air and heat will be transferred from this layer to the outer layers of air.

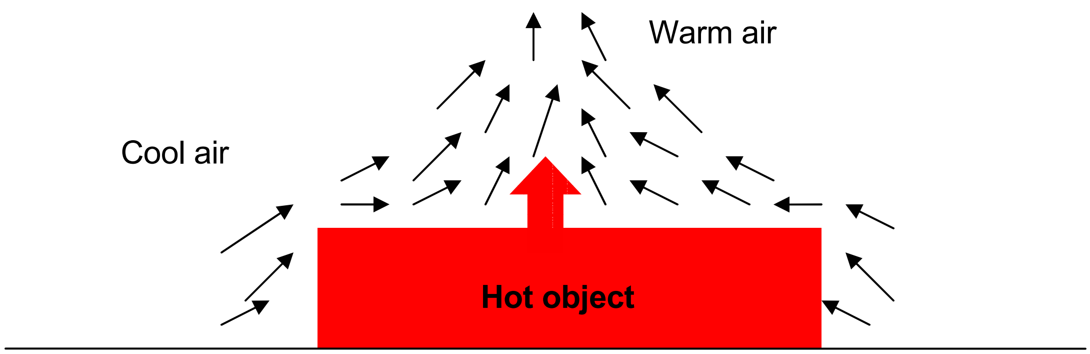


Fig. 14-1: Natural convection heat transfer from a hot body.

The temperature of the air adjacent to the hot object is higher, thus its density is lower. As a result, the heated air rises. This movement is called the *natural convection current*. Note that in the absence of this movement, heat transfer would be by conduction only and its rate would be much lower.

In a gravitational field, there is a net force that pushes a light fluid placed in a heavier fluid upwards. This force is called the *buoyancy force*.

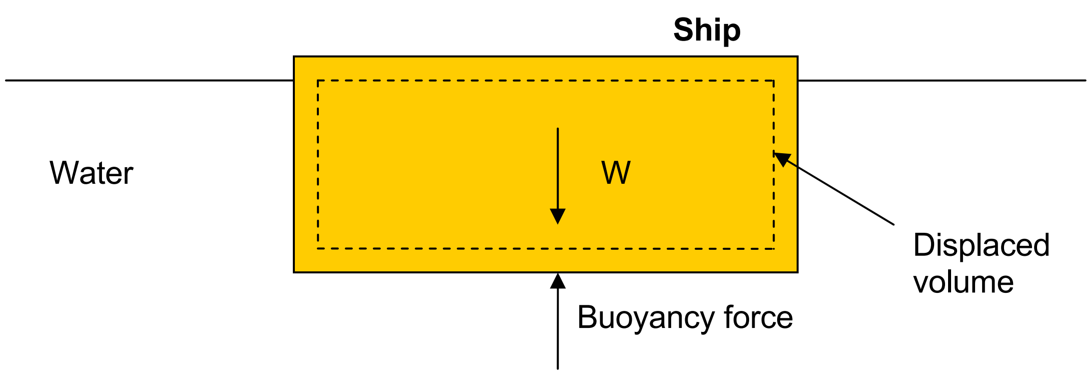


Fig. 14-2: Buoyancy force keeps the ship float in water.

.

The magnitude of the buoyancy force is the weight of the fluid displaced by the body.

$$F_{buoyancy} = \rho_{fluid}\ g\ V_{body}$$

where V_{body} is the volume of the portion of the body immersed in the fluid. The net force is:

$$F_{net} = W - F_{buoyancy}$$

$$F_{net} = (\rho_{body} - \rho_{fluid})\ g\ V_{body}$$

Note that the net force is proportional to the *difference in the densities* of the fluid and the body. This is known as *Archimedes' principle*.

We all encounter the feeling of "weight loss" in water which is caused by the buoyancy force. Other examples are hot balloon rising, and the chimney effect.

Note that the buoyancy force needs the gravity field, thus in space (where no gravity exists) the buoyancy effects does not exist.

Density is a function of temperature, the variation of density of a fluid with temperature at constant pressure can be expressed in terms of the volume expansion coefficient β, defined as:

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad \left(\frac{1}{K}\right)$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} \to \Delta\rho \approx -\rho\beta\,\Delta T \quad (\text{at constant P})$$

It can be shown that for an ideal gas

$$\beta_{\text{ideal gas}} = \frac{1}{T}$$

where T is the absolute temperature. Note that the parameter $\beta\Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change ΔT at constant pressure.

Since the buoyancy force is proportional to the density difference, the larger the temperature difference between the fluid and the body, the larger the buoyancy force will be.

Whenever two bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to that of the motion. Under steady conditions, the air flow rate driven by buoyancy is established by balancing the buoyancy force with the frictional force.

Grashof Number

Grashof number is a dimensionless group. It represents the ratio of the buoyancy force to the viscous force acting on the fluid:

$$Gr = \frac{\text{buoyancy forces}}{\text{viscous forces}} = \frac{\text{g}\Delta\rho \text{V}}{\rho\nu^2} = \frac{g\beta\Delta TV}{\rho\nu}$$

It is also expressed as

$$Gr = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2}$$

where g = gravitational acceleration, m / s^2
β = coefficient of volume expansion, 1 / K
δ = characteristic length of the geometry, m
v = kinematics viscosity of the fluid, m^2 / s

The role played by Reynolds number in forced convection is played by the Grashof number in natural convection. The critical Grashof number is observed to be about 10^9 for vertical plates. Thus, the flow regime on a vertical plate becomes turbulent at Grashof number greater than10^9. The heat transfer rate in natural convection is expressed by Newton's law of cooling as: $Q'_{conv} = h A (T_s - T_\infty)$

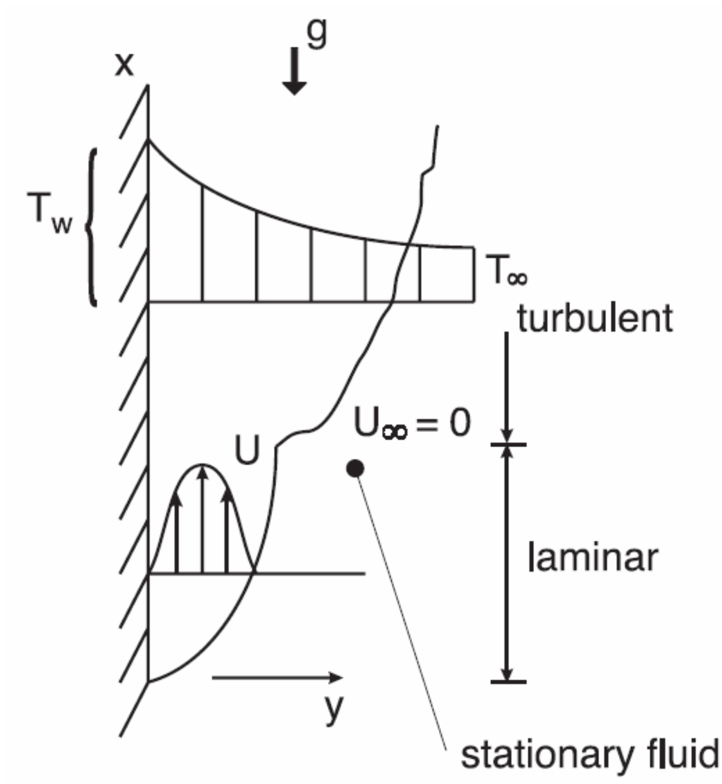


Fig. 14-3: Velocity and temperature profile for natural convection flow over a hot vertical plate.

Natural Convection over Surfaces

Natural convection on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid.

The velocity and temperature distribution for natural convection over a hot vertical plate are shown in Fig. 14-3.

Note that the velocity at the edge of the boundary layer becomes zero. It is expected since the fluid beyond the boundary layer is stationary.

The shape of the velocity and temperature profiles, in the cold plate case, remains the same but their direction is reversed.

Natural Convection Correlations

The complexities of the fluid flow make it very difficult to obtain simple analytical relations for natural convection. Thus, most of the relationships in natural convection are based on experimental correlations.

The Rayleigh number is defined as the product of the Grashof and Prandtl numbers:

$$Ra = Gr \, \mathrm{Pr} = \frac{g\beta(T_s - T_\infty)\delta^3}{\nu^2} \mathrm{Pr}$$

The Nusselt number in natural convection is in the following form:

$$Nu = \frac{h\delta}{k} = CRa^n$$

where the constants C and n depend on the geometry of the surface and the flow. Table 14-1 in Cengel book lists these constants for a variety of geometries.

These relationships are for isothermal surfaces, but could be used approximately for the case of non-isothermal surfaces by assuming surface temperature to be constant at some average value.

Isothermal Vertical Plate

For a vertical plate, the characteristic length is L.

$$Nu = \begin{cases} 0.59Ra^{1/4} & 10^4 < Ra < 10^9 \\ 0.1Ra^{1/3} & 10^9 < Ra < 10^{13} \end{cases}$$

Note that for ideal gases, $\beta = 1 / T_\infty$

Isothermal Horizontal Plate

The characteristics length is A/p where the surface area is A, and perimeter is p.

a) Upper surface of a hot plate

$$Nu = \begin{cases} 0.54Ra^{1/4} & 10^4 < Ra < 10^7 \\ 0.15Ra^{1/3} & 10^7 < Ra < 10^{11} \end{cases}$$

b) Lower surface of a cold plate

$$Nu = 0.27Ra^{1/4} \qquad 10^5 < Ra < 10^{11}$$

Example 14-1: isothermal vertical plate

A large vertical plate 4 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer if the plate is 10 m wide.

Solution:

We first determine the film temperature as

$$T_f = (T_s + T_\infty) / 2 = 35°C = 308 \text{ K}$$

The properties are

β = 1 / 308 = 3.25x10^{-3}, k = 0.02685 (W / mK), v = 16.5x10^{-6}, Pr = 0.7

The Rayleigh number can be found:

$$Ra = Gr\,\mathrm{Pr} = \frac{9.8 m/s^2 \left(3.25\times10^{-3} K^{-1}\right)\left(60 - 10^\circ C\right)(4m)^3}{\left(16.5\times10^{-6}\right)^2} 0.7 = 3.743\times10^{11}$$

The Nusselt number can be found from:

$$Nu = 0.1Ra^{1/3} = 0.1\left(3.743\times10^{11}\right)^{1/3} = 720.7$$

The heat-transfer coefficient is

$$h = \frac{Nu\,k}{L} = \frac{720.7\times0.02685}{4} = 4.84 \quad \left(W/m^2K\right)$$

The heat transfer is

$$q = h\,A\,(T_s - T_\infty) = 7.84 \text{ W/m°C}^2 \text{ (4 x 10 m2) (60 – 10 °C)} = 9.675 \text{ kW}$$

Natural Convection from Finned Surfaces

Finned surfaces of various shapes (heat sinks) are used in microelectronics cooling.

One of most crucial parameters in designing heat sinks is the *fin spacing*, *S*. Closely packed fins will have greater surface area for heat transfer, but a smaller heat transfer coefficient (due to extra resistance of additional fins). A heat sink with widely spaced fins will have a higher heat transfer coefficient but smaller surface area. Thus, an *optimum spacing* exists that maximizes the natural convection from the heat sink.

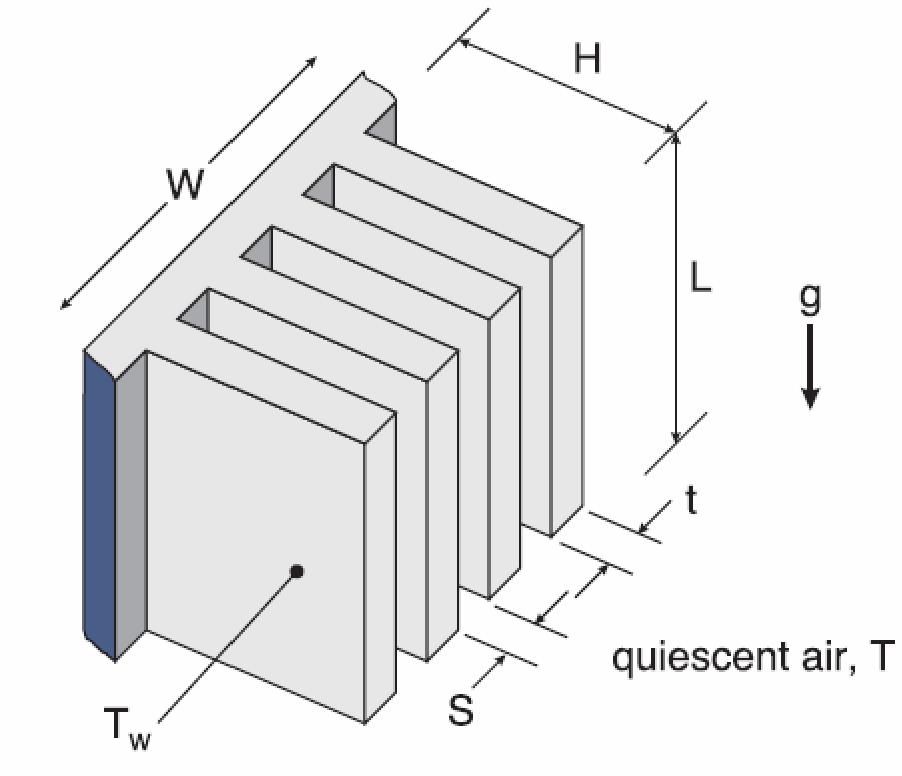


Fig. 14-4: A vertical heat sink.

Consider a heat sink with base dimension W (width) and L (length) in which the fins are assumed to be isothermal and the fin thickness t is small relative to fin spacing S. The optimum fin spacing for a vertical heat sink is given by Rohsenow and Bar-Cohen as

$$S_{opt} = 2.714 \frac{L}{Ra^{1/4}}$$

where L is the characteristic length in Ra number. All the fluid property are determined at the film temperature. The heat transfer coefficient for the optimum spacing can be found from

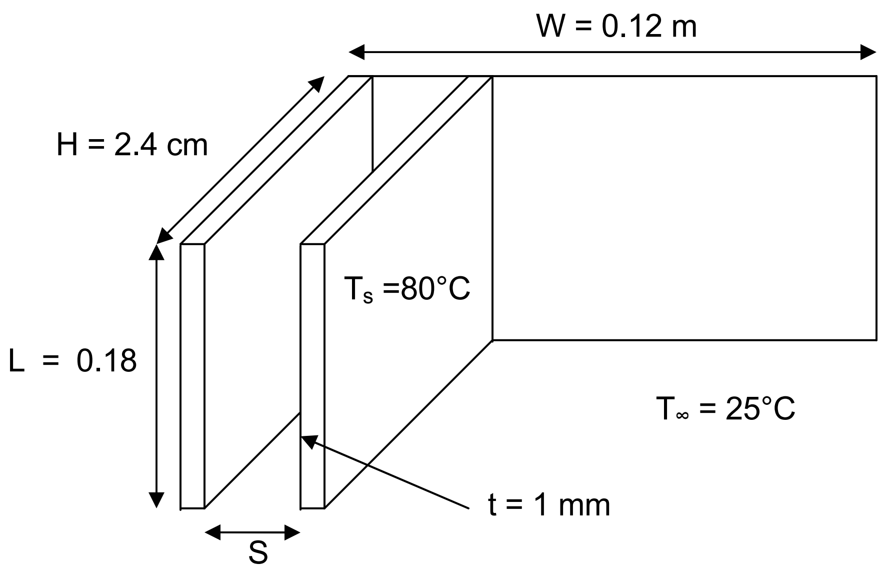
$$h = 1.31 \frac{k}{S_{opt}}$$

Note: as a result of above-mentioned "two opposing forces" (buoyancy and friction), heat sinks with *closely spaced* fins *are not suitable* for natural convection.

Example 14-2: Heat sink

A 12-cm wide and 18-cm-high vertical hot surface in 25°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick, 18 cm long in the vertical direction, and have a height of 2.4 cm from the base.

Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.



Assumptions:

The fin thickness t is much smaller than the fin spacing S.

Solution:

The properties of air are evaluated at the film temperature:

$$T_f = (T_\infty + T_s) / 2 = 52.5°C = 325.5 \text{ K}$$

At this temperature, k = 0.0279 W / (m.K), ν = 1.82 x 10^{-5} m^2 / s, Pr = 0.709, and assuming ideal gas β = 1 / T_f = 1 / 325.5 K = 0.003072 1 / K.

The characteristic length is L = 0.18 m.

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr = 2.067 \times 10^7$$

The optimum fin spacing is determined

$$S_{opt} = 2.714\frac{L}{Ra^{1/4}} = 0.0072\, m = 7.2\ mm$$

The number of fins and the heat transfer coefficient for the optimum fin spacing case are

$$n = \frac{W}{S+t} \approx 15 \text{ fins}$$

$$h = 1.31 \frac{k}{S_{opt}} = 5.08 \frac{W}{mK}$$

The rate of natural convection heat transfer becomes:

$$\dot{Q} = h(2nLH)(T_s - T_\infty) = 36.2\, W$$

.