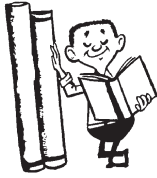


Introduction & Basic Concepts of Thermodynamics



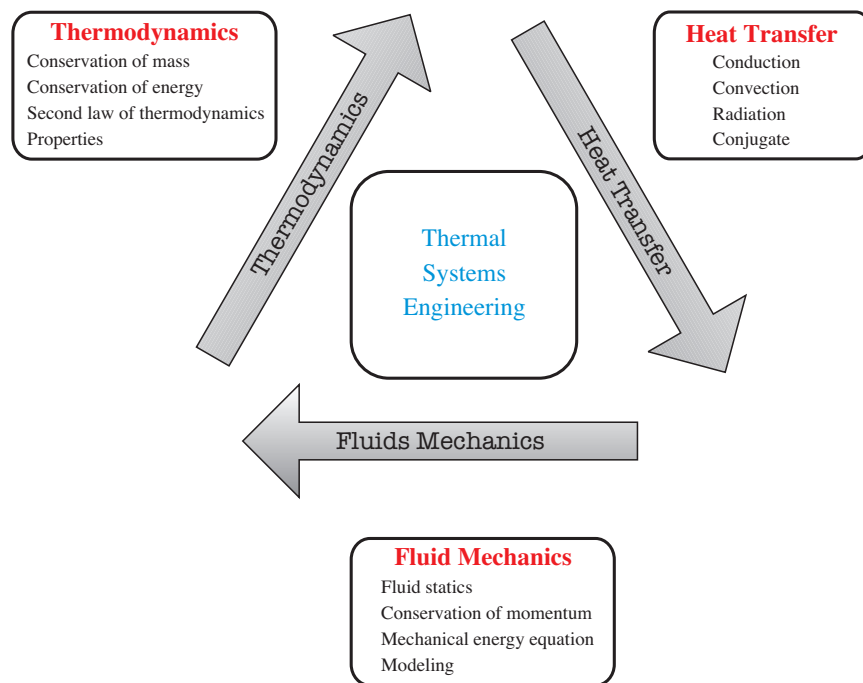
Reading

2-1 → 2-8

Problems

2-53, 2-67, 2-85, 2-96

Introduction to Thermal Sciences

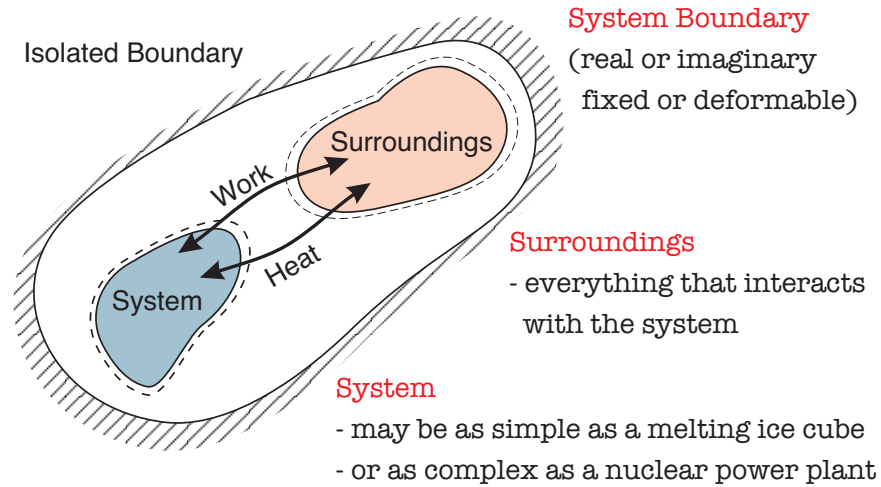


Thermodynamics: the study of energy, energy conversion and its relation to matter. The analysis of thermal systems is achieved through the application of the governing conservation equations, namely *Conservation of Mass*, *Conservation of Energy* (1st law of thermodynamics), the 2nd law of thermodynamics and the property relations.

Heat Transfer: the study of energy in transit including the relationship between energy, matter, space and time. The three principal modes of heat transfer examined are conduction, convection and radiation, where all three modes are affected by thermophysical properties, geometrical constraints and the temperatures associated with the heat sources and sinks used to drive heat transfer.

Fluid Mechanics: the study of fluids at rest or in motion. While this course will not deal extensively with fluid mechanics we will be influenced by the governing equations for fluid flow, namely *Conservation of Momentum* and *Conservation of Mass*.

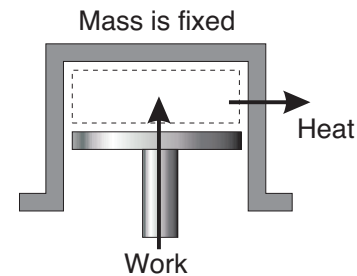
Thermodynamic Systems



System: thermodynamic systems are classified as either *closed* or *open*

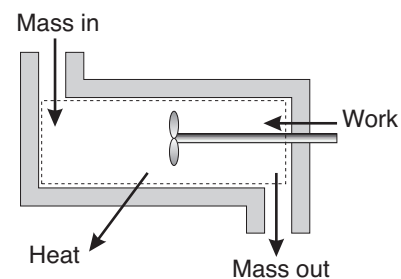
Closed System:

- consists of a fixed mass
- NO mass crosses the boundary



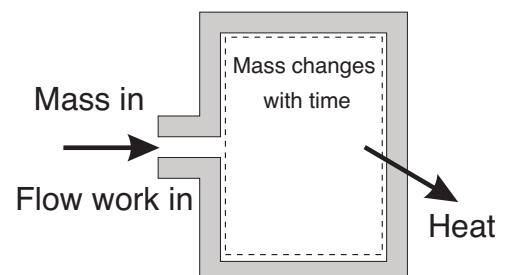
Open System: Steady

- consists of a fixed volume
- mass crosses the boundary
- no change with respect to time



Open System: Unsteady

- changes may occur with respect to time
- flow work in does not equal flow work out
- energy crosses the boundary as enthalpy and heat



Thermodynamic Properties of Systems

Thermodynamic Property: Any observable or measurable characteristic of a system or any mathematical combination of measurable characteristics

Extensive Properties: Properties that are dependent of the size or extent of the system, i.e. mass

- they are additive $\Rightarrow X_{A+B} = X_A + X_B$

Intensive Properties: Properties that are independent of the size (or mass) of the system

- they are **not** additive $\Rightarrow X_{A+B} \neq X_A + X_B$

Specific Properties: Extensive properties expressed per unit mass to make them intensive properties

- specific property (intensive) $\longrightarrow \frac{\text{extensive property}}{\text{mass}}$

Measurable Properties

- P , V , T , and m are important because they are measurable quantities
 - pressure (P) and temperature (T) are easily measured intensive properties.
Note: They are not always independent of one another.
 - volume (V) and mass (m) are easily measured extensive properties

Pressure

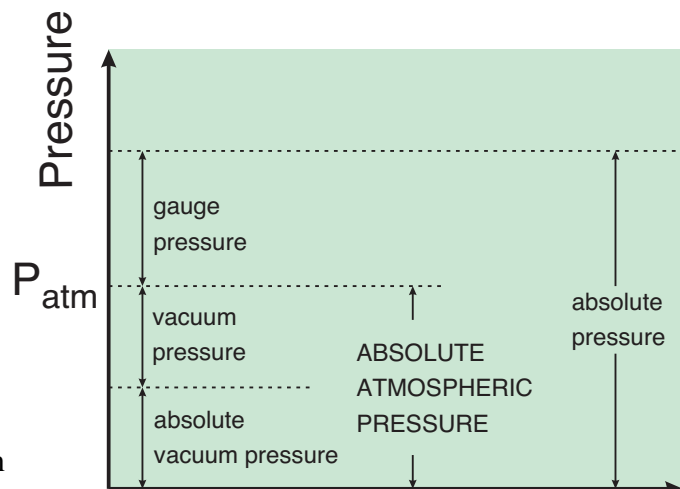
- $Pressure = \frac{Force}{Area}$
- gauge = absolute - atmospheric
 $\Rightarrow P_{abs} > P_{atm}$

$$P_{gauge} = P_{abs} - P_{atm}$$

- vacuum pressure $\Rightarrow P_{abs} < P_{atm}$

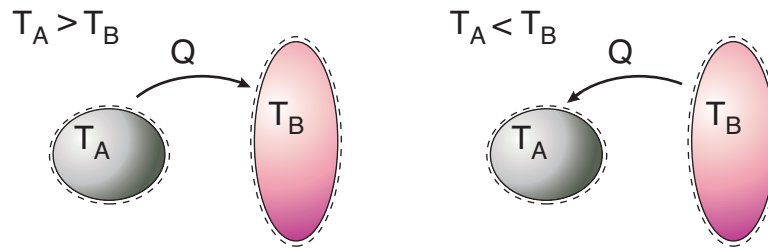
$$P_{vac} = P_{atm} - P_{abs}$$

- thermodynamics properties depend on absolute pressure



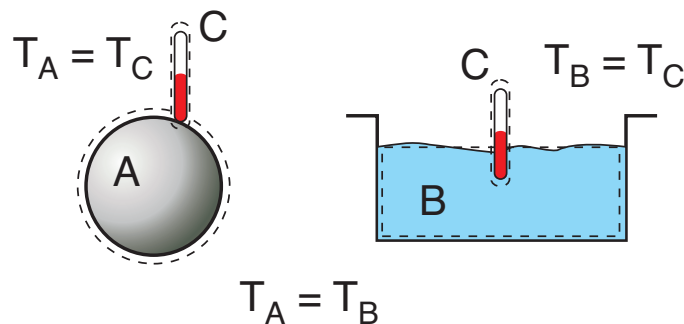
Temperature

- temperature is a pointer for the direction of energy transfer as heat



0th Law of Thermodynamics: when two objects are in thermal equilibrium with a third object they are in thermal equilibrium with each other.

- the 0th law makes a thermometer possible



- in accordance with the zeroth law, any system that possesses an equation of state that relates T to other accurately measurable properties can be used as a thermometer

Other Properties

- energy within a system can be stored as a combination of kinetic energy, potential energy or internal energy

- Internal energy U [kJ]:**

- associated with molecular motion i.e. translational, rotational or vibrational
- extensive since it depends on the amount of matter in the system

- Kinetic energy KE :**

- the energy of motion relative to some reference frame $\Rightarrow KE = \frac{1}{2}m(\mathbf{v})^2$

- Potential energy PE :**

- the energy of position within a gravitational field $\Rightarrow PE = mgz$

State and Equilibrium

- the state of a system is its condition as described by a set of relevant energy related properties.
- a system at equilibrium is in a state of balance

Definitions

Simple Compressible System: A simple compressible system experiences negligible electrical, magnetic, gravitational, motion, and surface tension effects, and only PdV work is done.

State Postulate:

State Postulate (for a simple compressible system): The state of a simple compressible system is completely specified by 2 independent and intensive properties.

Processes and Cycles

Definitions

1. **Process:** a transformation from one equilibrium state to another through a change in properties
2. **Quasi-equilibrium Process:** changes occurs sufficiently slow to allow the system to transition in a uniform manner
3. **Cycle:** a sequence of processes that begin and end at the same state i.e. see the Carnot cycle
4. **Steady:** no change with respect to time
 - if the process is not steady, it is *unsteady* or *transient*
 - often steady flow implies both *steady flow* and *steady state*
5. **Uniform:** no change with respect to position
 - if the flow field in a process is not uniform, it is *distributed*.

Example 1-1: A glass tube is attached to a water pipe. If the water pressure at the bottom of the glass tube is 115 kPa and the local atmospheric pressure is 92 kPa , determine how high the water will rise in the tube, in m . Assume $g = 9.8 \text{ m/s}^2$ at that location and take the density of water to be 1000 kg/m^3 .

Step 1: Draw a clearly labeled diagram to represent the system

Step 2: State what the problem is asking you to determine.

Step 3: State all assumptions used during the solution process.

Step 4: Prepare a table of properties.

Step 5: Solve (start by writing a force balance on the system)

Step 6: Clearly identify your answer

Properties of Pure Substances



Reading

4-1 → 4-7

5-3 → 5-5

Problems

4-41, 4-54, 4-58, 4-79, 4-95

Pure Substances

- it has a fixed chemical composition throughout (chemically uniform)
- a homogeneous mixture of various chemical elements or compounds can also be considered as a pure substance (uniform chemical composition)

Phases of Pure Substances

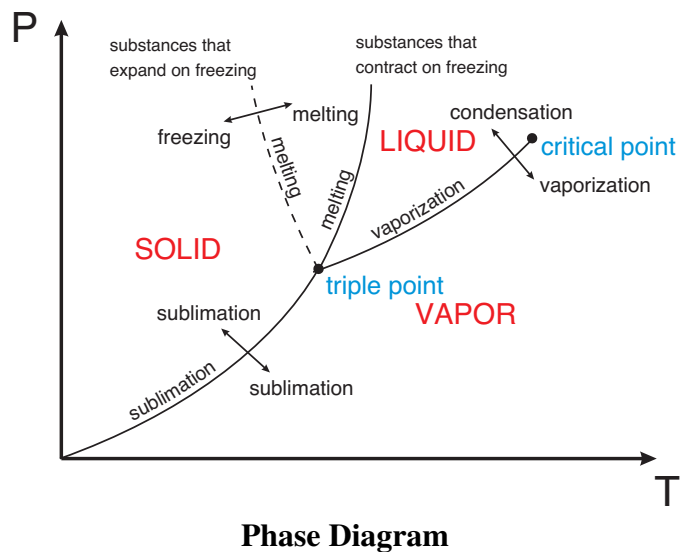
Solids: strong molecular bonds

Liquids: molecules are no longer in a fixed position relative to one another

Gases: there is no molecular order

Behavior of Pure Substances (Phase Change Processes)

- all pure substances exhibit the same general behavior.
 - one exception is that most contract on freezing. H_2O (and a few more) expand on freezing.
- phase change processes:
- **Critical point:** is the point at which the liquid and vapor phases are not distinguishable (we will talk about this more)
- **Triple point:** is the point at which the liquid, solid, and vapor phases can exist together

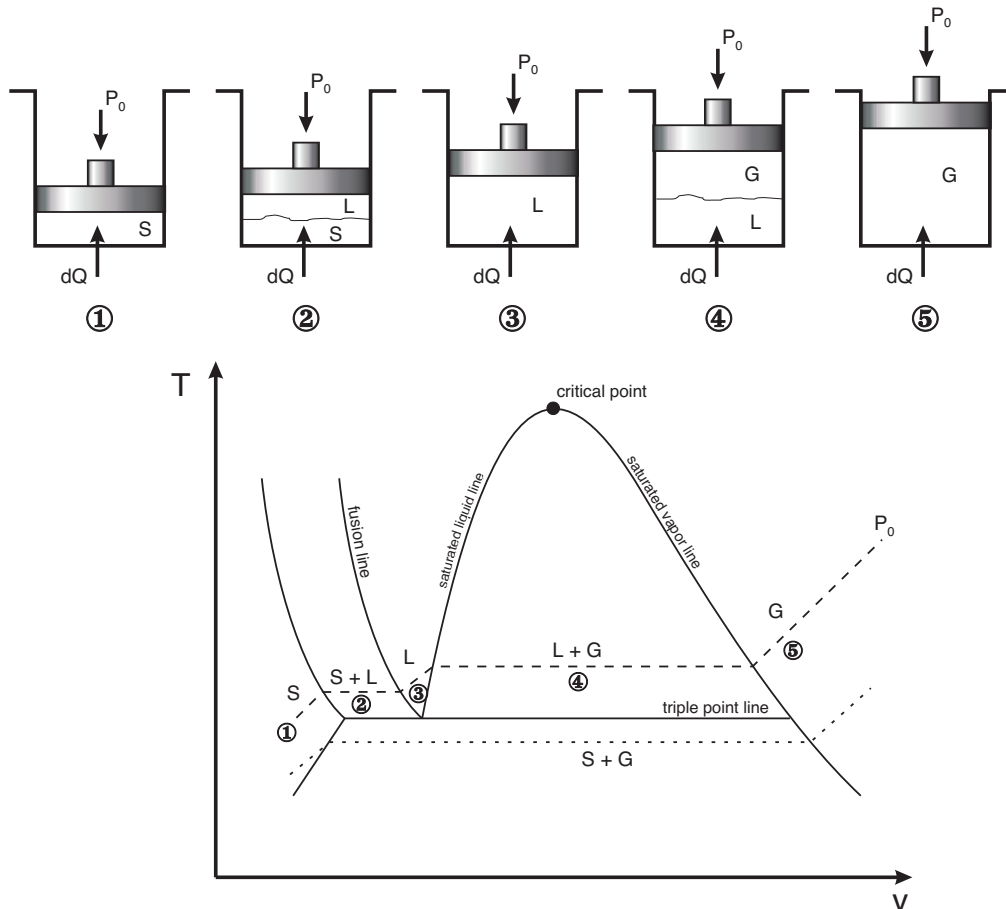


	Critical Point ¹		Triple Point ²	
	P (MPa)	T ($^{\circ}\text{C}$)	P (MPa)	T ($^{\circ}\text{C}$)
H_2O	22.06	373.95	0.00061	0.01
O_2	5.08	-118.35	0.000152	-218.79
CO_2	7.39	31.05	0.517	56.6
N	3.39	-146.95	0.0126	-209.97
He	0.23	-267.85	0.0051	-270.96

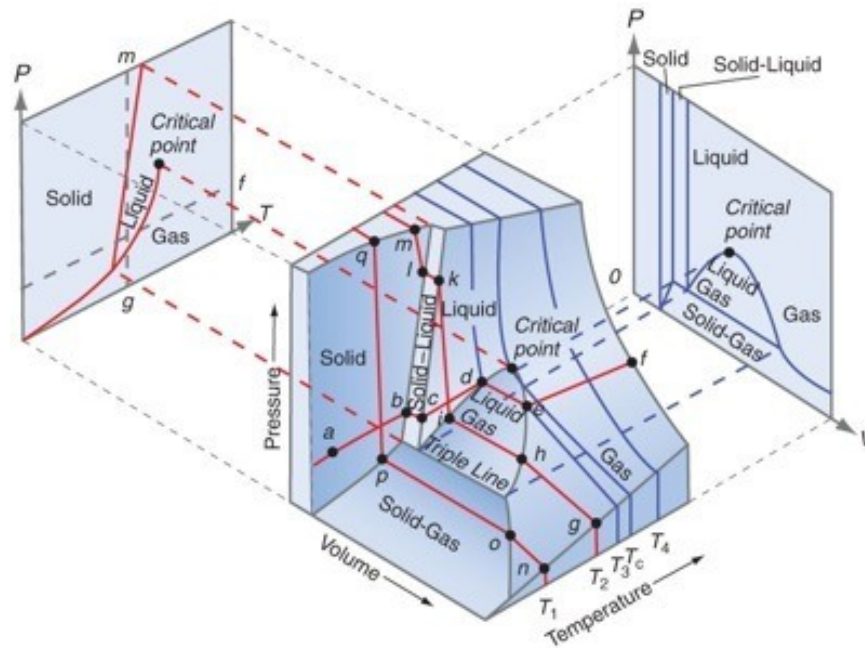
1 - see Table A-1 for other substances 2 - see Table 4.3 for other substances

$T - v$ Diagram for a Simple Compressible Substance

consider an experiment in which a substance starts as a solid and is heated up at constant pressure until it all becomes as gas



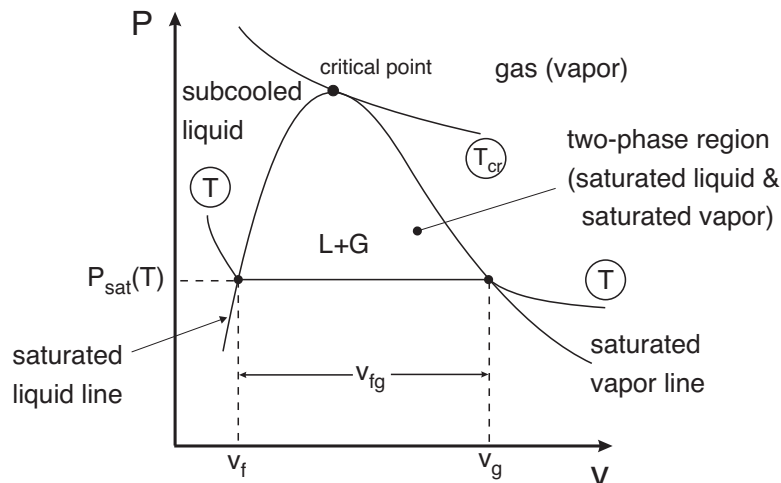
The P-v-T Surface Diagram



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- if P-v-T is projected onto a pressure-temperature plane we obtain a phase diagram
- if we project onto a pressure-volume plane we obtain the familiar P-v diagram for a substance (likewise for T-v)

The Vapor Dome (Two Phase Region)



specific volume:

$$v_f, v_g \text{ and } v_{fg} = v_g - v_f$$

internal energy:

$$u_f, u_g \text{ and } u_{fg} = u_g - u_f$$

specific enthalpy:

$$h_f, h_g \text{ and } h_{fg} = h_g - h_f$$

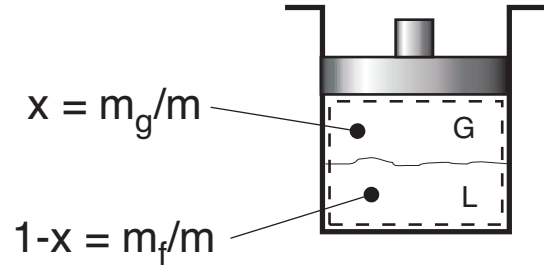
specific entropy:

$$s_f, s_g \text{ and } s_{fg} = s_g - s_f$$

- the subscripts denote: **f** - saturated liquid (fluid) and **g** - saturated vapor (gas)
- given the shape of the vapor dome, as $T \uparrow$ or $P \uparrow \Rightarrow h_{fg} \downarrow$

Quality

We need to define a new thermodynamic property: **Quality** $\equiv x = \frac{m_g}{m} = \frac{m_g}{m_g + m_f}$



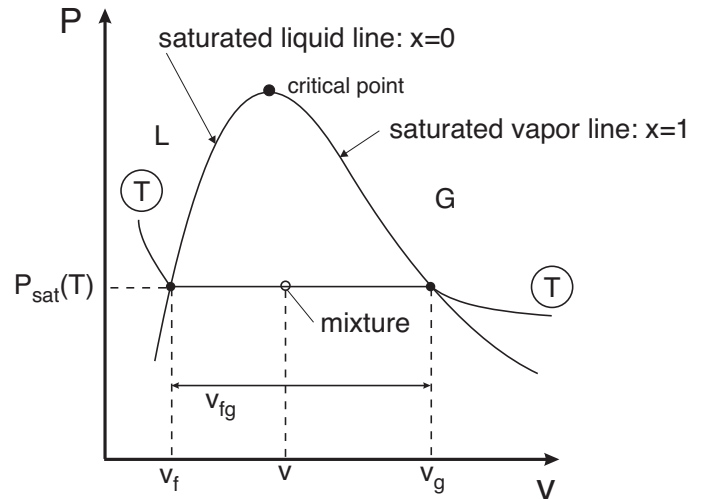
total mass of mixture $\Rightarrow m = m_g + m_f$

mass fraction of the gas $\Rightarrow x = m_g/m$

mass fraction of the liquid $\Rightarrow 1 - x = m_f/m$

Properties of Saturated Mixtures

- To fix the state we require (T, x) , (T, v) , $(P, x) \dots$
- all the calculations done in the vapor dome can be performed using Tables.
 - in Table A-4, the properties are listed under Temperature
 - in Table A-5, the properties are listed under Pressure
- other properties are calculated as mass-average values of the saturated quantities



$$v = \frac{V}{m} = \frac{V_f + V_g}{m} = \frac{m_f v_f + m_g v_g}{m} = (1 - x) v_f + x v_g$$

$$= v_f + x(v_g - v_f) = v_f + x v_{fg}$$

$$x = \frac{v - v_f}{v_{fg}}$$

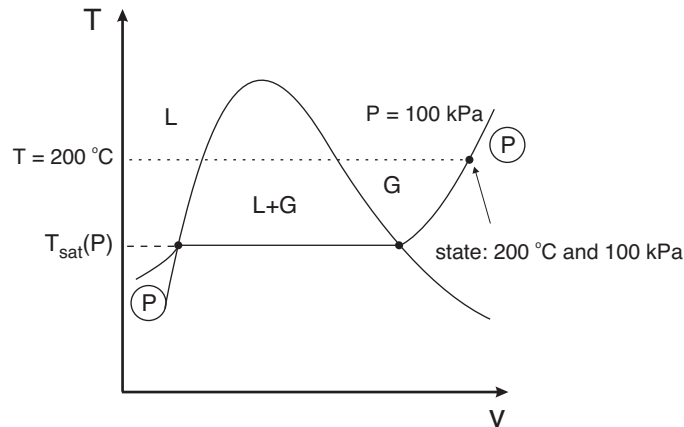
$$u = (1 - x)u_f + x u_g = u_f + x u_{fg} \Rightarrow x = \frac{u - u_f}{u_{fg}}$$

$$h = (1 - x)h_f + x h_g = h_f + x h_{fg} \Rightarrow x = \frac{h - h_f}{h_{fg}}$$

$$s = (1 - x)s_f + x s_g = s_f + x s_{fg} \Rightarrow x = \frac{s - s_f}{s_{fg}}$$

Properties of Superheated Vapor

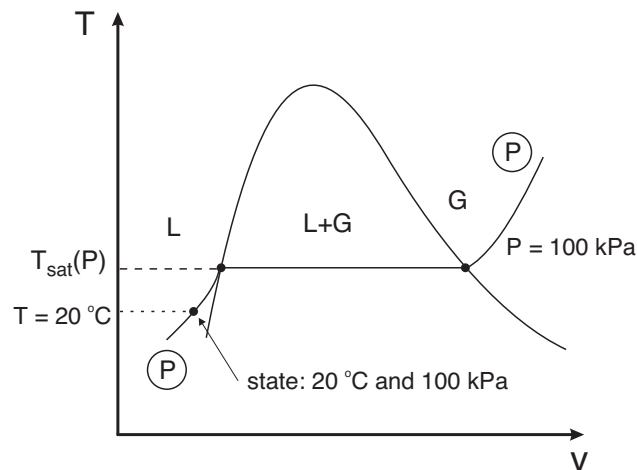
- superheated means $T > T_{sat}$ at the prevailing P , eg. water at 100 kPa has a saturation temperature of $T_{sat}(P) = 99.63^\circ\text{C}$.
- To fix the state we require $(T, v), (P, T) \dots$



- superheated vapor is a single phase
 - T and P are independent of each other in the single-phase region
 - properties are typically calculated as a function of T and P
- if $T \gg T_{cr}$ or $P \ll P_{cr}$ then ideal gas behavior applies and the ideal gas equations of state can be used
- Table A-6 for superheated water

Properties of Sub-cooled (Compressed) Liquid

- sub-cooled liquid means $T < T_{sat}$ at the prevailing P , eg. water at 20°C and 100 kPa has a saturation temperature of $T_{sat}(P) = 99.63^\circ\text{C}$.



- can be treated as incompressible (independent of P)

$$v = v(T, P) \Rightarrow v \approx v(T) = v_f(T)$$

$$u = u(T, P) \Rightarrow u \approx u(T) = u_f(T)$$

$$s = s(T, P) \Rightarrow s \approx s(T) = s_f(T)$$

- for enthalpy:

$$\begin{aligned}
 h(T, P) &= u(T, P) + v(T, P)P \approx u_f(T) + v_f(T)P \\
 &= \underbrace{u_f(T) + v_f(T)P_{sat}(T)}_{h_f(T)} + v_f(T)[P - P_{sat}(T)] \\
 &= h_f(T) + v_f(T)[P - P_{sat}(T)]
 \end{aligned}$$

Example 2-1: A saturated mixture of R-134a with a liquid volume fraction of 10% and a pressure of 200 kPa fills a rigid container whose volume is 0.5 m^3 . Find the quality of the saturated mixture and the total mass of the R134-a.

Equations of State: Calculate the Properties of Gaseous Pure Substances

Tables: water (Tables A-4 \rightarrow A-8, A-15), R134a (Tables A-11 \rightarrow A-13, A-16)

Graphs: water (Figures A-9 & A-10), R134a (Figure A-14)

Equations: (Table A-2c), (Table 4-4)

- *a better alternative for gases:* use the Equation of State which is a functional relationship between P , v , and T (3 measurable properties)

Ideal Gases

- gases that adhere to a pressure, temperature, volume relationship

$$Pv = RT \quad \text{or} \quad PV = mRT$$

are referred to as ideal gases

- where R is the gas constant for the specified gas of interest ($R = R_u/\tilde{M}$)

$$R_u = \text{Universal gas constant, } \equiv 8.314 \text{ kJ}/(\text{kmol} \cdot \text{K})$$

$$\tilde{M} = \text{molecular weight (or molar mass) of the gas (see Table A-1))}$$

Real Gases

- experience shows that **real gases** obey the following equation closely:

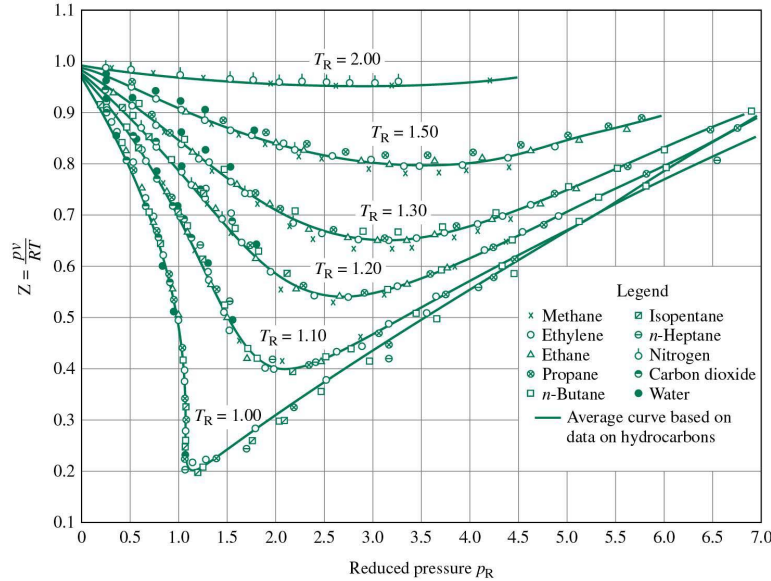
$$Pv = ZRT \quad (T \text{ and } P \text{ are in absolute terms})$$

where Z is the compressibility factor

- if we “reduce” the properties with respect to the values at the critical point, i.e.

$$\text{reduced pressure} = P_r = \frac{P}{P_c} \quad P_c = \text{critical pressure}$$

$$\text{reduced temperature} = T_r = \frac{T}{T_c} \quad T_c = \text{critical temperature}$$



Reference Values for u , h , s

- values of enthalpy, h and entropy, s listed in the tables are with respect to a datum where we arbitrarily assign the zero value. For instance:

Table A-4, A-5: saturated liquid - the reference for both h_f and s_f is taken at the triple point and $T_{ref} = 0.01^\circ\text{C}$. This is shown as follows:

$$u_f(@T = 0.01^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$h_f(@T = 0.01^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$s_f(@T = 0.01^\circ\text{C}) = 0 \text{ kJ/kg} \cdot \text{K}$$

Table A-11, A-12, & A-13: saturated R134a - the reference for both h_f and s_f is taken as -40°C . This is shown as follows:

$$h_f(@T = -40^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$h_f(@T = -40^\circ\text{C}) = 0 \text{ kJ/kg}$$

$$s_f(@T = -40^\circ\text{C}) = 0 \text{ kJ/kg} \cdot \text{K}$$

Others: sometimes tables will use 0 K as the reference for all tables. While this standardizes the reference, it tends to lead to larger values of enthalpy and entropy.

- for the most part the choice of a reference temperature in the property tables does not affect us because we generally deal with Δh , Δu , or Δs . The calculated difference of properties will be identical regardless of the reference temperature. Note: Be careful not to use values from different tables in the same calculation in the event that a difference reference temperature is used in the tables.

Calculation of the Stored Energy

- how do we calculate ΔU and ΔH ?
 1. one can often find u_1 and u_2 in the thermodynamic tables (like those examined for the states of water).
 2. we can also explicitly relate ΔU and ΔH to ΔT (as mathematical expressions) by using the thermodynamic properties C_p and C_v .
- the calculation of Δu and Δh for an ideal gas is given as

$$\Delta u = u_2 - u_1 = \int_1^2 C_v(T) dT \quad (kJ/kg)$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_p(T) dT \quad (kJ/kg)$$

- to carry out the above integrations, we need to know $C_v(T)$ and $C_p(T)$. These are available from a variety of sources

Table A-2a: for various materials at a fixed temperature of $T = 300\text{ K}$

Table A-2b: various gases over a range of temperatures $250\text{ K} \leq T \leq 1000\text{ K}$

Table A-2c: various common gases in the form of a third order polynomial

Specific Heats: Ideal Gases

- for an ideal gas $Pv = RT$

$$C_v = \frac{du}{dT} \Rightarrow C_v = C_v(T) \text{ only}$$

$$C_p = \frac{dh}{dT} \Rightarrow C_p = C_p(T) \text{ only}$$

$$h = u + Pv \Rightarrow h = u + RT \Rightarrow RT = h(T) - u(T)$$

If we differentiate with respect to T

$$R = \frac{dh}{dT} - \frac{du}{dT}$$

$$\boxed{R = C_p - C_v}$$

Specific Heats: Solids and Liquids

- solids and liquids are incompressible. (i.e., $\rho = \text{constant}$) and it can be shown (mathematically) that:

$$C_p = C_v = C$$

Ideal gases: $\Delta u = \int_1^2 C(T) dT \approx C_{avg} \Delta T$

$$\Delta h = \Delta u + \Delta(Pv) = \Delta u + v\Delta P + P\Delta v \approx C_{avg}\Delta T + v\Delta P$$

Solids: $\Delta h = \Delta u = C_{avg}\Delta T$ ($v\Delta P$ is negligible)

Liquids: If $P = \text{const}$, $\Delta h = C_{avg}\Delta T$

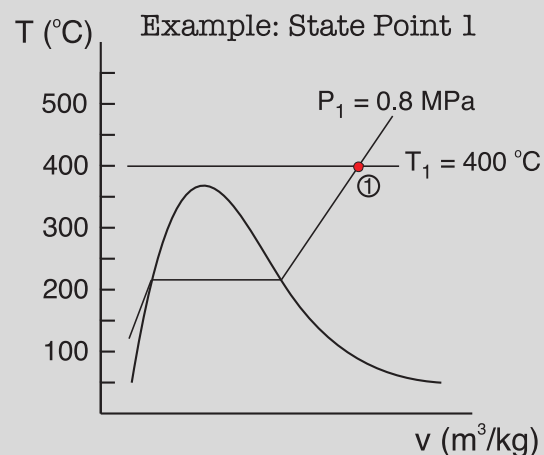
If $T = \text{const}$, $\Delta h = v\Delta P$

Example 2-2: Using water as the working fluid, complete the table shown below. Using graph paper, create a separate $T - v$ diagram for each state point that clearly shows how you have located the state point (approximately to scale). For instance, using State Point 1 as an example, show a curve of $T = 400^\circ\text{C}$ and a curve of $P = 0.8 \text{ MPa}$. State Point 1 will be at the intersection of these two curves. The intersection should have the correct relationship to the vapor dome (i.e. inside or outside).

For each case, show all calculations necessary to establish the actual location of the state point. Identify the relevant charts or tables used to perform your calculations

In the table, under the column for quality (x), indicate the quality of the state point is under the dome, **CL** if the state point is in the compressed liquid region and **SV** if the state point is in the superheated vapor region.

State	T $^\circ\text{C}$	P MPa	v m^3/kg	x
1	400	0.8		
2	200		0.005	
3		3.5	0.05	
4		0.01	20.0	
5		0.2		0.30
6	100	5.0		
7	300		0.5	
8	70.0			0.60
9	250	0.125		



First Law of Thermodynamics



Reading

3-2 → 3-7

5-1 → 5-2

6-1 → 6-5

Problems

3-40, 3-54, 3-105

5-8, 5-25, 5-29, 5-37, 5-40, 5-42, 5-63, 5-74, 5-84, 5-109

6-44, 6-51, 6-60, 6-80, 6-94, 6-124, 6-168, 6-173

Control Mass (Closed System)

In this section we will examine the case of a control surface that is closed to mass flow, so that no mass can escape or enter the defined control region.

Conservation of Mass

Conservation of Mass, which states that mass cannot be created or destroyed, is implicitly satisfied by the definition of a control mass.

Conservation of Energy

The first law of thermodynamics states “Energy cannot be created or destroyed it can only change forms”.

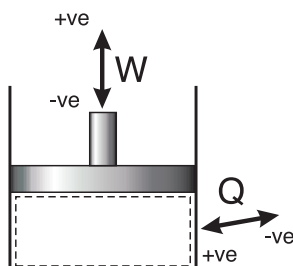
$$\boxed{\text{energy entering}} - \boxed{\text{energy leaving}} = \boxed{\text{change of energy within the system}}$$

Sign Convention

Cengel Approach

Heat Transfer: heat transfer to a system is positive and heat transfer from a system is negative.

Work Transfer: work done by a system is positive and work done on a system is negative.

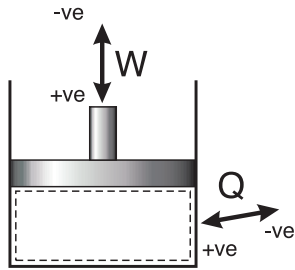


For instance: moving boundary work is defined as:

$$W_b = \int_1^2 P dV$$

During a compression process, work is done on the system and the change in volume goes negative, i.e. $dV < 0$. In this case the boundary work will also be negative.

Culham Approach

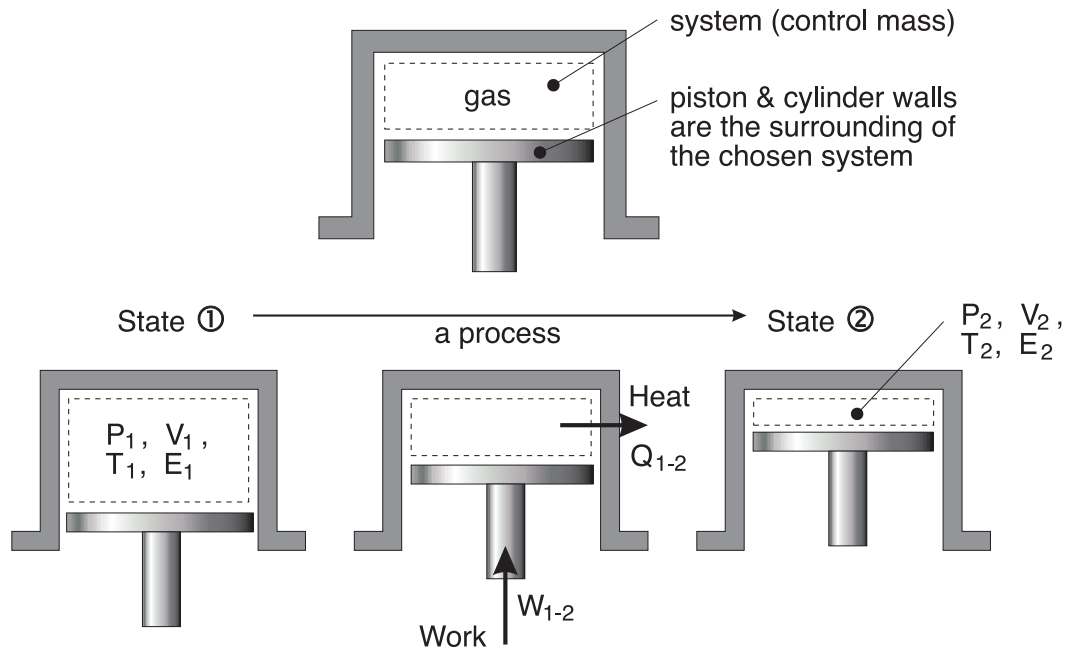


Using my sign convention, the boundary work is defined as:

$$W_b = - \int_1^2 P dV$$

During a compression process, the change in volume is still negative but because of the negative sign on the right side of the boundary work equation, the boundary work directed into the system is considered positive. Any form of energy that adds to the system is considered positive.

Example: A Gas Compressor



Performing a 1st law energy balance:

$$\left\{ \begin{array}{c} \text{Initial} \\ \text{Energy} \\ E_1 \end{array} \right\} + \left\{ \begin{array}{c} \text{Energy gain } W_{1-2} \\ \text{Energy loss } Q_{1-2} \end{array} \right\} = \left\{ \begin{array}{c} \text{Final} \\ \text{Energy} \\ E_2 \end{array} \right\}$$

$$E_1 + W_{1-2} - Q_{1-2} = E_2 \quad \Rightarrow \quad \Delta E = Q - W$$

A first law balance for a control mass can also be written in differential form as follows:

$$dE = \delta Q - \delta W$$

Note: d or Δ for a change in property and δ for a path function

The differential form of the energy balance can be written as a rate equation by dividing through by dt , a differential time, and then letting $dt \rightarrow 0$ in the limit to give

$$\frac{dE}{dt} = \frac{\delta Q}{dt} - \frac{\delta W}{dt} \Rightarrow \frac{dE}{dt} = \dot{Q} - \dot{W}$$

where

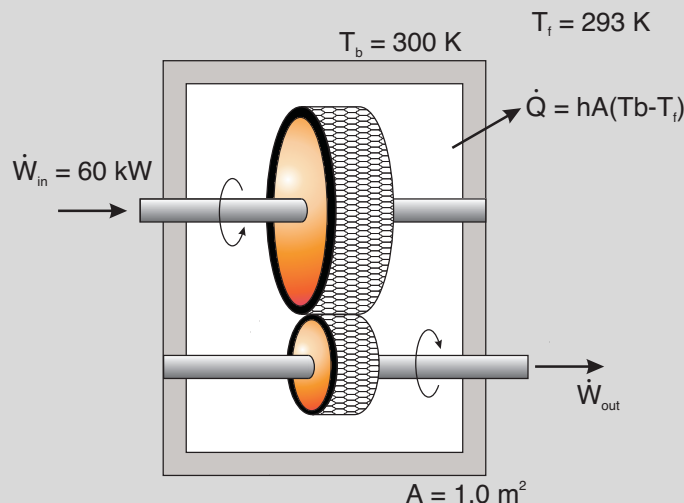
$$\frac{dE}{dt} = \text{rate of energy increase within the system,} \equiv \frac{dU}{dt} + \frac{dKE}{dt} + \frac{dPE}{dt}$$

$$\dot{Q} = \text{rate of heat transfer}$$

$$\dot{W} = \text{rate of work done,} \equiv \text{power}$$

- most closed systems encountered in practice are stationary i.e. the velocity and the elevation of the center of gravity of the system remain constant during the process
- for stationary systems we can assume that $\frac{dKE}{dt} = 0$ and $\frac{dPE}{dt} = 0$

Example 3-1: During steady-state operation, a gearbox receives $\dot{W}_{in} = 60 \text{ kW}$ through the input shaft and delivers power through the output shaft. For the gearbox as the system, the rate of energy transfer by heat is given by Newton's Law of Cooling as $\dot{W} = hA(T_b - T_f)$, where h , the heat transfer coefficient, is constant ($h = 0.171 \text{ kW/m}^2 \cdot \text{K}$) and the outer surface area of the gearbox is $A = 1.0 \text{ m}^2$. The temperature of the outer surface of the gearbox is $T_b = 300 \text{ K}$ and the ambient temperature surrounding the gearbox is $T_\infty = 293 \text{ K}$. Evaluate the rate of heat transfer, \dot{Q} and the power delivered through the output shaft, \dot{W}_{out} .



Forms of Energy Transfer

Work Versus Heat

- Work is macroscopically organized energy transfer.
 - Heat is microscopically disorganized energy transfer.
- } more on this when we discuss entropy

Heat Energy

- heat is defined as a form of energy that is transferred solely due to a temperature difference (without mass transfer)
- heat transfer is a directional (or vector) quantity with magnitude, direction and point of action
- modes of heat transfer:
 - **conduction:** diffusion of heat in a stationary medium (Chapters 10, 11 & 12)
 - **convection:** it is common to include convective heat transfer in traditional heat transfer analysis. However, it is considered mass transfer in thermodynamics. (Chapters 13 & 14)
 - **radiation:** heat transfer by photons or electromagnetic waves (Chapter 15)

Work Energy

- work is a form of energy in transit. One should not attribute work to a system.
- work (like heat) is a “path function” (magnitude depends on the process path)
- work transfer mechanisms in general, are a force acting over a distance

Mechanical Work

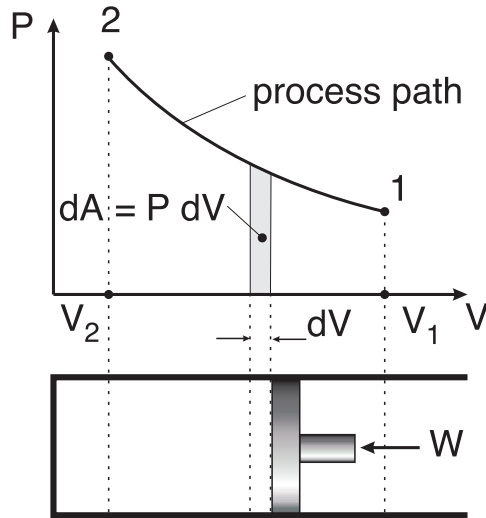
$$W_{12} = \int_1^2 F ds$$

- if there is no driving or resisting force in the process (e.g. expansion in a vacuum) or the boundaries of the system do not move or deform, $W_{12} = 0$.

Moving Boundary Work

$$W_{12} = - \int_1^2 F ds = - \int_1^2 P \cdot A ds = - \int_1^2 P dV$$

- a decrease in the volume, $dV \rightarrow -ve$ results in work addition ($+ve$) on the system



- consider compression in a piston/cylinder, where A is the piston cross sectional area (frictionless)
- the area under the process curve on a $P - V$ diagram is proportional to $\int_1^2 P dV$
- the work is:
 - $+ve$ for compression
 - $-ve$ for expansion
- sometimes called $P dV$ work or compression /expansion work

- *polytropic processes:* where $PV^n = C$
- examples of polytropic processes include:

Isobaric process: if $n = 0$ then $P = C$ and we have a constant pressure process

Isothermal process: if $n = 1$ then from the ideal gas equation $PV = RT$ and PV is only a function of temperature

Isometric process: if $n \rightarrow \infty$ then $P^{1/n}V = C^{1/n}$ and we have a constant volume process

Isentropic process: if $n = k = C_p/C_v$ then we have an isentropic process. (tabulated values for k are given in Table A-2) If we combine

$$Pv^k = C \quad \text{with} \quad Pv = RT$$

we get the isentropic equations, given as:

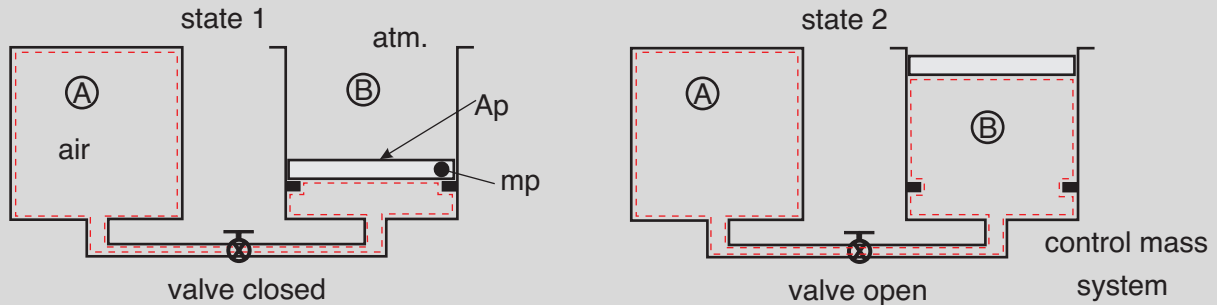
$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

Case 1: for an ideal gas with $n = 1$ \rightarrow
$$W_{12} = -C \ln \frac{V_2}{V_1}$$

Case 2: for $n \neq 1$ \rightarrow
$$W_{12} = \frac{P_1 V_1 - P_2 V_2}{1 - n} \quad (\text{in general})$$

\rightarrow
$$W_{12} = \frac{mR(T_1 - T_2)}{1 - n} \quad (\text{ideal gas})$$

Example 3-2: A pneumatic lift as shown in the figure below undergoes a quasi-equilibrium process when the valve is opened and air travels from tank A to tank B.



The initial conditions are given as follows and the final temperatures can be assumed to be the same as the initial temperatures.

$$\begin{aligned} P_{atm} &= 100 \text{ kPa} \\ m_p &= 500 \text{ kg} \\ V_{A,1} &= 0.4 \text{ m}^3 \\ P_{A,1} &= 500 \text{ kPa} \\ T_{A,1} &= 298 \text{ K} \end{aligned}$$

$$\begin{aligned} 1 \text{ Pa} &= 1 \text{ N/m}^2 \\ A_p &= 0.0245 \text{ m}^2 \\ V_{B,1} &= 0.1 \text{ m}^3 \\ P_{B,1} &= 100 \text{ kPa} \\ T_{B,1} &= 298 \text{ K} = 25 \text{ }^\circ\text{C} \end{aligned}$$

Find the final pressures $P_{A,2}$ and $P_{B,2}$ and the work, W_{12} , in going from state 1 to state 2.

Control Volume (Open System)

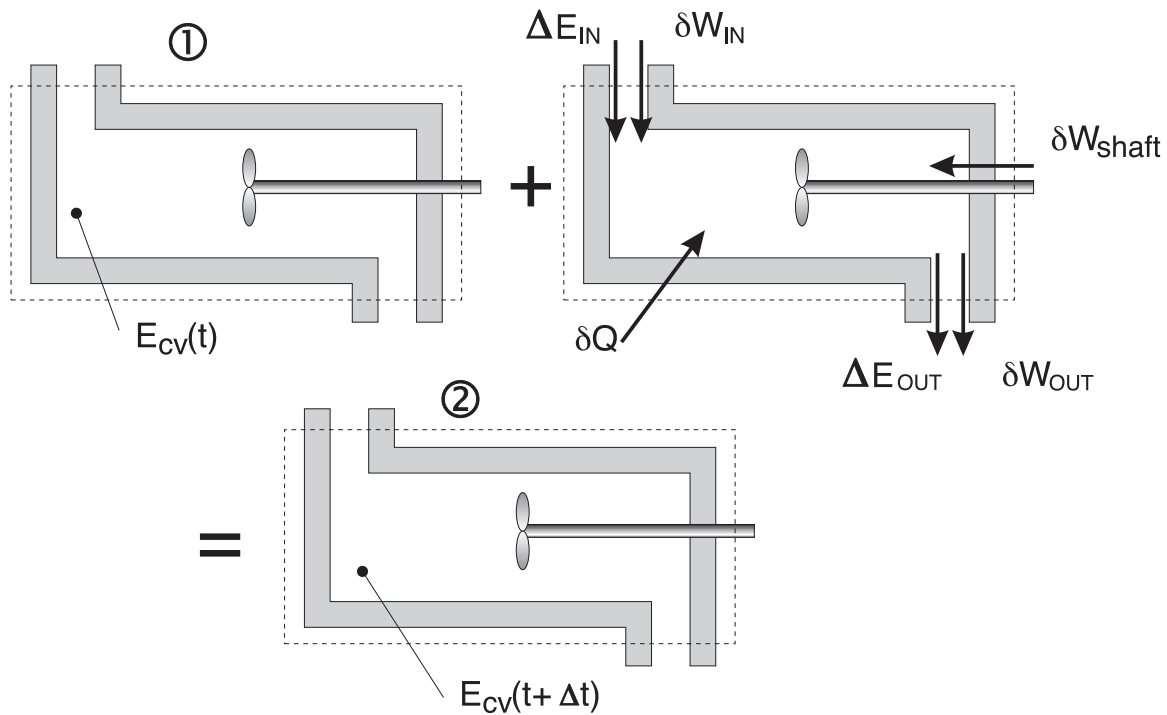
The major difference between a *Control Mass* and *Control Volume* is that mass crosses the system boundary of a control volume.

CONSERVATION OF MASS:

Unlike a control mass approach, the control volume approach does not implicitly satisfy conservation of mass, therefore we must make sure that mass is neither created nor destroyed in our process.

$$\left\{ \begin{array}{l} \text{rate of increase of} \\ \text{mass within the CV} \end{array} \right\} = \left\{ \begin{array}{l} \text{net rate of} \\ \text{mass flow IN} \end{array} \right\} - \left\{ \begin{array}{l} \text{net rate of} \\ \text{mass flow OUT} \end{array} \right\}$$

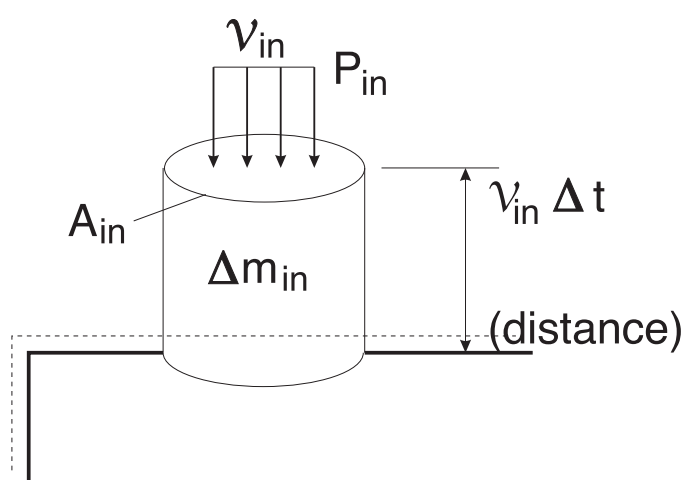
CONSERVATION OF ENERGY:



$$E_{CV}(t) + \delta Q + \delta W_{shaft} + (\Delta E_{IN} - \Delta E_{OUT}) + (\delta W_{IN} - \delta W_{OUT}) = E_{CV}(t + \Delta t) \quad (1)$$

What is flow work?

This is the work required to pass the flow across the system boundaries.



$$\Delta m_{IN} = \rho_{IN} \overbrace{A_{IN} v_{IN} \Delta t}^{\text{volume}}$$

$$\begin{aligned} \delta W_{IN} &= F \cdot \text{distance} \\ &= \underbrace{P_{IN} A_{IN}}_F \cdot \underbrace{v_{IN} \Delta t}_{\Delta s} \\ &= \frac{P_{IN} \Delta m_{IN}}{\rho_{IN}} \end{aligned}$$

$$\text{since } v = 1/\rho$$

$$\delta W_{IN} = (P v \Delta m)_{IN} \rightarrow \text{flow work} \quad (2)$$

Similarly

$$\delta W_{OUT} = (P v \Delta m)_{OUT} \quad (3)$$

Substituting Eqs. 2 and 3 into Eq. 1 gives the 1st law for a control volume

$$\begin{aligned} E_{CV}(t + \Delta t) - E_{CV}(t) = & \delta Q + \delta W_{shaft} + \Delta m_{IN}(e + Pv)_{IN} \\ & - \Delta m_{OUT}(e + Pv)_{OUT} \end{aligned} \quad (4)$$

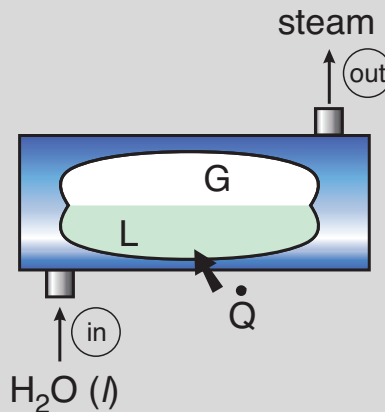
Equation 4 can also be written as a rate equation \rightarrow divide through by Δt and take the limit as $\Delta t \rightarrow 0$

$$\frac{d}{dt}E_{CV} = \dot{Q} + \dot{W}_{shaft} + [\dot{m}(e + Pv)]_{IN} - [\dot{m}(e + Pv)]_{OUT}$$

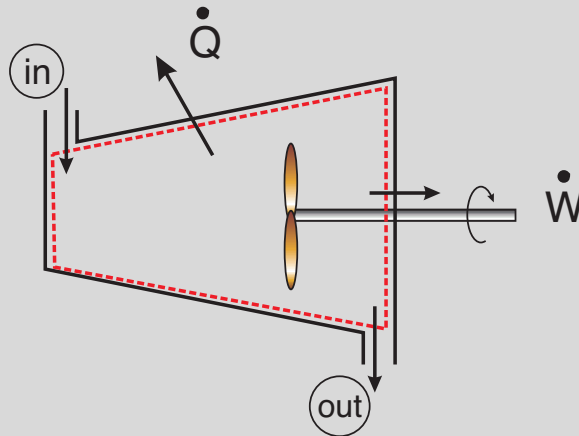
where:

$$\begin{aligned} e + Pv &= \underbrace{u + Pv}_{h} + \frac{(V)^2}{2} + gz \\ &= h(\text{enthalpy}) + KE + PE \end{aligned}$$

Example 3-3: Determine the heat flow rate, \dot{Q} , necessary to sustain a steady flow process where liquid water enters a boiler at 120°C and 10 MPa and exits the boiler at 10 MPa and a quality of 1 for a mass flow rate is 1 kg/s . The effects of potential and kinetic energy are assumed to be negligible.

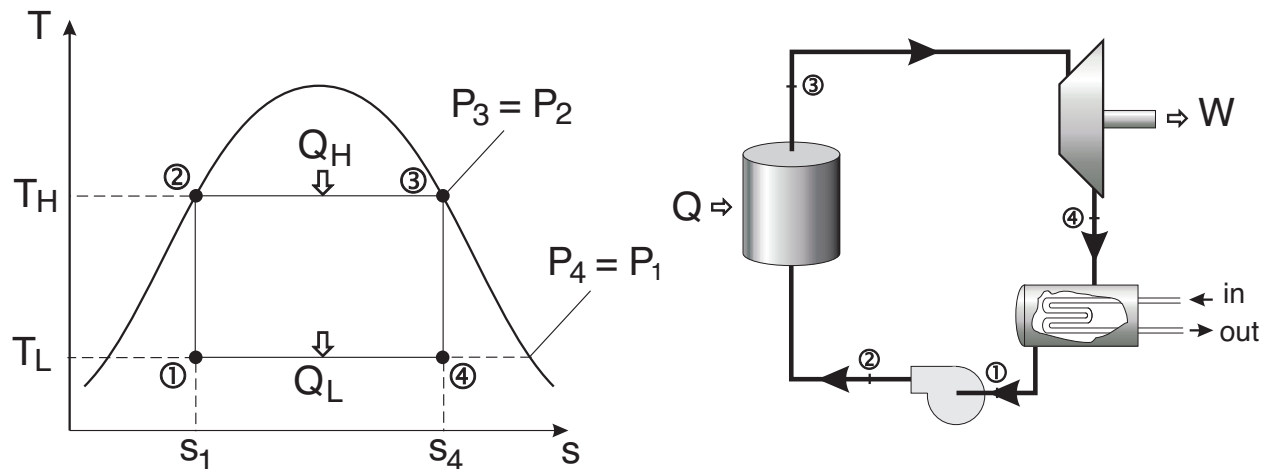


Example 3-4: Steam with a mass flow rate of 1.5 kg/s enters a steady-flow turbine with a flow velocity of 50 m/s at 2 MPa and 350°C and leaves at 0.1 MPa , a quality of 1, and a velocity of 200 m/s . The rate of heat loss from the uninsulated turbine is 8.5 kW . The inlet and exit to the turbine are positioned 6 m and 3 m above the reference position, respectively. Determine the power output from the turbine.
Note: include the effects of kinetic and potential energy in the calculations.



The Carnot Cycle

If the heat engine is a reversible system where no entropy is generated internally, we refer to the cycle as the Carnot cycle.

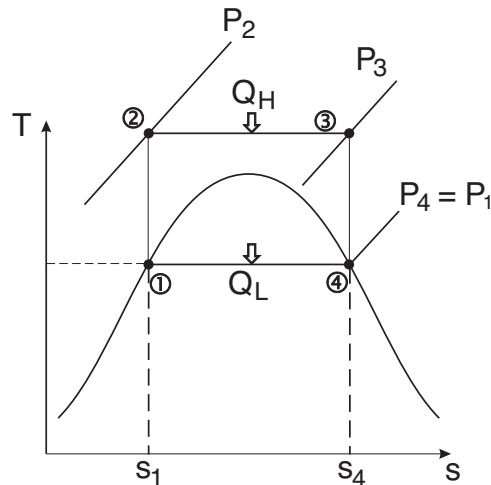


where the efficiency is given as:

$$\eta = 1 - \frac{T_L}{T_H} \Leftarrow \text{Carnot efficiency}$$

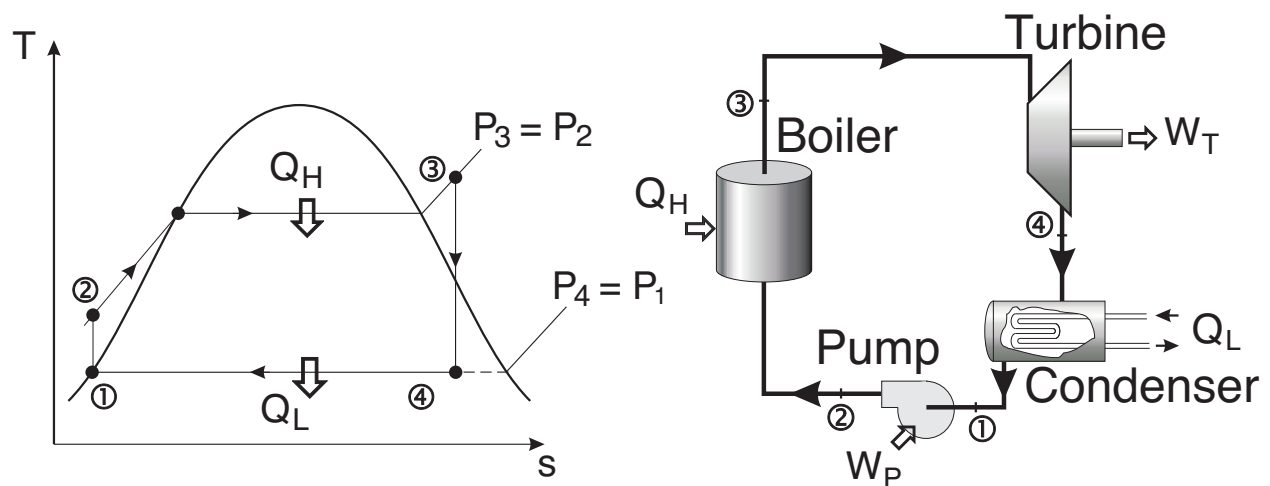
Practical Problems

- at state point 1 the steam is wet at T_L and it is difficult to pump water/steam (two phase) to state point 2
- can we devise a Carnot cycle to operate outside the wet vapor region



- between state points 2 and 3 the vapor must be isothermal and at different pressures - this is difficult to achieve
- the high temperature and pressure at 2 and 3 present metallurgical limitations

The Ideal Rankine Cycle



- water is typically used as the working fluid because of its low cost and relatively large value of enthalpy of vaporization

Device	1st Law Balance
Boiler	$h_2 + q_H = h_3 \Rightarrow q_H = h_3 - h_2$ (in)
Turbine	$h_3 = h_4 + w_T \Rightarrow w_T = h_3 - h_4$ (out)
Condenser	$h_4 = h_1 + q_L \Rightarrow q_L = h_4 - h_1$ (out)
Pump	$h_1 + w_P = h_2 \Rightarrow w_P = h_2 - h_1$ (in)

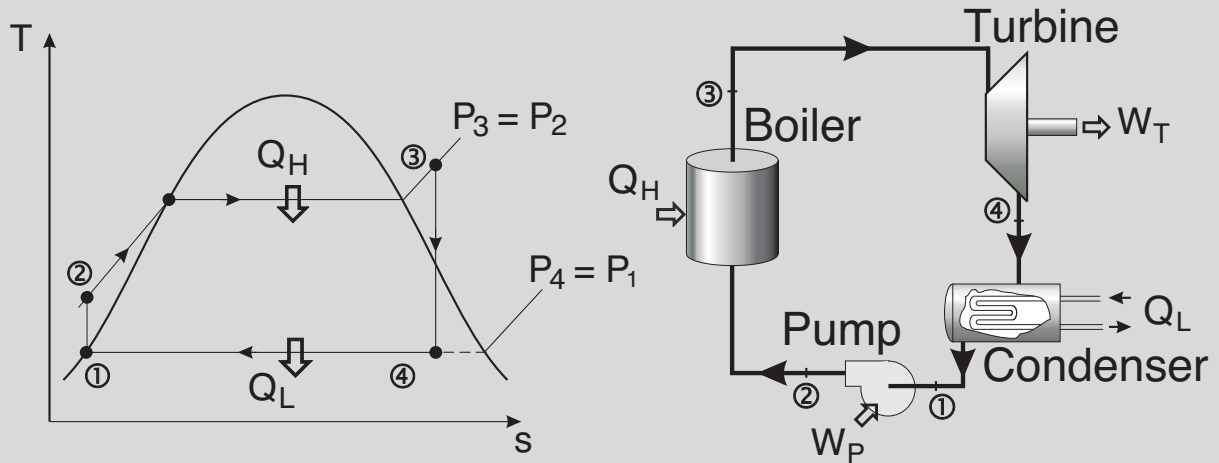
The net work output is given as

$$w_T - w_p = (h_3 - h_4) - (h_2 - h_1) = (h_3 - h_4) + (h_1 - h_2)$$

The Rankine efficiency is

$$\eta_R = \frac{\text{net work output}}{\text{heat supplied to the boiler}} = \frac{(h_3 - h_4) + (h_1 - h_2)}{(h_3 - h_2)}$$


Example 3-5: For the steam power plant shown below,



find: $\dot{Q}_H, \dot{Q}_L, \dot{W}_{net} = \dot{W}_T - \dot{W}_P$, and the overall cycle efficiency, η_R given the following conditions:

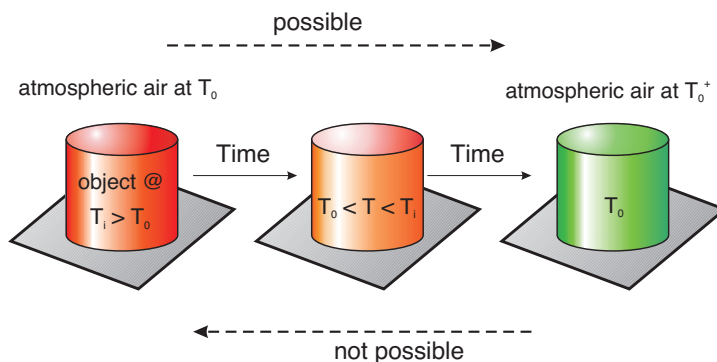
$P_1 = 10 \text{ kPa}$	$P_2 = 10 \text{ MPa}$
$P_3 = 10 \text{ MPa}$	$P_4 = 10 \text{ kPa}$
$T_1 = 40 \text{ }^\circ\text{C}$	$T_3 = 530 \text{ }^\circ\text{C}$
$T_2 = 110 \text{ }^\circ\text{C}$	
$x_4 = 0.9$	$\dot{m} = 5 \text{ kg/s}$

Entropy and the Second Law of Thermodynamics

	Reading	Problems
	7-1 → 7-3	7-88, 7-131, 7-135
	7-6 → 7-10	8-24, 8-44, 8-46, 8-60, 8-73, 8-99, 8-128, 8-132,
	8-1 → 8-10, 8-13	8-135, 8-148, 8-152, 8-166, 8-168, 8-189

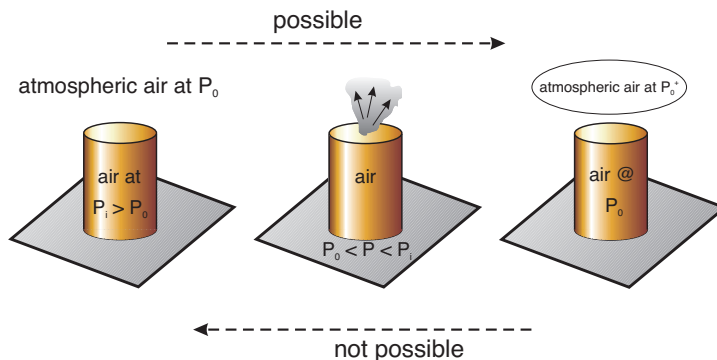
The 1st law allows many processes to occur spontaneously in either direction, but our experience tells us that they are not reversible.

Case 1: Hot object cooling down in atmosphere



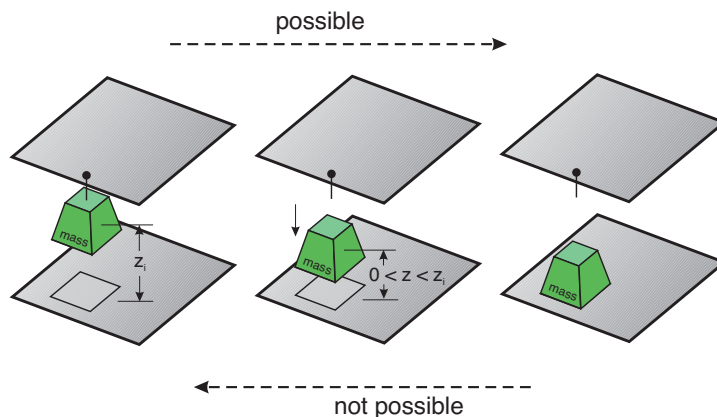
- object at T_i dissipates heat to the atmosphere at T_0 over time
- this process is not reversible i.e. energy dissipated to the atmosphere cannot be used to heat the object back to T_i

Case 2: Pressurized cylinder releases air to the environment



- air held at P_i in a closed tank is spontaneously released to the surroundings
- this process is not reversible i.e. atmospheric air will not spontaneously charge the tank even though energy is conserved

Case 3: A mass is allowed to fall from z_i

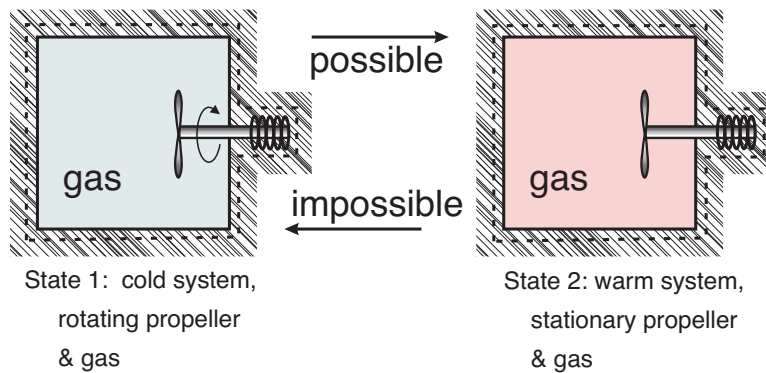


- a mass suspended by a cable at elevation z_i is released and allowed to fall
- this process is not reversible i.e. potential energy lost to internal energy via heating and compression cannot be used to raise the object

While the 1st law allowed us to determine the quantity of energy transfer in a process it does not provide any information about the direction of energy transfer nor the quality of the energy transferred in the process. In addition, we can not determine from the 1st law alone whether the process is possible or not. The second law will provide answers to these unanswered questions.

1. Direction:

Consider an isolated system where $Q = W = 0$ (Assume energy to drive the propeller is a tensioned spring, and no work crosses the boundary.)

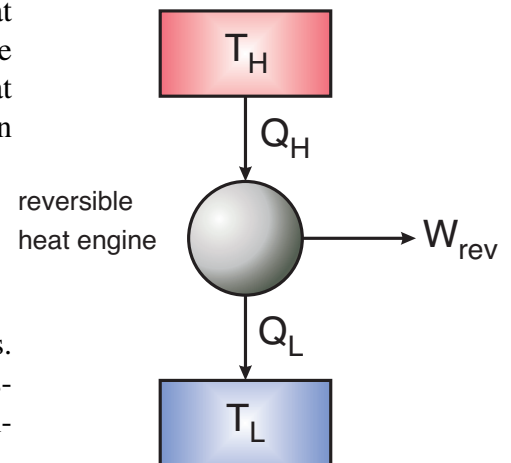


2. Quality of Energy:

A heat engine produces reversible work as it transfers heat from a high temperature reservoir at T_H to a low temperature reservoir at T_L . If we fix the low temperature reservoir at $T_L = 300\text{ K}$, we can determine the relationship between the efficiency of the heat engine,

$$\eta = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

as the temperature of the high temperature reservoir changes. In effect we are determining the quality of the energy transferred at high temperature versus that transferred at low temperature.



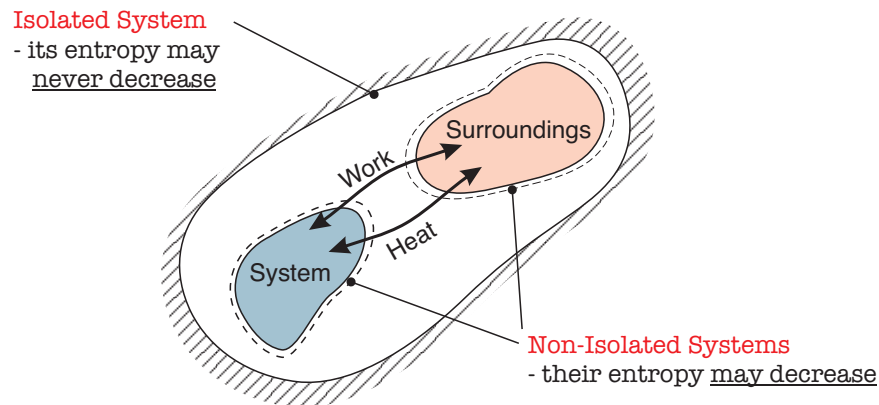
Second Law of Thermodynamics

Definition

The entropy of an isolated system can never decrease. When an isolated system reaches equilibrium, its entropy attains the maximum value possible under the constraints of the system

- we have conservation of mass and energy, but not entropy. Entropy is not conserved.

- the 2nd law dictates why processes occur in a specific direction i.e., S_{gen} cannot be $-ve$
- the second law states: $(\Delta S)_{system} + (\Delta S)_{surr.} \geq 0$. If heat is leaving the system, then ΔS_{system} can be negative but the combined entropy of the system and the surroundings can never be negative.



Entropy

1. Like mass and energy, every system has entropy.

Entropy is a measure of the degree of microscopic disorder and represents our uncertainty about the microscopic state.

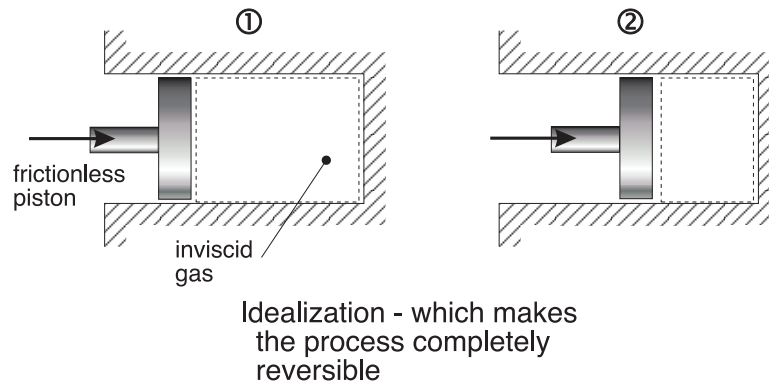
2. **Reference:** At $T = 0 \text{ K}$ $S = 0$. There is no uncertainty about the microscopic state.
3. **Relationship to Work:** Microscopic disorder results in a loss of ability to do useful work.
4. **Work:** Energy transfer by work is microscopically organized and therefore entropy-free.
5. **Heat:** Energy transfer as heat takes place as work at the microscopic level but in a random, disorganized way. This results in entropy flow in or out of the system.

Reversible Process

In a reversible process things happen very slowly, without any resisting force, without any space limitation \rightarrow *Everything happens in a highly organized way* (it is not physically possible - it is an idealization).

An internally reversible system is one in which there are no irreversibilities in the system itself but there may be irreversibilities in the surroundings such as in the transfer of heat across a finite temperature difference between the system and the surroundings.

Example: Slow adiabatic compression of a gas



→ idealization where $S_2 = S_1 \Rightarrow S_{gen} = 0$

$T_2 > T_1 \Rightarrow$ increased microscopic disorder

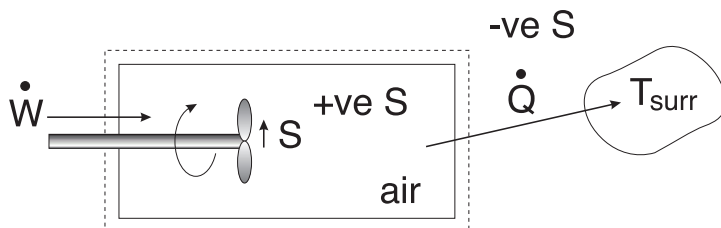
$V_2 < V_1 \Rightarrow$ reduced uncertainty about the whereabouts of molecules

$$\underbrace{\text{Reversible}}_{S_{gen}=0} + \underbrace{\text{Adiabatic Process}}_{Q=0} \Rightarrow \underbrace{\text{Isentropic Process}}_{S_1=S_2}$$

Does:

$$\text{Isentropic Process} \Rightarrow \text{Reversible} + \text{Adiabatic}$$

NOT ALWAYS - the entropy increase of a substance during a process as a result of irreversibilities may be offset by a decrease in entropy as a result of heat losses.



$$\text{If } \dot{S}_{gen} = \frac{\dot{Q}}{T_{sur}}, \text{ and}$$

$$\dot{S}_{gen} > 0 \text{ (fluid friction)} \\ \Rightarrow \text{not reversible}$$

$$\dot{Q} > 0 \Rightarrow \text{(non-adiabatic system)}$$

If \dot{S}_{gen} and \dot{Q}/T_{TER} and $\Delta S = 0$, i.e. the system is isentropic, however, the system is not adiabatic nor is it reversible.

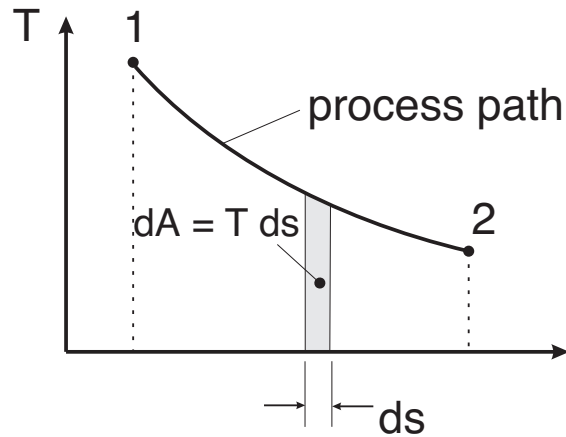
Evaluating Entropy

For a simple compressible system, Gibb's equations are given as:

$$Tds = du + Pdv$$

$$Tds = dh - vdP$$

- these relations are valid for open and closed, and reversible and irreversible processes



- the area under the curve on a $T - s$ diagram is the heat transfer for internally reversible processes

$$q_{int,rev} = \int_1^2 T ds$$

$$q_{int,rev,isothermal} = T\Delta s$$

- one can use Tds relations to calculate ds and Δs

Tabulated Calculation of Δs for Pure Substances

Calculation of the Properties of Wet Vapor:

Use Tables A-4 and A-5 to find s_f , s_g and/or s_{fg} for the following

$$s = (1 - x)s_f + xs_g \qquad s = s_f + xs_{fg}$$

Calculation of the Properties of Superheated Vapor:

Given two properties or the state, such as temperature and pressure, use Table A-6.

Calculation of the Properties of a Compressed Liquid:

Use Table A-7. In the absence of compressed liquid data for a property $s_{T,P} \approx s_{f@T}$

Calculation of Δs for Incompressible Materials

$$Tds = du + Pdv \qquad \text{for an incompressible substance, } dv = 0, \text{ and } C_p = C_v = C$$

$$ds = \frac{du}{T} = C \frac{dT}{T} \Rightarrow s_2 - s_1 = \int_1^2 C(T) \frac{dT}{T}$$

$$\Delta s = C_{avg} \ln \frac{T_2}{T_1} \qquad \text{where } C_{avg} = [C(T_1) + C(T_2)]/2$$

- if the process is isentropic, then $T_2 = T_1$, and $\Delta s = 0$

Calculation of Δs for Ideal Gases

$$\text{Ideal Gas Equation} \Rightarrow P v = R T$$

$$du = C_v dT \Rightarrow u_2 - u_1 = C_v(T_2 - T_1)$$

$$dh = C_p dT \Rightarrow h_2 - h_1 = C_p(T_2 - T_1)$$

There are 3 forms of a change in entropy as a function of T & v , T & P , and P & v .

$$\begin{aligned} s_2 - s_1 &= C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \\ &= C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\ &= C_p \ln \frac{v_2}{v_1} + C_v \ln \frac{P_2}{P_1} \end{aligned}$$

Variable Specific Heat

If we don't assume C_v and C_p to be constant i.e. independent of T , then we can use the ideal gas tables (Table A-21) as follows

$$s_2 - s_1 = s^o(T_2) - s^o(T_1) + R \ln(v_2/v_1) \quad \text{or}$$

$$s_2 - s_1 = s^o(T_2) - s^o(T_1) - R \ln(P_2/P_1)$$

Non-Reversible Processes and Isentropic Efficiencies

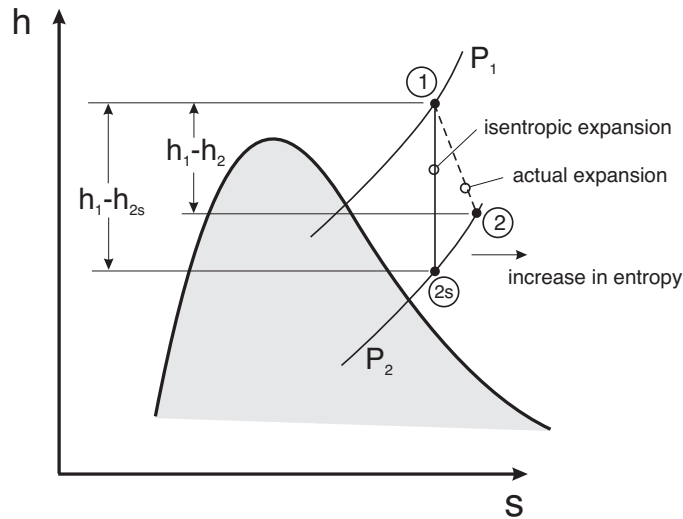
Real processes, such as expansion of steam in a turbine, produce irreversibilities associated with friction, mixing, etc. The net result is that the performance is downgraded, resulting in a loss of net work output attributed to an increase in entropy.

For a turbine under steady flow conditions, the theoretical maximum work output assuming no internal irreversibilities for an adiabatic process, i.e. isentropic expansion is

$$w_{isentropic} = h_1 - h_{2s}$$

However for a real process where expansion leads to an increase in entropy, the actual work output is degraded

$$w_{actual} = h_1 - h_2$$



The ratio of the actual work to the isentropic work is called the isentropic efficiency and is defined as follows:

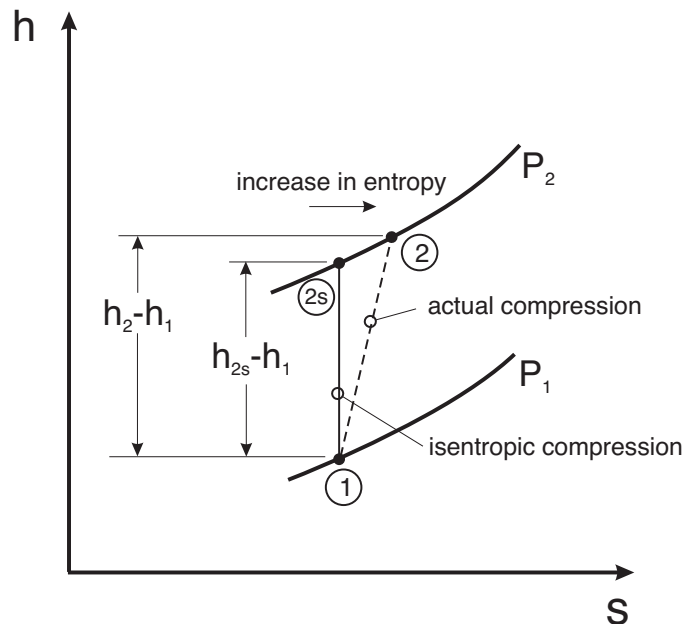
Turbine:

$$\eta_T = \frac{\text{actual turbine work}}{\text{isentropic turbine work}}$$

$$= \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Work Output Devices: i.e. Turbines and Nozzles

In a similar manner, the isentropic efficiency for other steady flow devices can be defined as follows:



Compressor:

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{actual compressor work}}$$

$$= \frac{h_{2s} - h_1}{h_2 - h_1}$$

Pump:

$$\eta_P = \frac{\text{Isentropic pump work}}{\text{actual pump work}}$$

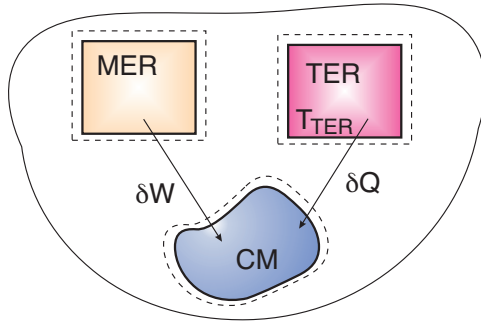
$$= \frac{h_{2s} - h_1}{h_2 - h_1}$$

Work Input Devices: i.e. Compressors and Pumps

Nozzle:

$$\eta_N = \frac{\text{actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_2^2}{V_{2s}^2} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

Entropy Balance for a Closed System (Control Mass)



We can first perform a 1st law energy balance on the system shown above.

$$dU = \delta Q + \delta W \quad (1)$$

For a simple compressible system

$$\delta W = -PdV \quad (2)$$

From Gibb's equation we know

$$T_{TER} dS = dU + PdV \quad (3)$$

Combining (1), (2) and (3) we get

$$T_{TER} dS = \delta Q$$

$$\text{net in-flow} \quad dS = \frac{\delta Q}{T_{TER}}$$

$$\text{net out-flow} \quad dS = -\frac{\delta Q}{T_{TER}}$$

From the Clausius inequality we can show that the irreversibilities in the system lead to entropy production and the entropy balance equation becomes

$$\underbrace{(dS)_{CM}}_{\equiv \text{storage}} = \underbrace{\frac{\delta Q}{T_{TER}}}_{\equiv \text{entropy flow}} + \underbrace{dS_{gen}}_{\equiv \text{production}}$$

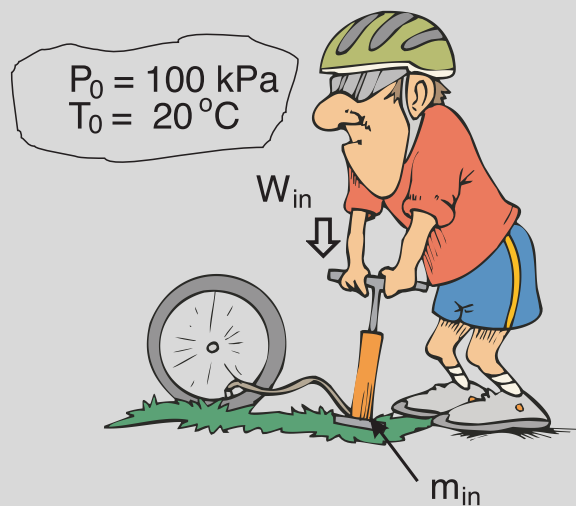
$$(S_2 - S_1)_{CM} = \frac{Q_{1-2}}{T_{TER}} + \underbrace{S_{gen}}_{\geq 0}$$

$$\text{accumulation} = (OUT - IN) + \text{generation}$$

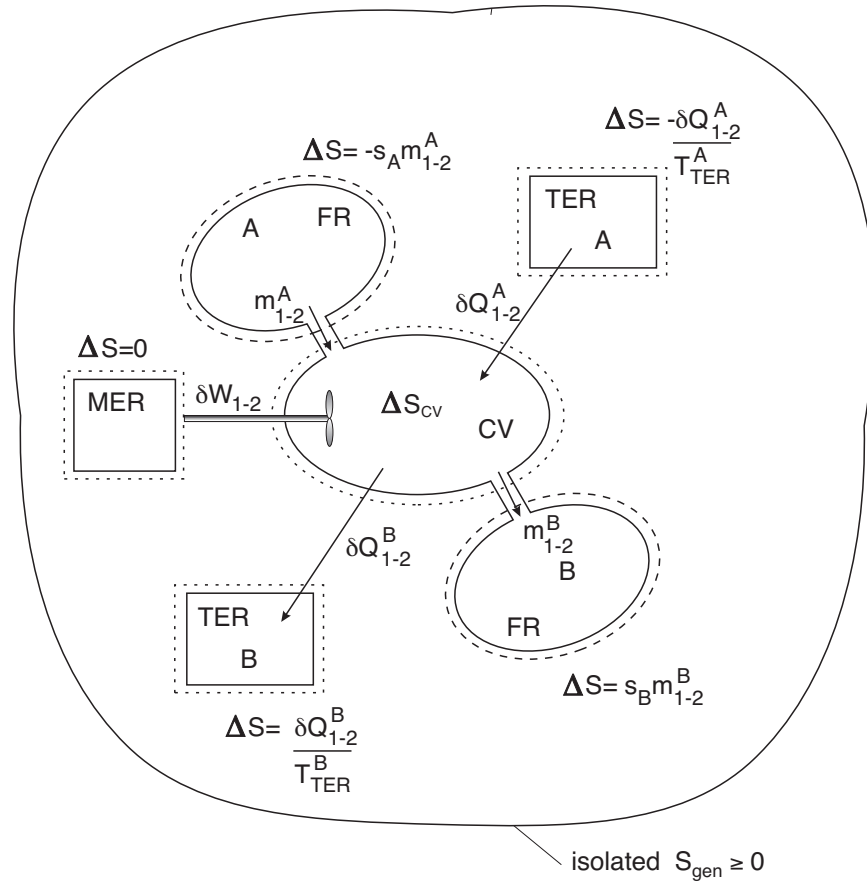
$\frac{Q_{1-2}}{T_{TER}}$ - the entropy associated with heat transfer across a finite temperature difference, i.e. $T > 0$

Example 4-1: A bicycle tire has a volume of 1200 cm^3 which is considered to be constant during “inflation”. Initially the tire contains air at atmospheric conditions given as $P_0 = 100 \text{ kPa}$ and $T_0 = 20^\circ\text{C}$. A student then hooks up a bicycle pump and begins to force air from the atmosphere into the tire. After pumping stops and a new equilibrium is reached, the tire pressure is 600 kPa and the air temperature in the tire is 20°C .

- Determine the mass $[\text{kg}]$ of air added to the tire.
- Determine the minimum amount of work $[\text{kJ}]$ required to reach this end state.
- If more than the minimum work is actually used, where does this extra energy go? That is what becomes of the extra energy?



Entropy Rate Balance for Open Systems (Control Volume)



For the isolated system going through a process from $1 \rightarrow 2$

$$\delta S_{gen} = (\Delta S)_{sys} + (\Delta S)_{sur}$$

$$\delta S_{gen} = \underbrace{\Delta S_{CV}}_{system} + \underbrace{\left(-s_A m_{1-2}^A + s_B m_{1-2}^B - \frac{\delta Q_{1-2}^A}{T_{TER}^A} + \frac{\delta Q_{1-2}^B}{T_{TER}^B} \right)}_{surroundings}$$

or as a rate equation

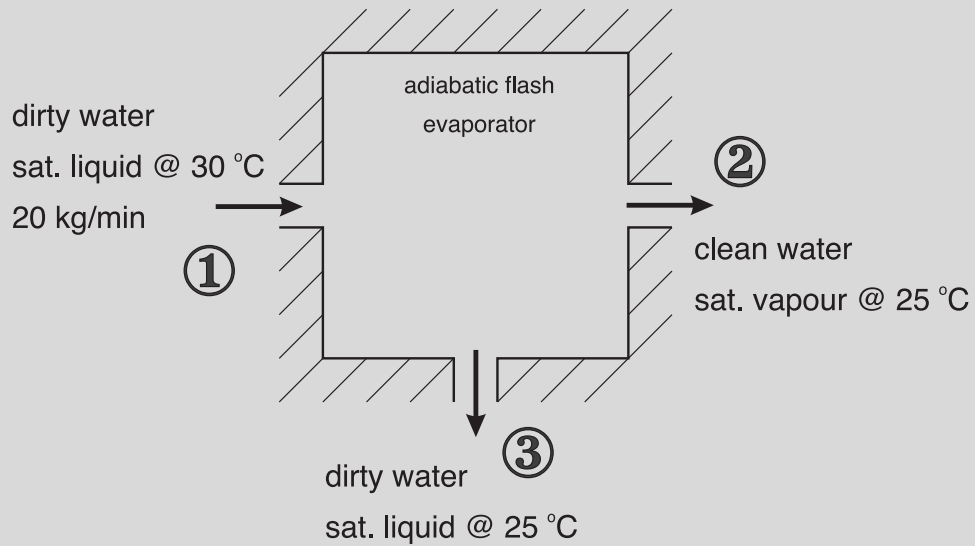
$$\dot{S}_{gen} = \left(\frac{dS}{dt} \right)_{CV} + \left(s\dot{m} + \frac{\dot{Q}}{T_{TER}} \right)_{OUT} - \left(s\dot{m} + \frac{\dot{Q}}{T_{TER}} \right)_{IN}$$

This can be thought of as

$$generation = accumulation + OUT - IN$$

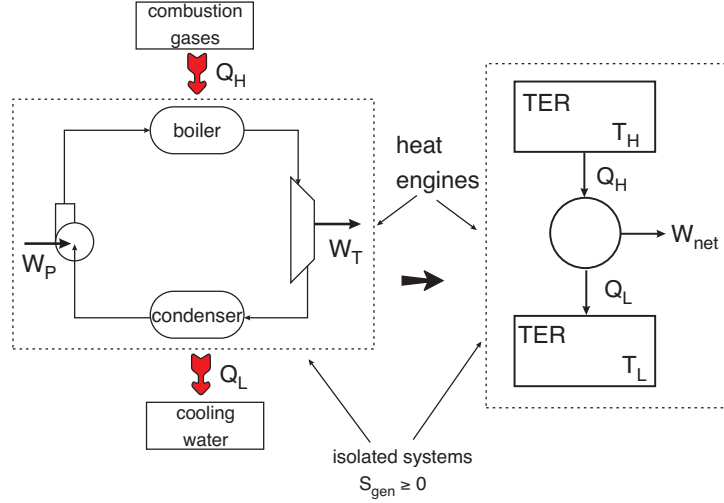
Example 4-2: An adiabatic flash evaporator is used to make a small amount of clean water from dirty water. The dirty water enters as saturated liquid at $30\text{ }^{\circ}\text{C}$ and at a rate of 20 kg/min . Clean water leaves as a saturated vapour at $25\text{ }^{\circ}\text{C}$. Dirty water leaves the evaporator as saturated liquid at $25\text{ }^{\circ}\text{C}$.

- Determine the mass flow rate [kg/min] of the clean water vapour.
- Determine the rate of entropy generation [kW/K] within the evaporator.



Entropy Generation in a System of Components: Heat Engine

A heat engine is a device in which a working substance (control mass) undergoes a cyclic process while operating between two temperature reservoirs (TER).



For the general case, the 1st law energy balance gives

$$\Delta E^0 = Q_H - Q_L - W_{net} = 0 \quad (1)$$

The 2nd law gives

$$S_{gen} \geq 0 \quad \text{for an isolated system}$$

where $S_{gen} = 0$ implies a reversible system and $S_{gen} > 0$ implies a real system.

An entropy balance gives

$$S_{gen} = \underbrace{(\Delta S)_{CM}}_{\equiv 0 \text{ (cyclic)}} + \underbrace{(\Delta S)_{TER-H}}_{\equiv -Q_H/T_H} + \underbrace{(\Delta S)_{TER-L}}_{\equiv +Q_L/T_L} + (\Delta S)_{ME}^0$$

Therefore

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} + \frac{S_{gen} T_L}{Q_H} \quad (2)$$

Combining Eqs. (1) and (2)

$$\begin{aligned} W_{net} &= Q_H - Q_L = Q_H \left(1 - \frac{Q_L}{Q_H} \right) \\ &= Q_H \left(1 - \frac{T_L}{T_H} - \frac{S_{gen} T_L}{Q_H} \right) \\ &= \underbrace{Q_H \left(1 - \frac{T_L}{T_H} \right)}_{W_{max} \text{ possible}} - \underbrace{T_L S_{gen}}_{W_{lost} \text{ due to irreversibilities}} \end{aligned}$$

The engine efficiency is defined as the benefit over the cost

$$\eta = \frac{\text{benefit}}{\text{cost}} = \frac{W_{net}}{Q_H} = 1 - \frac{T_L}{T_H} - \frac{T_L S_{gen}}{Q_H}$$

Conduction Heat Transfer



Reading

10-1 → 10-6

11-1 → 11-2

Problems

10-20, 10-35, 10-49, 10-54, 10-59, 10-69,
10-71, 10-92, 10-126, 10-143, 10-157, 10-162

11-14, 11-17, 11-36, 11-41, 11-46, 11-97, 11-104

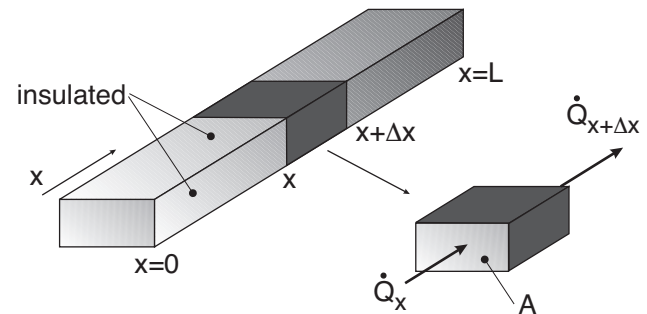
General Heat Conduction

From a 1st law energy balance:

$$\frac{\partial E}{\partial t} = \dot{Q}_x - \dot{Q}_{x+\Delta x}$$

If the volume to the element is given as $V = A \cdot \Delta x$, then the mass of the element is

$$m = \rho \cdot A \cdot \Delta x$$



The energy term ($KE = PE = 0$) is

$$E = m \cdot u = (\rho \cdot A \cdot \Delta x) \cdot u$$

For an incompressible substance the internal energy is $du = C dT$ and we can write

$$\frac{\partial E}{\partial t} = \rho C A \Delta x \frac{\partial T}{\partial t}$$

Heat flow along the x -direction is a product of the temperature difference.

$$\dot{Q}_x = \frac{kA}{\Delta x} (T_x - T_{x+\Delta x})$$

where k is the thermal conductivity of the material. In the limit as $\Delta x \rightarrow 0$

$$\dot{Q}_x = -kA \frac{\partial T}{\partial x}$$

This is *Fourier's law of heat conduction*. The $-ve$ in front of k guarantees that we adhere to the 2nd law and that heat always flows in the direction of lower temperature.

We can write the heat flow rate across the differential length, Δx as a truncated Taylor series expansion as follows

$$\dot{Q}_{x+\Delta x} = \dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x} \Delta x$$

when combined with Fourier's equation gives

$$\dot{Q}_{x+\Delta x} = \underbrace{-kA \frac{\partial T}{\partial x}}_{\dot{Q}_x} - \frac{\partial}{\partial x} \left(kA \frac{\partial T}{\partial x} \right) \Delta x$$

Noting that

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} = \frac{\partial E}{\partial t} = \rho C A \Delta x \frac{\partial T}{\partial t}$$

By removing the common factor of $A\Delta x$ we can then write the general 1-D conduction equation as

$$\underbrace{\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right)}_{\text{longitudinal conduction}} = \underbrace{\rho C \frac{\partial T}{\partial t}}_{\text{thermal inertia}}$$



Steady Conduction

- $\frac{\partial T}{\partial t} \rightarrow 0$
- properties are constant
- temperature varies in a linear manner
- heat flow rate defined by Fourier's equation
- resistance to heat flow: $R = \frac{\Delta T}{\dot{Q}}$

Transient Conduction

- properties are constant
- therefore $\frac{\partial^2 T}{\partial x^2} = \frac{\rho C}{k} \frac{\partial T}{\partial t} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$

where thermal diffusivity is defined as

$$\alpha = \frac{k}{\rho C}$$

- exact solution is complicated
- partial differential equation can be solved using approximate or graphical methods

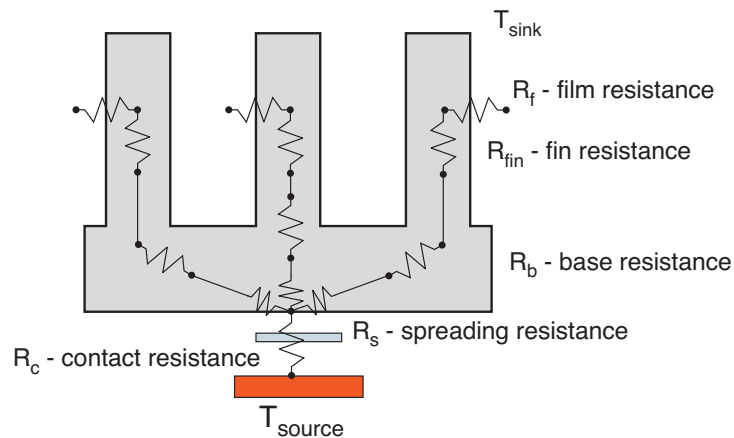
Steady Heat Conduction

Thermal Resistance Networks

Thermal circuits based on heat flow rate, \dot{Q} , temperature difference, ΔT and thermal resistance, R , enable analysis of complex systems.

Thermal Resistance

The thermal resistance to heat flow ($^{\circ}\text{C}/\text{W}$) can be constructed for all heat transfer mechanisms, including conduction, convection, and radiation as well as contact resistance and spreading resistance.



Conduction: $R_{\text{cond}} = \frac{L}{kA}$

Convection: $R_{\text{conv}} = \frac{1}{hA}$

Radiation: $R_{\text{rad}} = \frac{1}{h_{\text{rad}}A} \longrightarrow h_{\text{rad}} = \epsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$

Contact: $R_c = \frac{1}{h_c A} \longrightarrow h_c \text{ see Table 10-2}$

Cartesian Systems

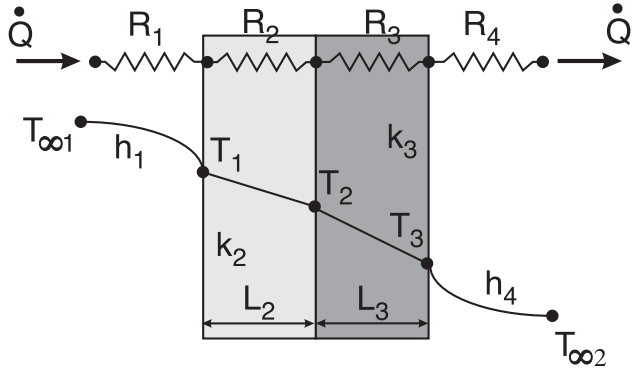
Resistances in Series

The heat transfer across the fluid/solid interface is based on *Newton's law of cooling*

$$\dot{Q} = hA(T_{\text{in}} - T_{\text{out}}) = \frac{T_{\text{in}} - T_{\text{out}}}{R_{\text{conv}}} \quad \text{where} \quad R_{\text{conv}} = \frac{1}{hA}$$

The heat flow through a solid material of conductivity, k is

$$\dot{Q} = \frac{kA}{L}(T_{in} - T_{out}) = \frac{T_{in} - T_{out}}{R_{cond}} \quad \text{where } R_{cond} = \frac{L}{kA}$$



By summing the temperature drop across each section, we can write:

$$\begin{aligned} \dot{Q} R_1 &= (T_{\infty 1} - T_1) \\ \dot{Q} R_2 &= (T_1 - T_2) \\ \dot{Q} R_3 &= (T_2 - T_3) \\ \dot{Q} R_4 &= (T_3 - T_{\infty 2}) \end{aligned}$$

$$\dot{Q} \left(\sum_{i=1}^4 R_i \right) = (T_{\infty 1} - T_{\infty 2})$$

The total heat flow across the system can be written as

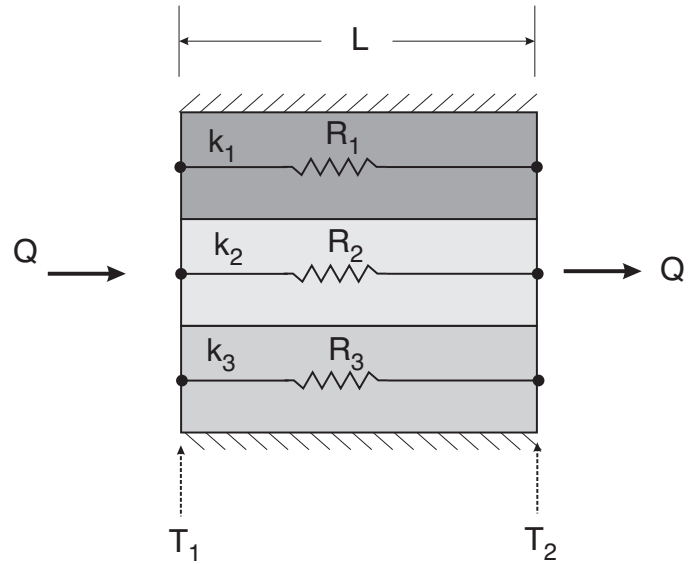
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \quad \text{where } R_{total} = \sum_{i=1}^4 R_i$$

Resistances in Parallel

For systems of parallel flow paths as shown above, we can use the 1st law to preserve the total energy

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

where we can write



$$\dot{Q}_1 = \frac{T_1 - T_2}{R_1}$$

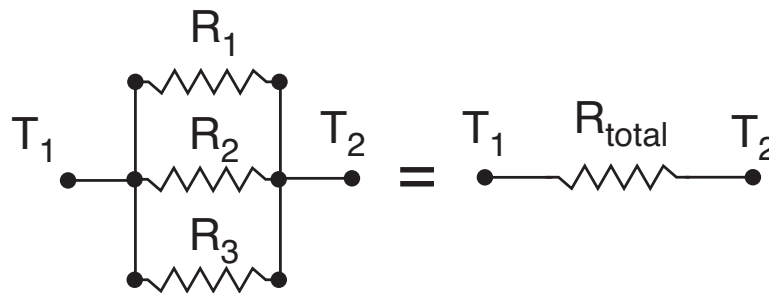
$$R_1 = \frac{L}{k_1 A_1}$$

$$\dot{Q}_2 = \frac{T_1 - T_2}{R_2}$$

$$R_2 = \frac{L}{k_2 A_2}$$

$$\dot{Q} = \sum \dot{Q}_i = (T_1 - T_2) \left(\sum \frac{1}{R_i} \right) \quad \text{where} \quad \frac{1}{R_{total}} = \sum \frac{1}{R_i} = UA$$

In general, for parallel networks we can use a parallel resistor network as follows:

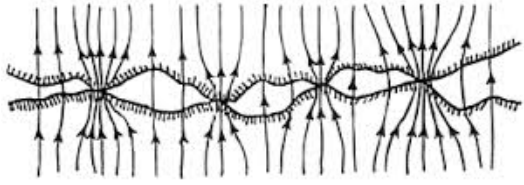


$$\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

and

$$\dot{Q} = \frac{T_1 - T_2}{R_{total}}$$

Thermal Contact Resistance



- real surfaces have microscopic roughness, leading to non-perfect contacts where

- 1 - 4% of the surface area is in solid-solid contact, the remainder consists of air gaps

- the total heat flow rate can be written as

$$\dot{Q}_{total} = h_c A \Delta T_{interface}$$

where:

h_c = thermal contact conductance

A = apparent or projected area of the contact

$\Delta T_{interface}$ = average temperature drop across the interface

The conductance, h_c and the contact resistance, R_c can be written as

$$h_c A = \frac{\dot{Q}_{total}}{\Delta T_{interface}} = \frac{1}{R_c}$$

Table 10-2 can be used to obtain some representative values for contact conductance

Table 10-2: Contact Conductances

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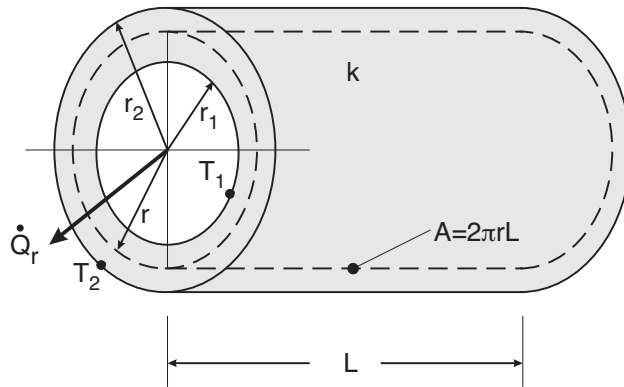
TABLE 10-2

Thermal contact conductance of some metal surfaces in air (from various sources)

Material	Surface condition	Roughness, μm	Temperature, $^{\circ}\text{C}$	Pressure, MPa	h_c^* $\text{W/m}^2 \cdot ^{\circ}\text{C}$
Identical Metal Pairs					
416 Stainless steel	Ground	2.54	90–200	0.17–2.5	3800
304 Stainless steel	Ground	1.14	20	4–7	1900
Aluminum	Ground	2.54	150	1.2–2.5	11,400
Copper	Ground	1.27	20	1.2–20	143,000
Copper	Milled	3.81	20	1–5	55,500
Copper (vacuum)	Milled	0.25	30	0.17–7	11,400
Dissimilar Metal Pairs					
Stainless steel–				10	2900
Aluminum		20–30	20	20	3600
Stainless steel–				10	16,400
Aluminum		1.0–2.0	20	20	20,800
Steel Ct-30–				10	50,000
Aluminum	Ground	1.4–2.0	20	15–35	59,000
Steel Ct-30–				10	4800
Aluminum	Milled	4.5–7.2	20	30	8300
				5	42,000
Aluminum-Copper	Ground	1.17–1.4	20	15	56,000
				10	12,000
Aluminum-Copper	Milled	4.4–4.5	20	20–35	22,000

*Divide the given values by 5.678 to convert to $\text{Btu/h} \cdot \text{ft}^2 \cdot ^{\circ}\text{F}$.

Cylindrical Systems

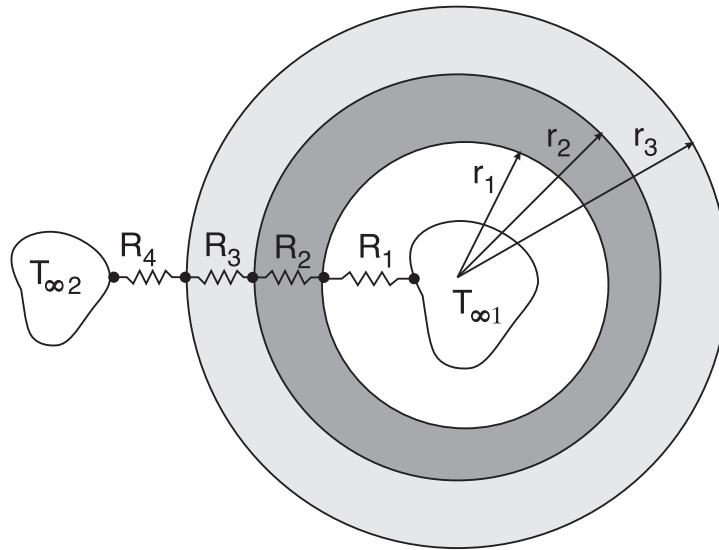


Steady, 1D heat flow from T_1 to T_2 in a cylindrical system occurs in a radial direction where the lines of constant temperature (isotherms) are concentric circles, as shown by the dotted line and $T = T(r)$.

Performing a 1st law energy balance on a *control mass* from the annular ring of the cylindrical cylinder gives:

$$\dot{Q}_r = \frac{T_1 - T_2}{\left(\frac{\ln(r_2/r_1)}{2\pi k \mathcal{L}} \right)} \quad \text{where } R = \left(\frac{\ln(r_2/r_1)}{2\pi k \mathcal{L}} \right)$$

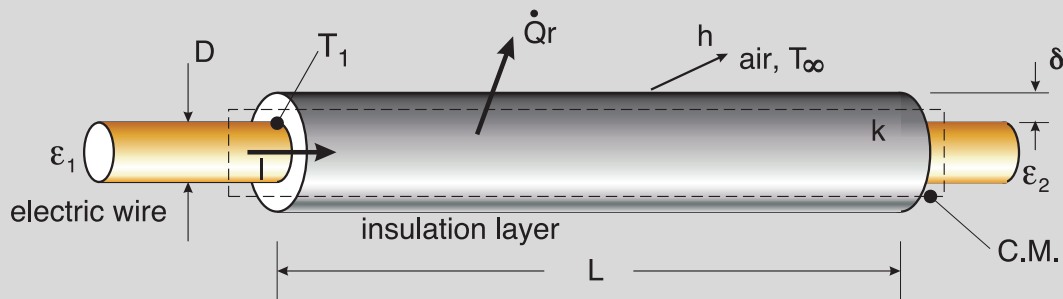
Composite Cylinders



Then the total resistance can be written as

$$\begin{aligned} R_{total} &= R_1 + R_2 + R_3 + R_4 \\ &= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi k_2 \mathcal{L}} + \frac{\ln(r_3/r_2)}{2\pi k_3 \mathcal{L}} + \frac{1}{h_4 A_4} \end{aligned}$$

Example 5-1: Determine the temperature (T_1) of an electric wire surrounded by a layer of plastic insulation with a thermal conductivity of 0.15 W/mK when the thickness of the insulation is a) 2 mm and b) 4 mm , subject to the following conditions:



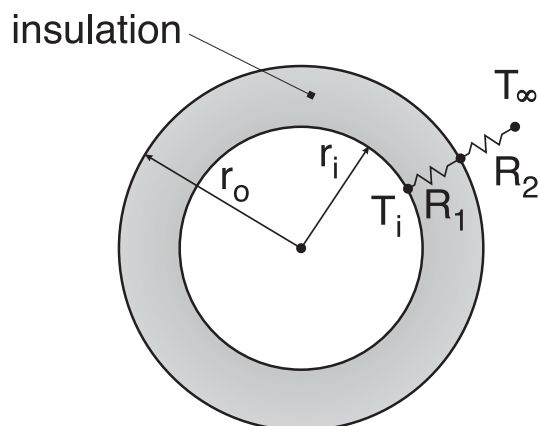
Given:

$$\begin{aligned} I &= 10 \text{ A} \\ \Delta\epsilon &= \epsilon_1 - \epsilon_2 = 8 \text{ V} \\ D &= 3 \text{ mm} \\ \mathcal{L} &= 5 \text{ m} \\ k &= 0.15 \text{ W/mK} \\ T_\infty &= 30^\circ \text{C} \\ h &= 12 \text{ W/m}^2 \cdot \text{K} \end{aligned}$$

Find:

$$\begin{aligned} T_1 &= ??? \\ \text{when:} \\ \delta &= 2 \text{ mm} \\ \delta &= 4 \text{ mm} \end{aligned}$$

Critical Radius of Insulation



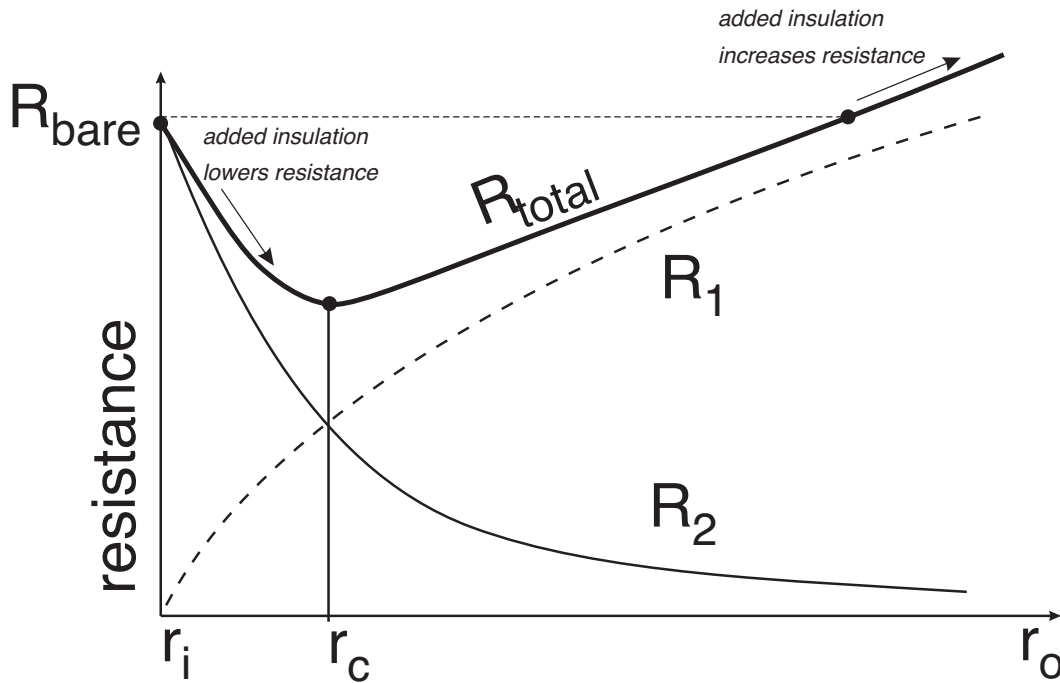
Consider a steady, 1-D problem where an insulation cladding is added to the outside of a tube with constant surface temperature T_i . What happens to the heat transfer as insulation is added, i.e. we increase the thickness of the insulation?

The resistor network can be written as a series combination of the resistance of the insulation, R_1 and the convective resistance, R_2

$$R_{total} = R_1 + R_2 = \frac{\ln(r_o/r_i)}{2\pi k\mathcal{L}} + \frac{1}{h2\pi r_o\mathcal{L}}$$

Could there be a situation in which adding insulation increases the overall heat transfer?

$$\frac{dR_{total}}{dr_o} = \frac{1}{2\pi k r_o \mathcal{L}} - \frac{1}{h 2\pi r_o^2 \mathcal{L}} = 0 \quad \Rightarrow \quad r_{cr,cyl} = \frac{k}{h} \quad [m]$$



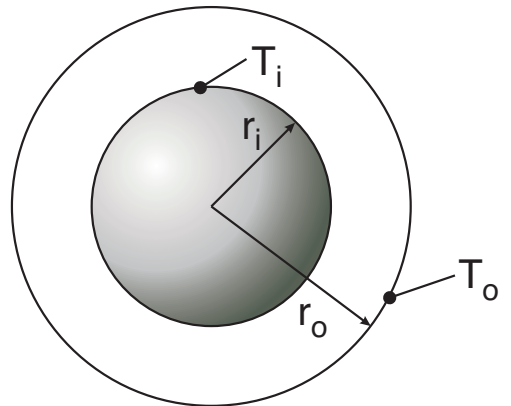
There is always a value of $r_{cr,cal}$, but there is a minimum in heat transfer only if $r_{cr,cal} > r_i$

Spherical Systems

For steady, 1D heat flow in spherical geometries we can write the heat transfer in the radial direction as

$$\dot{Q} = \frac{4\pi k r_i r_o}{(r_o - r_i)} (T_i - T_o) = \frac{(T_i - T_o)}{R}$$

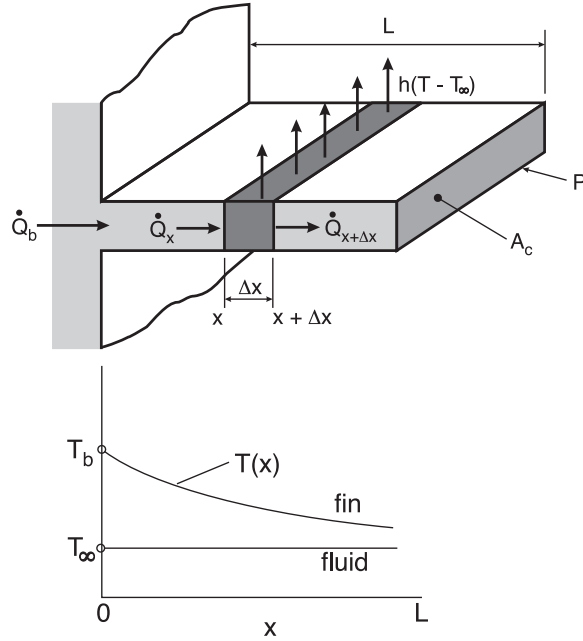
where: $R = \frac{r_o - r_i}{4\pi k r_i r_o}$



The critical radius of insulation for a spherical shell is given as

$$r_{cr,sphere} = \frac{2k}{h} \quad [m]$$

Heat Transfer from Finned Surfaces



We can establish a 1st law balance over the thin slice of the fin between x and $x + \Delta x$ such that

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} - \underbrace{P\Delta x}_{A_{surface}} h(T - T_\infty) = 0$$

From Fourier's law we know

$$\dot{Q}_x - \dot{Q}_{x+\Delta x} = kA_c \frac{d^2 T}{dx^2} \Delta x$$

Therefore the conduction equation for a fin with constant cross section is

$$\underbrace{kA_c \frac{d^2 T}{dx^2}}_{\text{longitudinal conduction}} - \underbrace{hP(T - T_\infty)}_{\text{lateral convection}} = 0$$

Let the temperature difference between the fin and the surroundings (temperature excess) be $\theta = T(x) - T_\infty$ which allows the 1-D fin equation to be written as

$$\frac{d^2 \theta}{dx^2} - m^2 \theta = 0 \quad \text{where} \quad m = \left(\frac{hP}{kA_c} \right)^{1/2}$$

The solution to the differential equation for θ is

$$\theta(x) = C_1 \sinh(mx) + C_2 \cosh(mx) \quad [\equiv \theta(x) = C_1 e^{mx} + C_2 e^{-mx}]$$

Potential boundary conditions include:

Base: $\rightarrow @x = 0 \quad \theta = \theta_b$

Tip: $\rightarrow @x = L \quad \theta = \theta_L$ [T-specified tip]

$$\theta = \left. \frac{d\theta}{dx} \right|_{x=L} = 0 \quad \text{[adiabatic (insulated) tip]}$$

$$\theta \rightarrow 0 \quad \text{[infinitely long fin]}$$

Substituting the boundary conditions to find the constants of integration, the temperature distribution and fin heat transfer rate can be determined as follows:

Case 1: Prescribed temperature ($\theta @ x=L = \theta_L$)

$$\frac{\theta(x)}{\theta_b} = \frac{(\theta_L/\theta_b) \sinh mx + \sinh m(L-x)}{\sinh mL}$$

$$\dot{Q}_b = M \frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$$

Case 2: Adiabatic tip $\left(\frac{d\theta}{dx}\Big|_{x=L} = 0\right)$

$$\frac{\theta(x)}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL} \quad \dot{Q}_b = M \tanh mL$$

Case 3: Infinitely long fin ($\theta \rightarrow 0$)

$$\frac{\theta(x)}{\theta_b} = e^{-mx} \quad \dot{Q}_b = M$$

where

$$m = \sqrt{hP/(kA_c)}$$

$$M = \sqrt{hPkA_c} \theta_b$$

$$\theta_b = T_b - T_\infty$$

Fin Efficiency and Effectiveness

The dimensionless parameter that compares the actual heat transfer from the fin to the ideal heat transfer from the fin is the *fin efficiency*

$$\eta = \frac{\text{actual heat transfer rate}}{\text{maximum heat transfer rate when the entire fin is at } T_b} = \frac{\dot{Q}_b}{hPL\theta_b}$$

If the fin has a constant cross section then

$$\eta = \frac{\tanh(mL)}{mL}$$

An alternative figure of merit is the *fin effectiveness* given as

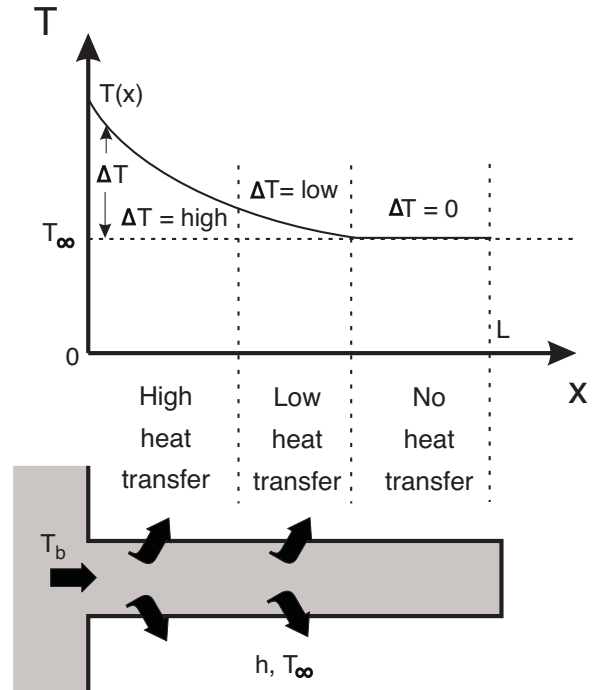
$$\epsilon_{fin} = \frac{\text{total fin heat transfer}}{\text{the heat transfer that would have occurred through the base area in the absence of the fin}} = \frac{\dot{Q}_b}{hA_c\theta_b}$$

How to Determine the Appropriate Fin Length

- theoretically an infinitely long fin will dissipate the most heat
- but practically, an extra long fin is inefficient given the exponential temperature decay over the length of the fin
- so what is a realistic fin length in order to optimize performance and cost

If we determine the ratio of heat flow for a fin with an insulated tip (Case 2) versus an infinitely long fin (Case 3) we can assess the relative performance of a conventional fin

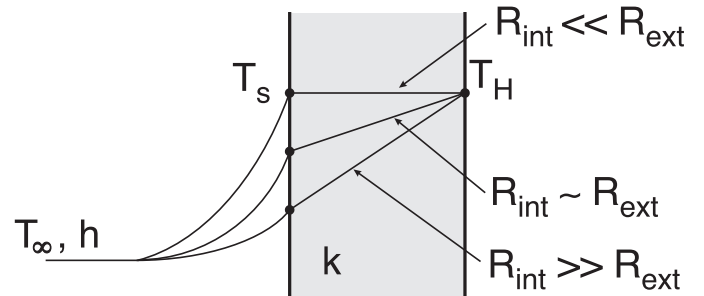
$$\frac{\dot{Q}_{Case\ 2}}{\dot{Q}_{Case\ 3}} = \frac{M \tanh mL}{M} = \tanh mL$$



Transient Heat Conduction

Performing a 1st law energy balance on a plane wall gives

$$\begin{aligned} \dot{Q}_{cond} &= \frac{T_H - T_s}{L/(k \cdot A)} \\ &= \dot{Q}_{conv} = \frac{T_s - T_\infty}{1/(h \cdot A)} \end{aligned}$$



where the Biot number can be obtained as follows:

$$\frac{T_H - T_s}{T_s - T_\infty} = \frac{L/(k \cdot A)}{1/(h \cdot A)} = \frac{\text{internal resistance to H.T.}}{\text{external resistance to H.T.}} = \frac{hL}{k} = Bi$$

$R_{int} \ll R_{ext}$: the Biot number is small and we can conclude

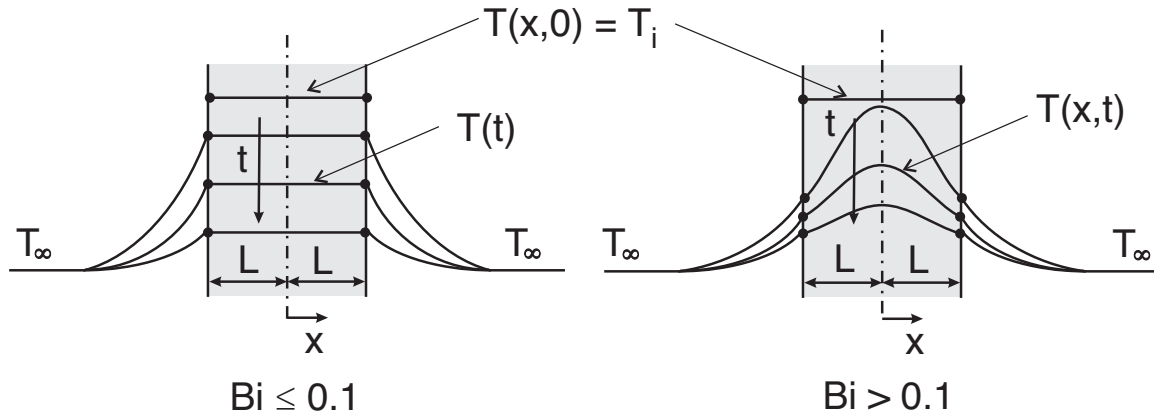
$$T_H - T_s \ll T_s - T_\infty \quad \text{and in the limit } T_H \approx T_s$$

$R_{ext} \ll R_{int}$: the Biot number is large and we can conclude

$$T_s - T_\infty \ll T_H - T_s \quad \text{and in the limit } T_s \approx T_\infty$$

Transient Conduction Analysis

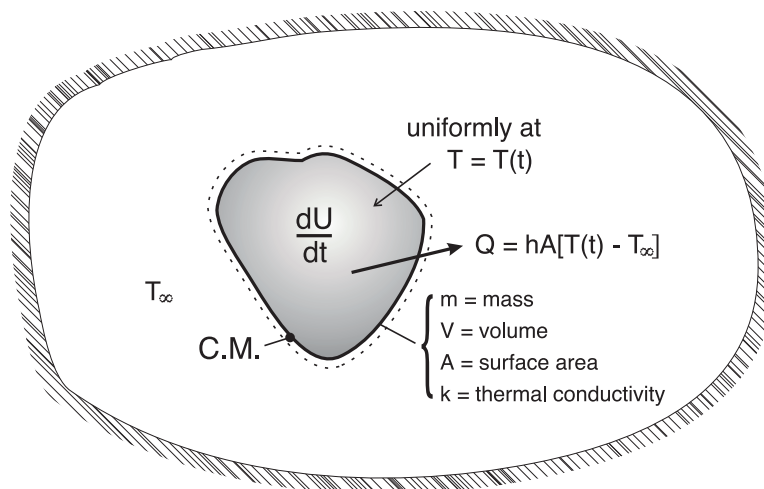
- if the internal temperature of a body remains relatively constant with respect to time
 - can be treated as a lumped system analysis
 - heat transfer is a function of time only, $T = T(t)$



$Bi \leq 0.1$: temperature profile is not a function of position
 temperature profile only changes with respect to time $\rightarrow T = T(t)$
 use lumped system analysis

$Bi > 0.1$: temperature profile changes with respect to time and position $\rightarrow T = T(x, t)$
 use approximate analytical or graphical solutions (Heisler charts)

Lumped System Analysis



At $t > 0$, $T = T(x, y, z, t)$, however, when $Bi \leq 0.1$ then we can assume $T \approx T(t)$.

Performing a 1st law energy balance on the control volume shown below

$$\frac{dE_{C.M.}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \rightarrow 0$$

If we assume PE and KE to be negligible then

$$\frac{dU}{dt} = -\dot{Q} \quad \Leftrightarrow \quad \frac{dU}{dt} < 0 \text{ implies } U \text{ is decreasing}$$

For an incompressible substance specific heat is constant and we can write

$$\underbrace{mC}_{\equiv C_{th}} \frac{dT}{dt} = - \underbrace{Ah}_{1/R_{th}} (T - T_{\infty})$$

where C_{th} = lumped capacitance

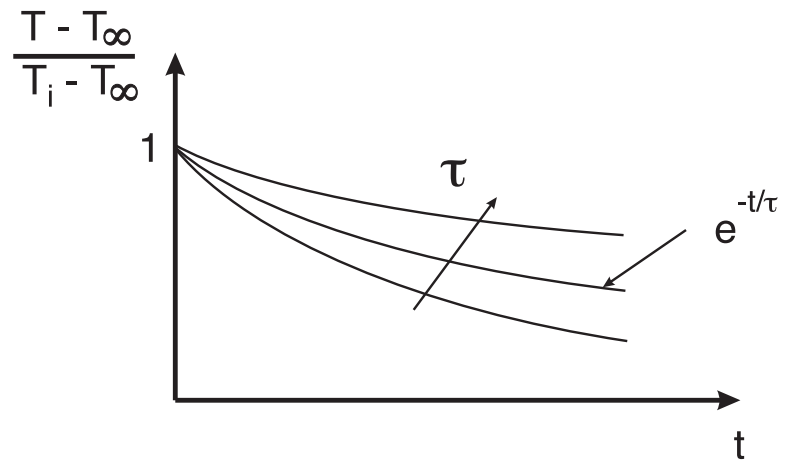
$$C_{th} \frac{dT}{dt} = -\frac{1}{R_{th}} (T - T_{\infty})$$

We can integrate and apply the initial condition, $T = T_i$ @ $t = 0$ to obtain

$$\frac{T(t) - T_{\infty}}{T_i - T_{\infty}} = e^{-t/(R_{th} \cdot C_{th})} = e^{-t/\tau} = e^{-bt}$$

where

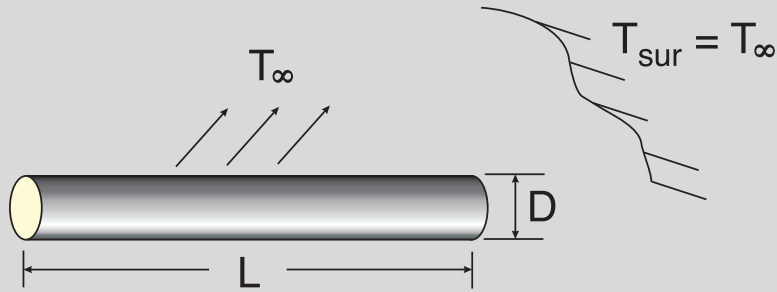
$$\begin{aligned} \frac{1}{b} &= \tau \\ &= R_{th} \cdot C_{th} \\ &= \text{thermal time constant} \\ &= \frac{mC}{Ah} = \frac{\rho VC}{Ah} \end{aligned}$$



The total heat transferred over the time period $0 \rightarrow t^*$ is

$$Q_{total} = mC(T_i - T_{\infty})[1 - e^{-t^*/\tau}]$$

Example 5-2: Determine the time it takes a fuse to melt if a current of 3 A suddenly flows through the fuse subject to the following conditions:



Given:

$$D = 0.1\text{ mm}$$

$$T_{\text{melt}} = 900\text{ }^{\circ}\text{C}$$

$$k = 20\text{ W/mK}$$

$$L = 10\text{ mm}$$

$$T_{\infty} = 30\text{ }^{\circ}\text{C}$$

$$\alpha = 5 \times 10^{-5}\text{ m}^2/\text{s} \equiv k/\rho C_p$$

Assume:

- constant resistance $\mathcal{R} = 0.2\text{ ohms}$
- the overall heat transfer coefficient is $h = h_{\text{conv}} + h_{\text{rad}} = 10\text{ W/m}^2\text{K}$
- neglect any conduction losses to the fuse support

Approximate Analytical and Graphical Solutions (Heisler Charts)

If $Bi > 0.1$

- need to solve the partial differential equation for temperature
- leads to an infinite series solution \Rightarrow difficult to obtain a solution (see pp. 481 - 483 for exact solution by separation of variables)

We must find a solution to the PDE

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \Rightarrow \quad \frac{T(x, t) - T_\infty}{T_i - T_\infty} = \sum_{n=1,3,5\dots}^{\infty} A_n e^{\left(-\frac{\lambda_n}{L}\right)^2 \alpha t} \cos\left(\frac{\lambda_n x}{L}\right)$$

By using dimensionless groups, we can reduce the temperature dependence to 3 dimensionless parameters

Dimensionless Group	Formulation
temperature	$\theta(x, t) = \frac{T(x, t) - T_\infty}{T_i - T_\infty}$
position	$X = x/L$
heat transfer	$Bi = hL/k$ Biot number
time	$Fo = \alpha t/L^2$ Fourier number

note: Cengel uses τ instead of Fo .

Now we can write

$$\theta(x, t) = f(X, Bi, Fo)$$

The characteristic length for the Biot number is

slab	$\mathcal{L} = L$
cylinder	$\mathcal{L} = r_o$
sphere	$\mathcal{L} = r_o$

contrast this versus the characteristic length for the lumped system analysis.

With this, two approaches are possible

1. use the first term of the infinite series solution. This method is only valid for $Fo > 0.2$
2. use the Heisler charts for each geometry as shown in Figs. 11-15, 11-16 and 11-17

First term solution: $Fo > 0.2 \rightarrow$ error about 2% max.

Plane Wall:
$$\theta_{wall}(x, t) = \frac{T(x, t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 Fo} \cos(\lambda_1 x / L)$$

Cylinder:
$$\theta_{cyl}(r, t) = \frac{T(r, t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 Fo} J_0(\lambda_1 r / r_o)$$

Sphere:
$$\theta_{sph}(r, t) = \frac{T(r, t) - T_{\infty}}{T_i - T_{\infty}} = A_1 e^{-\lambda_1^2 Fo} \frac{\sin(\lambda_1 r / r_o)}{\lambda_1 r / r_o}$$

λ_1, A_1 can be determined from Table 11-2 based on the calculated value of the Biot number (will likely require some interpolation). The Bessel function, J_0 can be calculated using Table 11-3.

Using Heisler Charts

- find T_0 at the center for a given time (Table 11-15 a, Table 11-16 a or Table 11-17 a)
- find T at other locations at the same time (Table 11-15 b, Table 11-16 b or Table 11-17 b)
- find Q_{tot} up to time t (Table 11-15 c, Table 11-16 c or Table 11-17 c)

Example 5-3: An aluminum plate made of Al 2024-T6 with a thickness of 0.15 m is initially at a temperature of 300 K . It is then placed in a furnace at 800 K with a convection coefficient of $500\text{ W/m}^2\text{ K}$.

- Find:
- i) the time (s) for the plate midplane to reach 700 K
 - ii) the surface temperature at this condition. Use both the Heisler charts and the approximate analytical, first term solution.

Convection Heat Transfer



Reading

12-1 → 12-8

13-1 → 13-6

14-1 → 14-4

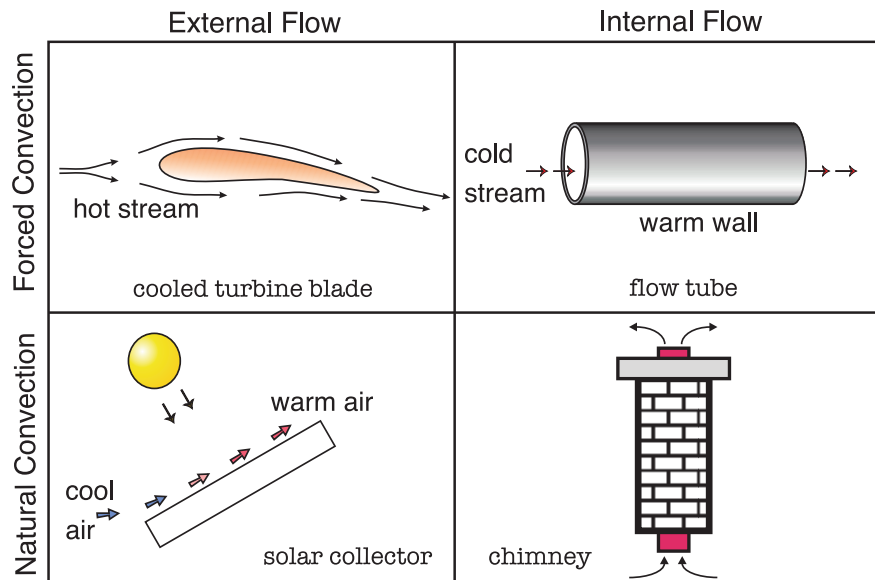
Problems

12-40, 12-49, 12-68, 12-70, 12-87, 12-98

13-39, 13-47, 13-59

14-18, 14-24, 14-45, 14-82

Introduction



Newton's Law of Cooling

$$\dot{Q}_{conv} = \frac{\Delta T}{R_{conv}} = hA(T_w - T_\infty)$$

$$\Rightarrow R_{conv} = \frac{1}{hA}$$

Typical Values of h ($W/m^2 K$)

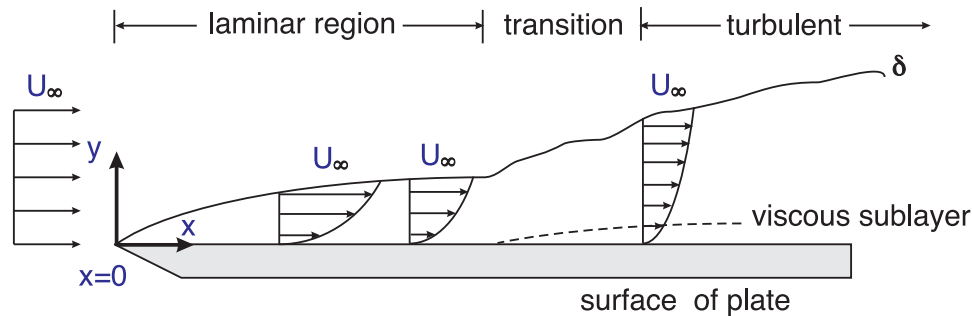
Natural Convection	gases: 3-20 water: 60 - 900
Forced Convection	gases: 30 - 300 oils: 60 - 1800 water: 100 -1500
Boiling	water: $3000 - 10^5$
Condensation	steam: $3000 - 10^5$

Controlling Factors

Geometry: shape, size, aspect ratio and orientation
Flow Type: forced, natural, laminar, turbulent, internal, external
Boundary: isothermal ($T_w = \text{constant}$) or isoflux ($\dot{q}_w = \text{constant}$)
Fluid Type: viscous oil, water, gases or liquid metals
Properties: all properties determined at film temperature
 $T_f = (T_w + T_\infty)/2$
 Note: ρ and $\nu \propto 1/P_{atm} \Rightarrow$ see Q12-40
 density: ρ (kg/m^3)
 specific heat: C_p ($J/kg \cdot K$)
 dynamic viscosity: μ , ($N \cdot s/m^2$)
 kinematic viscosity: $\nu \equiv \mu/\rho$ (m^2/s)
 thermal conductivity: k , ($W/m \cdot K$)
 thermal diffusivity: α , $\equiv k/(\rho \cdot C_p)$ (m^2/s)
 Prandtl number: Pr , $\equiv \nu/\alpha$ (—)
 volumetric compressibility: β , ($1/K$)

Forced Convection

The simplest forced convection configuration to consider is the flow of mass and heat near a flat plate as shown below.

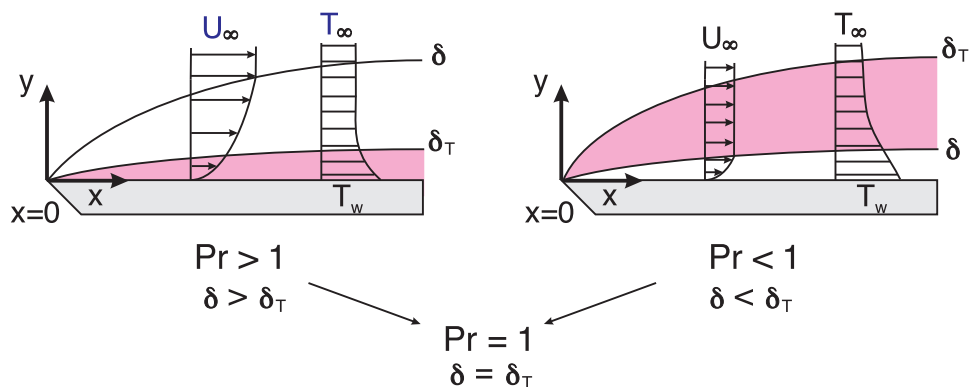


- flow forms thin layers that can slip past one another at different velocities
- as Reynolds number increases the flow has a tendency to become more chaotic resulting in disordered motion known as turbulent flow
 - transition from laminar to turbulent is called the critical Reynolds number, Re_{cr}

$$Re_{cr} = \frac{U_\infty x_{cr}}{\nu}$$

- for flow over a flat plate $Re_{cr} \approx 500,000$
- $x < x_{cr}$ the boundary layer is laminar; $x > x_{cr}$ the boundary layer is turbulent

Boundary Layers



Velocity Boundary Layer

- the region of fluid flow over the plate where viscous effects dominate is called the *velocity* or *hydrodynamic* boundary layer
- the velocity at the surface of the plate, $y = 0$, is set to zero, $U_{@y=0} = 0 \text{ m/s}$ because of the *no slip condition* at the wall

- the velocity of the fluid progressively increases away from the wall until we reach approximately $0.99 U_\infty$ which is denoted as the δ , the *velocity boundary layer thickness*.
- the region beyond the velocity boundary layer is denoted as the *inviscid flow* region, where frictional effects are negligible and the velocity remains relatively constant at U_∞

Thermal Boundary Layer

- the thermal boundary layer is arbitrarily selected as the locus of points where

$$\frac{T - T_w}{T_\infty - T_w} = 0.99$$

- for *low Prandtl number* fluids the velocity boundary layer is fully contained within the thermal boundary layer
- conversely, for *high Prandtl number* fluids the thermal boundary layer is contained within the velocity boundary layer

Flow Over Plates

1. Laminar Boundary Layer Flow, Isothermal (UWT)

All laminar formulations for $Re < 500,000$. The local value of the Nusselt number is given as

$$\boxed{Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}} \Rightarrow \text{local, laminar, UWT, } Pr \geq 0.6$$

An average value of the heat transfer coefficient for the full extent of the plate can be obtained by using the mean value theorem.

$$\boxed{Nu_L = \frac{h_L L}{k_f} = 0.664 Re_L^{1/2} Pr^{1/3}} \Rightarrow \text{average, laminar, UWT, } Pr \geq 0.6$$

For low Prandtl numbers, i.e. liquid metals

$$\boxed{Nu_x = 0.565 Re_x^{1/2} Pr^{1/2}} \Rightarrow \text{local, laminar, UWT, } Pr \leq 0.6$$

2. Turbulent Boundary Layer Flow, Isothermal (UWT)

All turbulent formulations for $500,000 \leq Re \leq 10^7$. The local Nusselt number is given as

$$\boxed{Nu_x = 0.0296 Re_x^{0.8} Pr^{1/3}} \Rightarrow \text{local, turbulent, UWT, } 0.6 < Pr < 60$$

and the average Nusselt number is

$$\boxed{Nu_L = 0.037 Re_L^{0.8} Pr^{1/3}} \Rightarrow \text{average, turbulent, UWT, } 0.6 < Pr < 60$$

3. Combined Laminar and Turbulent Boundary Layer Flow, Isothermal (UWT)

When $(T_w - T_\infty)$ is constant

$$h_L = \frac{1}{L} \int_0^L h dx = \frac{1}{L} \left\{ \int_0^{x_{cr}} h_x^{lam} dx + \int_{x_{cr}}^L h_x^{tur} dx \right\}$$

$$\boxed{Nu_L = \frac{h_L L}{k} = (0.037 Re_L^{0.8} - 871) Pr^{1/3}} \Rightarrow \text{average, combined, UWT, } 0.6 < Pr < 60, 500,000 \leq Re_L \leq 10^7$$

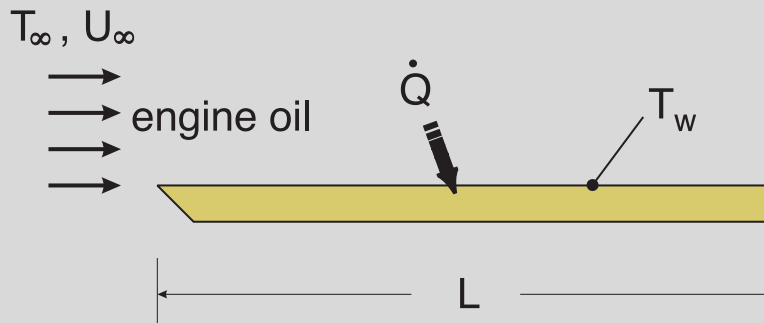
4. Laminar Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.453 Re_x^{1/2} Pr^{1/3}} \Rightarrow \text{local, laminar, UWF, } Pr \geq 0.6$$

5. Turbulent Boundary Layer Flow, Isoflux (UWF)

$$\boxed{Nu_x = 0.0308 Re_x^{4/5} Pr^{1/3}} \Rightarrow \text{local, turbulent, UWF, } Pr \geq 0.6$$

Example 6-1: Hot engine oil with a bulk temperature of 60°C flows over a horizontal, flat plate 5 m long with a wall temperature of 20°C . If the fluid has a free stream velocity of 2 m/s , determine the heat transfer rate from the oil to the plate if the plate is assumed to be of unit width.



Flow Over Cylinders and Spheres

1. Boundary Layer Flow Over Circular Cylinders, Isothermal (UWT)

The Churchill-Bernstein (1977) correlation for the average Nusselt number for long ($L/D > 100$) cylinders is

$$Nu_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282,000} \right)^{5/8} \right]^{4/5}$$

\Rightarrow average, UWT, $Re_D < 10^7$, $0 \leq Pr \leq \infty$, $Re_D \cdot Pr > 0.2$

All fluid properties are evaluated at $T_f = (T_w + T_\infty)/2$.

2. Boundary Layer Flow Over Non-Circular Cylinders, Isothermal (UWT)

The empirical formulations of Zhukauskas and Jakob given in Table 12-3 are commonly used, where

$$Nu_D \approx \frac{\bar{h}D}{k} = C Re_D^m Pr^{1/3} \Rightarrow \text{see Table 12-3 for conditions}$$

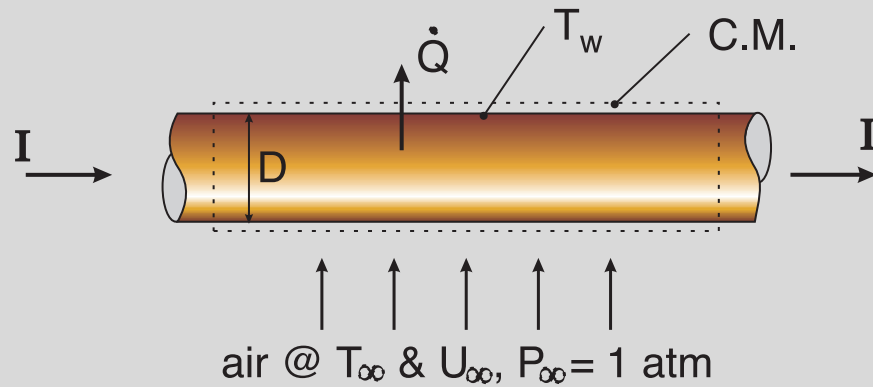
3. Boundary Layer Flow Over a Sphere, Isothermal (UWT)

For flow over an isothermal sphere of diameter D , Whitaker recommends

$$Nu_D = 2 + [0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s} \right)^{1/4} \Rightarrow \begin{array}{l} \text{average, UWT,} \\ 0.7 \leq Pr \leq 380 \\ 3.5 < Re_D < 80,000 \end{array}$$

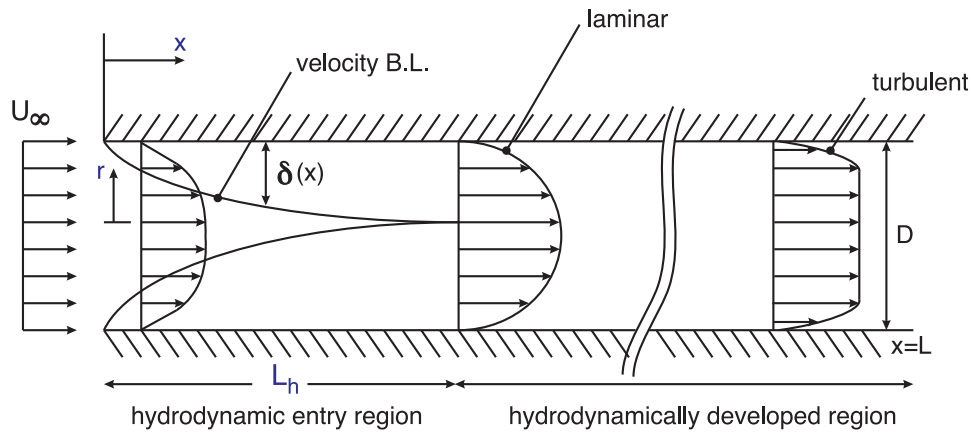
where the dynamic viscosity of the fluid in the bulk flow, μ_∞ is based on the free stream temperature, T_∞ and the dynamic viscosity of the fluid at the surface, μ_s , is based on the surface temperature, T_s . All other properties are based on T_∞ .

Example 6-2: An electric wire with a 1 mm diameter and a wall temperature of 325 K is cooled by air in cross flow with a free stream temperature of 275 K. Determine the air velocity required to maintain a steady heat loss per unit length of 70 W/m.



Internal Flow

Lets consider fluid flow in a duct bounded by a wall that is at a different temperature than the fluid. For simplicity we will examine a round tube of diameter D as shown below



The Reynolds number is given as: $Re_D = \frac{U_m D}{\nu}$. For flow in a tube:

$Re_D < 2300$ laminar flow

$2300 < Re_D < 4000$ transition to turbulent flow

$Re_D > 4000$ turbulent flow

For engineering calculations, we typically assume that $Re_{cr} \approx 2300$, therefore

$$Re_D \begin{cases} < Re_{cr} & \text{laminar} \\ > Re_{cr} & \text{turbulent} \end{cases}$$

Hydrodynamic (Velocity) Boundary Layer

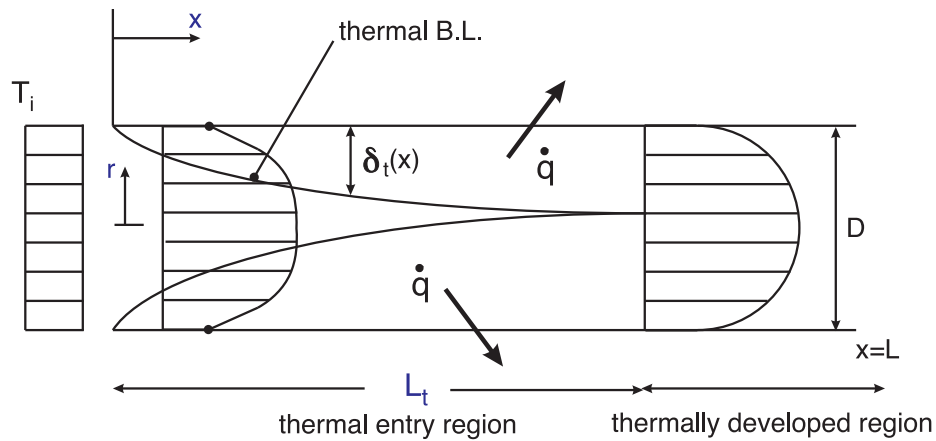
- when the boundary layer grows to the tube radius, r , the boundary layers merge
 - this flow length is called the flow entrance length, L_h
 - $0 \leq x \leq L_h$ is the hydrodynamic entrance region
 - $L_h < x \leq L$ is the fully developed region or hydrodynamically developed region
- the hydrodynamic boundary layer thickness can be approximated as

$$\delta(x) \approx 5x \left(\frac{U_m x}{\nu} \right)^{-1/2} = \frac{5x}{\sqrt{Re_x}}$$

- the hydrodynamic entry length can be approximated as

$$L_h \approx 0.05 Re_D D \quad (\text{laminar flow})$$

Thermal Boundary Layer



- a thermal entrance region develops from $0 \leq x \leq L_t$
- the thermal entry length can be approximated as

$$L_t \approx 0.05 Re_D Pr D = Pr L_h \quad (\text{laminar flow})$$

- for turbulent flow $L_h \approx L_t \approx 10D$

Wall Boundary Conditions

- 1. Uniform Wall Heat Flux:** The total heat transfer from the wall to the fluid stream can be determined by performing an energy balance over the tube. If we assume steady flow conditions, $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$ then the energy balance becomes

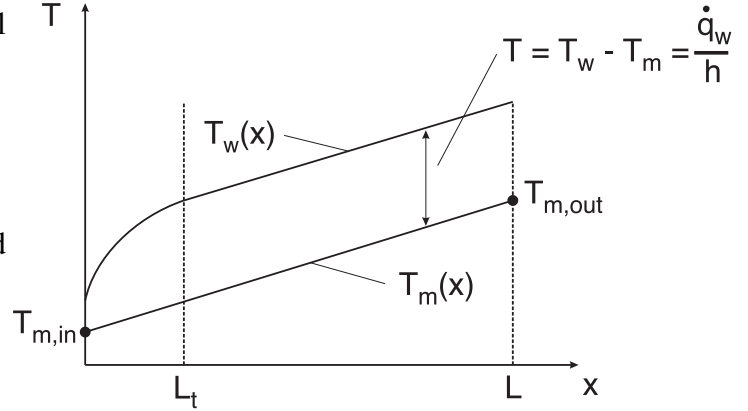
$$\dot{Q} = \dot{q}_w A = \dot{m}(h_{out} - h_{in}) = \dot{m}C_p(T_{out} - T_{in})$$

Since the wall flux \dot{q}_w is uniform, the local mean temperature is linear with x .

$$T_{m,x} = T_{m,i} + \frac{\dot{q}_w A}{\dot{m}C_p} x$$

The surface temperature can be determined from

$$T_w = T_m + \frac{\dot{q}_w}{h}$$



- 2. Isothermal Wall:** Using Newton's law of cooling we can determine the average rate of heat transfer to or from a fluid flowing in a tube

$$\dot{Q} = hA \underbrace{(T_w - T_m)}_{\text{average } \Delta T}$$

From an energy balance over a control volume in the fluid, we can determine

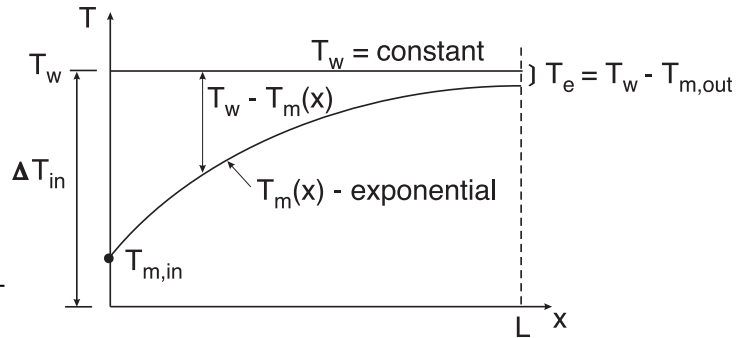
$$\dot{Q} = \dot{m}C_p dT_m$$

Equating the two equations above we find

$$\dot{m}C_p dT_m = hA \underbrace{(T_w - T_m)}_{\text{average } \Delta T}$$

By isolating the temperature terms and integrating we obtain

$$\ln \left(\frac{T_w - T_{out}}{T_w - T_{in}} \right) = - \frac{hA}{\dot{m}C_p}$$



Because of the exponential temperature decay within the tube, it is common to present the mean temperature from inlet to outlet as a log mean temperature difference where

$$\Delta T_{ln} = \frac{T_{out} - T_{in}}{\ln \left(\frac{T_w - T_{out}}{T_w - T_{in}} \right)} = \frac{T_{out} - T_{in}}{\ln(\Delta T_{out}/\Delta T_{in})} \Rightarrow \dot{Q} = hA \Delta T_{ln}$$

1. Laminar Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For laminar flow where $Re_D \leq 2300$

$$\boxed{Nu_D = 3.66} \Rightarrow \text{fully developed, laminar, UWT, } L > L_t \text{ \& } L_h$$

$$\boxed{Nu_D = 4.36} \Rightarrow \text{fully developed, laminar, UWF, } L > L_t \text{ \& } L_h$$

$$\boxed{Nu_D = 1.86 \left(\frac{Re_D Pr D}{L} \right)^{1/3} \left(\frac{\mu_b}{\mu_s} \right)^{0.14}} \Rightarrow \begin{array}{l} \text{developing laminar flow, UWT,} \\ Pr > 0.5 \\ L < L_h \text{ or } L < L_t \end{array}$$

For non-circular tubes the hydraulic diameter, $D_h = 4A_c/P$ can be used in conjunction with Table 13-1 to determine the Reynolds number and in turn the Nusselt number.

2. Turbulent Flow in Circular Tubes, Isothermal (UWT) and Isoflux (UWF)

For turbulent flow where $Re_D \geq 2300$ the Dittus-Boelter equation (Eq. 13-68) can be used

$$\begin{array}{l} \text{turbulent flow, UWT or UWF,} \\ 0.7 \leq Pr \leq 160 \\ Re_D > 2,300 \\ n = 0.4 \text{ heating} \\ \boxed{Nu_D = 0.023 Re_D^{0.8} Pr^n} \Rightarrow n = 0.3 \text{ cooling} \end{array}$$

For non-circular tubes, again we can use the hydraulic diameter, $D_h = 4A_c/P$ to determine both the Reynolds and the Nusselt numbers.

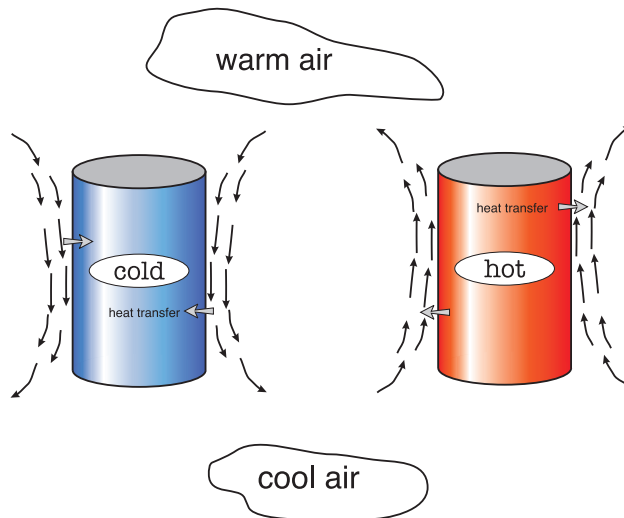
In all cases the fluid properties are evaluated at the mean fluid temperature given as

$$T_{mean} = \frac{1}{2} (T_{m,in} + T_{m,out})$$

except for μ_s which is evaluated at the wall temperature, T_s .

Natural Convection

What Drives Natural Convection?



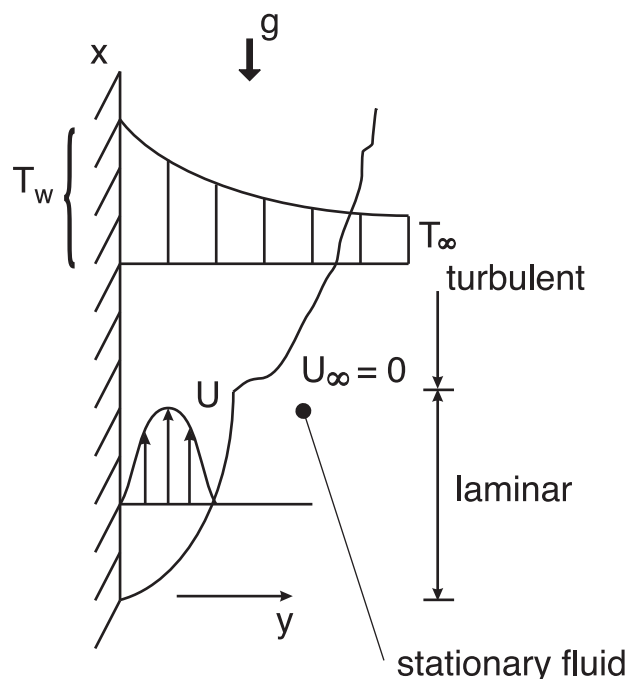
- fluids tend to expand when heated and contract when cooled at constant pressure
- therefore a fluid layer adjacent to a surface will become lighter if heated and heavier if cooled by the surface
- a lighter fluid will flow upward and a cooler fluid will flow downward
- as the fluid sweeps the wall, heat transfer will occur in a similar manner to boundary layer flow however in this case the bulk fluid is stationary as opposed to moving at a constant velocity in the case of forced convection

In natural convection, the *Grashof number* is analogous to the Reynolds number.

$$Gr = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$$

Natural Convection Over Surfaces

- natural convection heat transfer depends on geometry and orientation
- note that unlike forced convection, the velocity at the edge of the boundary layer goes to zero
- the velocity and temperature profiles within a boundary layer formed on a vertical plate in a stationary fluid looks as follows:



Natural Convection Heat Transfer Correlations

The general form of the Nusselt number for natural convection is as follows:

$$Nu = f(Gr, Pr) \equiv C Gr^m Pr^n \quad \text{where } Ra = Gr \cdot Pr$$

- C depends on geometry, orientation, type of flow, boundary conditions and choice of characteristic length.
- m depends on type of flow (laminar or turbulent)
- n depends on the type of fluid and type of flow
- Table 14-1 should be used to find Nusselt number for various combinations of geometry and boundary conditions
 - for ideal gases $\beta = 1/T_f$, $(1/K)$
 - all fluid properties are evaluated at the film temperature, $T_f = (T_w + T_\infty)/2$.

1. Laminar Flow Over a Vertical Plate, Isothermal (UWT)

The general form of the Nusselt number is given as

$$Nu_{\mathcal{L}} = \frac{h\mathcal{L}}{k_f} = C \left(\frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\nu^2} \right)^{1/4} \left(\frac{\nu}{\alpha} \right)^{1/4} = C \underbrace{Gr_{\mathcal{L}}^{1/4} Pr^{1/4}}_{Ra^{1/4}}$$

where

$$Ra_{\mathcal{L}} = Gr_{\mathcal{L}} Pr = \frac{g\beta(T_w - T_\infty)\mathcal{L}^3}{\alpha\nu}$$

2. Laminar Flow Over a Long Horizontal Circular Cylinder, Isothermal (UWT)

The general boundary layer correlation is

$$Nu_D = \frac{hD}{k_f} = C \left(\frac{g\beta(T_w - T_\infty)D^3}{\nu^2} \right)^{1/4} \left(\frac{\nu}{\alpha} \right)^{1/4} = C \underbrace{Gr_D^{1/4} Pr^{1/4}}_{Ra_D^{1/4}}$$

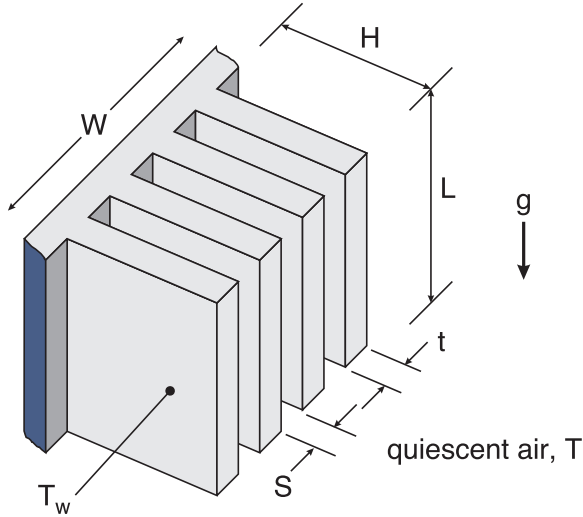
where

$$Ra_D = Gr_D Pr = \frac{g\beta(T_w - T_\infty)D^3}{\alpha\nu}$$

Natural Convection From Plate Fin Heat Sinks

The average Nusselt number for an isothermal plate fin heat sink with natural convection can be determined using

$$Nu_S = \frac{hS}{k_f} = \left[\frac{576}{(Ra_S S/L)^2} + \frac{2.873}{(Ra_S S/L)^{0.5}} \right]^{-0.5}$$



Two factors must be considered in the selection of the number of fins

- more fins results in added surface area and reduced boundary layer resistance,

$$R \downarrow = \frac{1}{hA \uparrow}$$

- more fins leads to a decrease fin spacing and a decrease in the heat transfer coefficient

$$R \uparrow = \frac{1}{h \downarrow A}$$

A basic optimization of the fin spacing can be obtained as follows:

$$\dot{Q} = hA(T_w - T_\infty)$$

where the fins are assumed to be isothermal and the surface area is $2nHL$, with the area of the fin edges ignored.

For isothermal fins with $t < S$

$$S_{opt} = 2.714 \left(\frac{L}{Ra_L^{1/4}} \right)$$

with

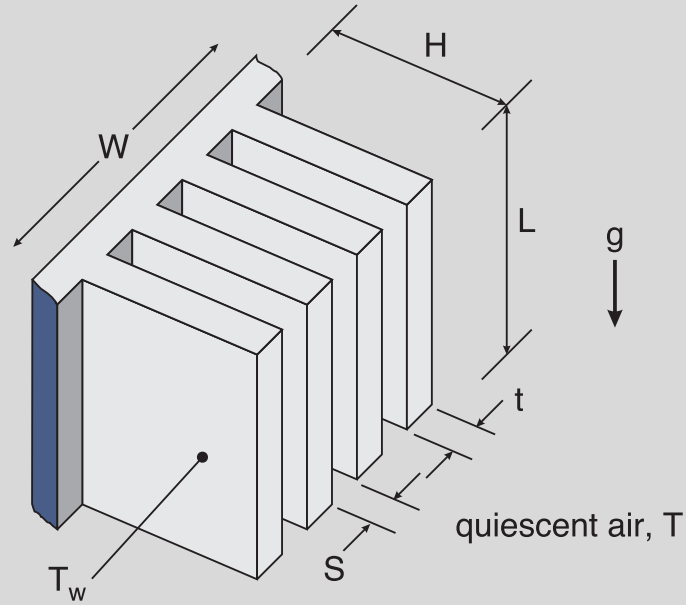
$$Ra_L = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} Pr$$

The corresponding value of the heat transfer coefficient is

$$h = 1.307k_f/S_{opt}$$

All fluid properties are evaluated at the film temperature.

Example 6-3: Find the optimum fin spacing, S_{opt} and the rate of heat transfer, \dot{Q} for the following plate fin heat sink cooled by natural convection.



Given:

$W = 120 \text{ mm}$	$H = 24 \text{ mm}$
$L = 18 \text{ mm}$	$t = 1 \text{ mm}$
$T_w = 80 \text{ }^\circ\text{C}$	$T_\infty = 25 \text{ }^\circ\text{C}$
$P_\infty = 1 \text{ atm}$	fluid = air

Find: S_{opt} and the corresponding heat transfer, \dot{Q}

Radiation Heat Transfer



Reading

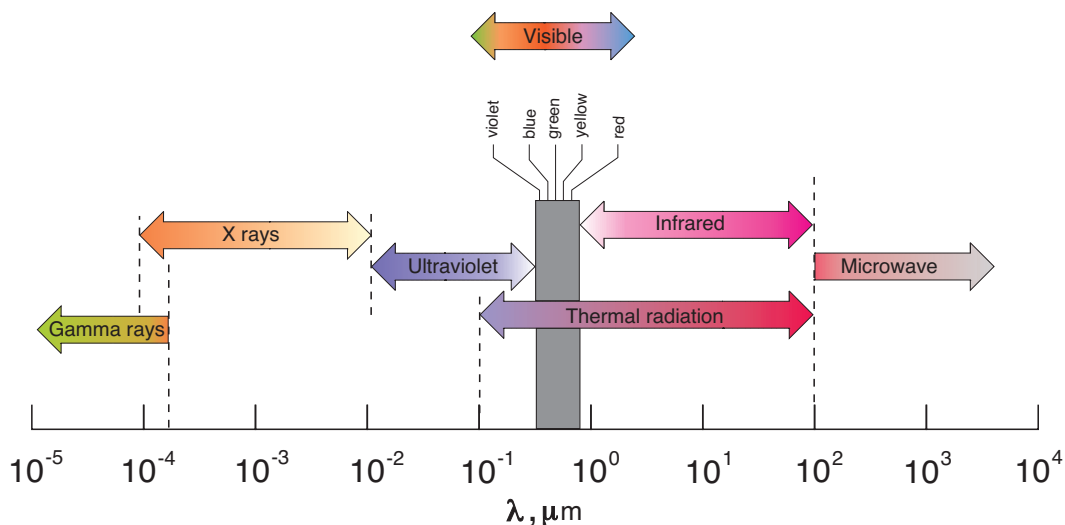
15-1 → 15-7

Problems

15-27, 15-33, 15-50, 15-57, 15-77, 15-79,
15-96, 15-107, 15-108

Introduction

A narrower band inside the thermal radiation spectrum is denoted as the visible spectrum, that is the thermal radiation that can be seen by the human eye. The visible spectrum occupies roughly $0.4 - 0.7 \mu\text{m}$. Thermal radiation is mostly in the infrared range. As objects heat up, their energy level increases, their frequency, ν , increases and the wavelength of the emitted radiation decreases.



Blackbody Radiation

A blackbody is an ideal radiator that *absorbs* all incident radiation regardless of wavelength and direction.

Definitions

1. **Blackbody emissive power:** the radiation emitted by a blackbody per unit time and per unit surface area

$$E_b = \sigma T^4 \quad [W/m^2] \quad \Leftarrow \text{Stefan-Boltzmann law}$$

where Stefan-Boltzmann constant $= 5.67 \times 10^{-8} W/(m^2 \cdot K^4)$ and the temperature T is given in K .

2. **Spectral blackbody emissive power:** the amount of radiation energy emitted by a blackbody per unit surface area and per unit wavelength about the wavelength λ . The following relationship between emissive power, temperature and wavelength is known as *Planck's distribution law*

$$E_{b,\lambda} = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad [W/(m^2 \cdot \mu m)]$$

where

$$C_1 = 2\pi h C_0^2 = 3.74177 \times 10^8 [W \cdot \mu m^4/m^2]$$

$$C_2 = h C_0 / K = 1.43878 \times 10^4 [\mu m \cdot K]$$

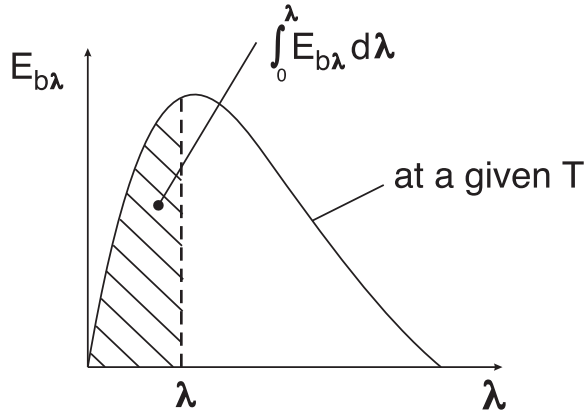
The wavelength at which the peak emissive power occurs for a given temperature can be obtained from *Wien's displacement law*

$$(\lambda T)_{max \text{ power}} = 2897.8 \mu m \cdot K$$

3. **Blackbody radiation function:** the fraction of radiation emitted from a blackbody at temperature, T in the wavelength band $\lambda = 0 \rightarrow \lambda$

$$f_{0 \rightarrow \lambda} = \frac{\int_0^\lambda E_{b,\lambda}(T) d\lambda}{\int_0^\infty E_{b,\lambda}(T) d\lambda} = \frac{\int_0^\lambda \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} d\lambda}{\sigma T^4}$$

let $t = \lambda T$ and $dt = T d\lambda$, then



$$\begin{aligned} f_{0 \rightarrow \lambda} &= \frac{\int_0^t \frac{C_1 T^5 (1/T) dt}{t^5 [\exp(C_2/t) - 1]}}{\sigma T^4} \\ &= \frac{C_1}{\sigma} \int_0^{\lambda T} \frac{dt}{t^5 [\exp(C_2/t) - 1]} \\ &= f(\lambda T) \end{aligned}$$

$f_{0 \rightarrow \lambda}$ is tabulated as a function λT in Table 15.2

We can easily find the fraction of radiation emitted by a blackbody at temperature T over a discrete wavelength band as

$$f_{\lambda_1 \rightarrow \lambda_2} = f(\lambda_2 T) - f(\lambda_1 T)$$

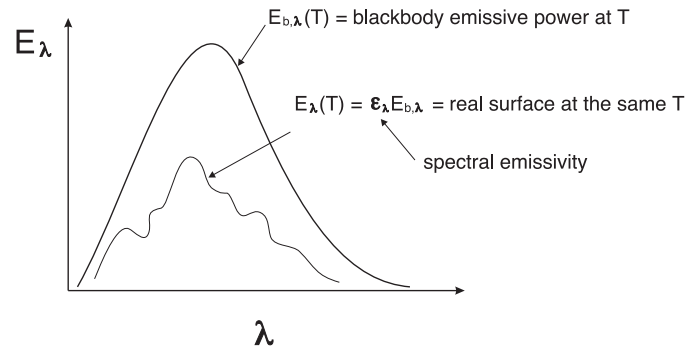
$$f_{\lambda \rightarrow \infty} = 1 - f_{0 \rightarrow \lambda}$$

Radiation Properties of Real Surfaces

The thermal radiation emitted by a real surface is a function of surface temperature, T , wavelength, λ , direction and surface properties.

$$E_{\lambda} = f(T, \lambda, \text{direction, surface properties})$$

\Rightarrow spectral emissive power



while for a blackbody, the radiation was only a function of temperature and wavelength

$$E_{b,\lambda} = f(T, \lambda) \rightarrow \text{diffuse emitter} \Rightarrow \text{independent of direction}$$

Definitions

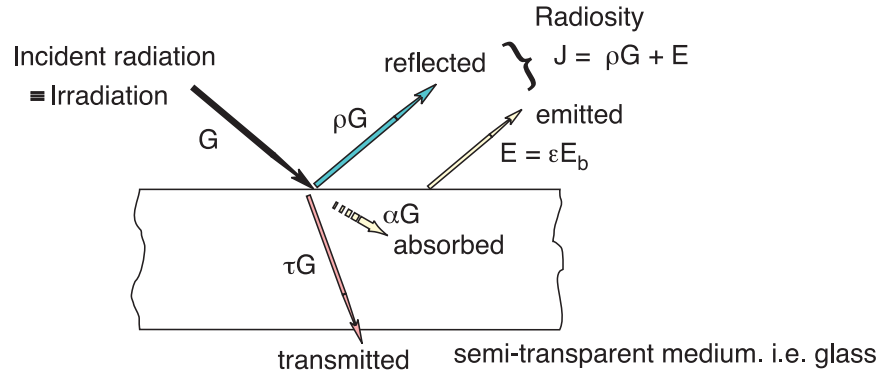
1. **Diffuse surface:** properties are independent of direction.
2. **Gray surface:** properties are independent of wavelength.
3. **Emissivity:** defined as the ratio of radiation emitted by a surface to the radiation emitted by a blackbody at the same surface temperature.

$$\begin{aligned} \epsilon(T) &= \frac{\text{radiation emitted by surface at temperature } T}{\text{radiation emitted by a black surface at } T} \\ &= \frac{\int_0^{\infty} E_{\lambda}(T) d\lambda}{\int_0^{\infty} E_{b\lambda}(T) d\lambda} = \frac{\int_0^{\infty} \epsilon_{\lambda}(T) E_{b\lambda}(T) d\lambda}{E_b(T)} = \frac{E(T)}{\sigma T^4} \end{aligned}$$

where ϵ changes rather quickly with surface temperature.

Typical Emissivity Values	
metal (polished)	$\epsilon \approx 0.1$
metal (oxidized)	$\epsilon \approx 0.3 - 0.4$
skin	$\epsilon \approx 0.9$
graphite	$\epsilon \approx 0.95$

4. **Irradiation, G :** the radiation energy incident on a surface per unit area and per unit time



An energy balance based on incident radiation gives

$$G = \rho G + \alpha G + \tau G$$

where

$$\left. \begin{array}{l} \rho = \text{reflectivity} \\ \alpha = \text{absorptivity} \\ \tau = \text{transmissivity} \end{array} \right\} \Rightarrow \text{function of } \lambda \text{ \& } T \text{ of the incident radiation } G$$

$$\epsilon = \text{emissivity} \Rightarrow \text{function of } \lambda \text{ \& } T \text{ of the emitting surface}$$

If we normalize with respect to the total irradiation

$$\alpha + \rho + \tau = 1$$

In general $\epsilon \neq \alpha$. However, for a diffuse-gray surface (properties are independent of wavelength and direction)

$$\epsilon = \alpha \quad \text{diffuse-gray surface}$$

5. **Radiosity, J :** the total radiation energy leaving a surface per unit area and per unit time.

For a surface that is gray and opaque, i.e. $\epsilon = \alpha$ and $\alpha + \rho = 1$, the radiosity is given as

$$\begin{aligned} J &= \text{radiation emitted by the surface} + \text{radiation reflected by the surface} \\ &= \epsilon E_b + \rho G \\ &= \epsilon \sigma T^4 + \rho G \end{aligned}$$

Since $\rho = 0$ for a blackbody, the radiosity of a blackbody is

$$J = \sigma T^4$$

Diffuse-Gray Surfaces, $\epsilon = \alpha$

Kirchhoff's Law

The absorptivity, $\alpha(\lambda, T, \text{direction})$ of a non-black surface is always equal to the emissivity, $\epsilon(\lambda, T, \text{direction})$ of the same surface when the surface is in thermal equilibrium with the radiation that impinges on it.

$$\epsilon(\lambda, T, \phi, \theta) = \alpha(\lambda, T, \phi, \theta)$$

To a lesser degree of certainty we can write a more restrictive form of Kirchhoff's law for diffuse-gray surfaces where

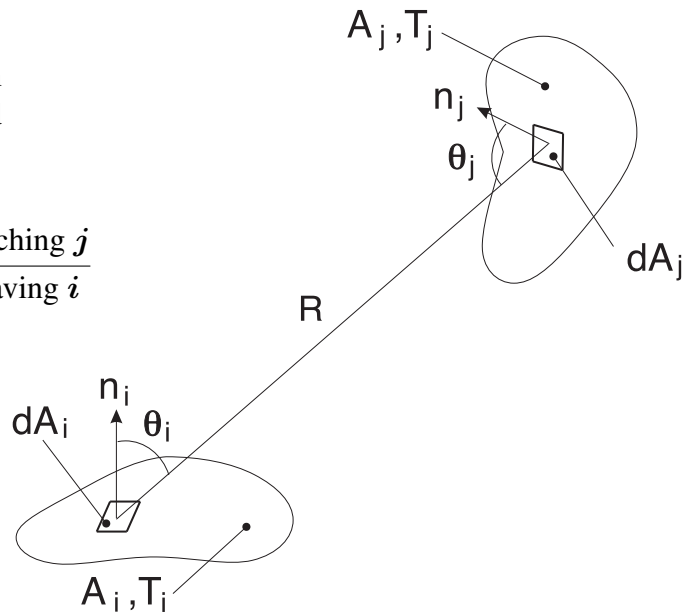
$$\epsilon(T) = \alpha(T)$$

While Kirchhoff's law requires that the radiant source and the surface be in thermal equilibrium, this is seldom the case. The law can still be used but you should proceed with caution when the two temperatures differ by more than 100 K.

View Factor (Shape Factor, Configuration Factor)

- **Definition:** The view factor, $F_{i \rightarrow j}$ is defined as the fraction of radiation leaving surface i which is intercepted by surface j . Hence

$$F_{i \rightarrow j} = \frac{\dot{Q}_{i \rightarrow j}}{A_i J_i} = \frac{\text{radiation reaching } j}{\text{radiation leaving } i}$$



$$F_{i \rightarrow j} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_j dA_i$$

$$F_{j \rightarrow i} = \frac{1}{A_j} \int_{A_j} \int_{A_i} \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i dA_j$$

View Factor Relations

Reciprocity Relation

The last two equations show that

$$A_i F_{i \rightarrow j} = A_j F_{j \rightarrow i}$$

Summation Relation

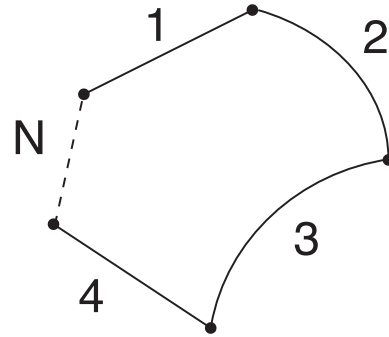
$$A_1 J_1 = \dot{Q}_{1 \rightarrow 1} + \dot{Q}_{1 \rightarrow 2} + \dots + \dot{Q}_{1 \rightarrow N}$$

Therefore

$$1 = \sum_{j=1}^N \left(\frac{\dot{Q}_{i \rightarrow j}}{A_i J_i} \right) = \sum_{j=1}^N F_{i \rightarrow j}$$

Hence

$$\sum_{j=1}^N F_{i \rightarrow j} = 1 \quad ; \quad i = 1, 2, \dots, N$$

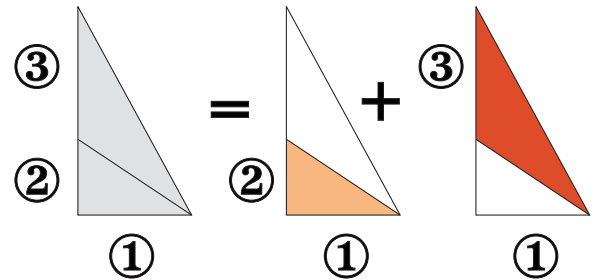


Note that $F_{i \rightarrow i} \neq 0$ for a concave surface. For a plane or convex surface $F_{i \rightarrow i} = 0$.

Superposition Relation

If the surface is not available in the tables sometimes it can be treated as the sum of smaller known surfaces to form the full extent of the surface of interest.

$$F_{1 \rightarrow (2,3)} = F_{1 \rightarrow 2} + F_{1 \rightarrow 3}$$

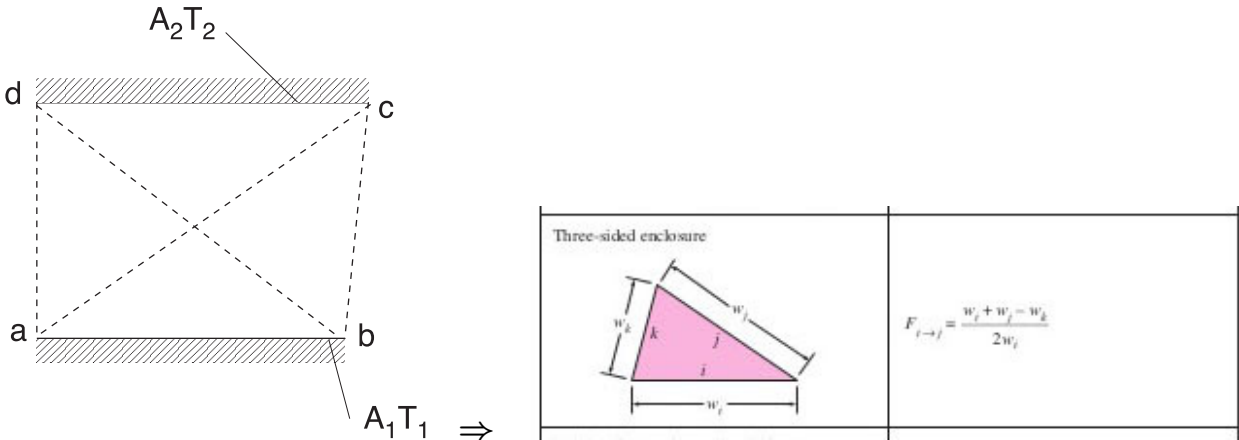


Symmetry Relation

If the problem is symmetric, then the view factors will also be symmetric.

Hottel Crossed String Method

Can be applied to 2D problems where surfaces are any shape, flat, concave or convex. Note for a 2D surface the area, A is given as a length times a unit width.



$$A_1 F_{12} = A_2 F_{21} = \frac{(\text{total crossed}) - (\text{total uncrossed})}{2}$$

A_1 and A_2 do not have to be parallel

$$A_1 F_{12} = A_2 F_{21} = \frac{1}{2} [\underbrace{(ac + bd)}_{\text{crossed}} - \underbrace{(bc + ad)}_{\text{uncrossed}}]$$

Radiation Exchange Between Surfaces

In general, radiation exchange between surfaces should include:

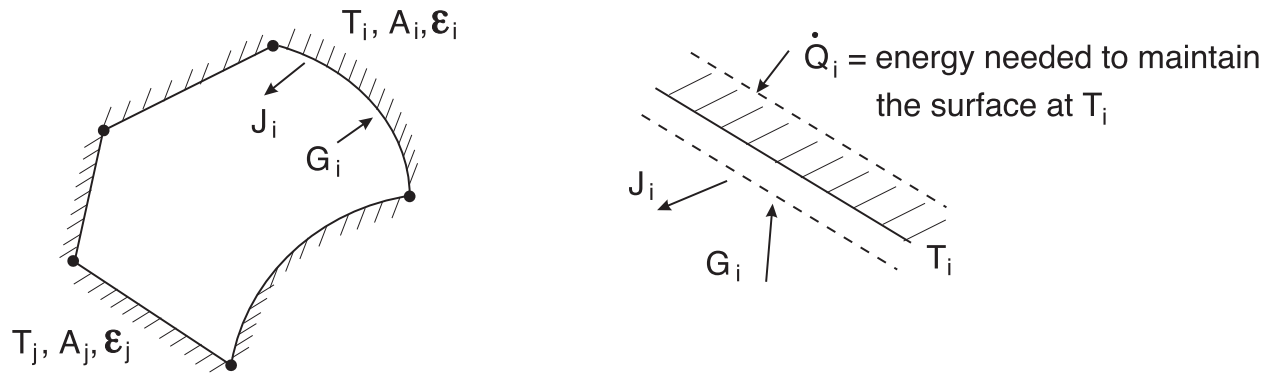
- irradiation of each surface accounting for all energy reflected from other surfaces
- multiple reflections may occur before all energy is absorbed

Diffuse-Gray Surfaces Forming an Enclosure

To help simplify radiation analyses in diffuse, gray enclosures we will assume

1. each surface of the enclosure is isothermal

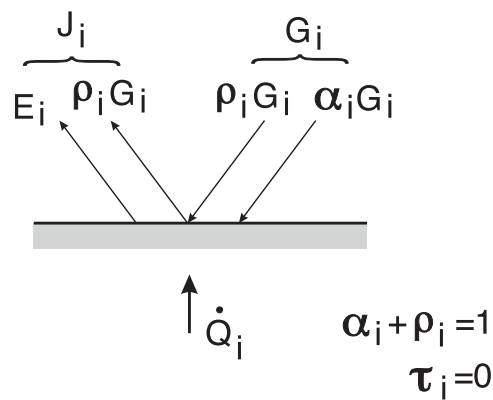
2. radiosity, J_i , and irradiation, G_i are uniform over each surface
3. the surfaces are opaque ($\tau_i = 0$) and diffuse-gray ($\alpha_i = \epsilon_i$)
4. the cavity is filled with a fluid which does not participate in the radiative exchange process



Radiation Heat Transfer To or From a Surface

- an energy balance on the i 'th surface gives:

$$\dot{Q}_i = \dot{q}_i A_i = A_i (J_i - G_i)$$



recasting the energy balance:

$$\dot{Q}_i = A_i \left[\underbrace{(E_i + \rho_i G_i)}_{J_i} - \underbrace{(\rho_i G_i + \alpha_i G_i)}_{G_i} \right] = A_i (E_i - \alpha_i G_i) \quad (1)$$

where:

$$J_i = E_i + \rho_i G_i \quad (2) \Rightarrow \text{radiosity}$$

$$E_i = \epsilon_i E_{b,i} = \epsilon_i \sigma T_i^4 \quad (3) \Rightarrow \text{emissive power}$$

$$\rho_i = 1 - \alpha_i = 1 - \epsilon_i \quad (4) \Rightarrow \text{since } \alpha_i + \rho_i + \tau_i = 1$$

and $\alpha_i = \epsilon_i$

Combining Eqs. 2, 3 and 4 gives

$$J_i = \epsilon_i E_{b,i} + (1 - \epsilon_i) G_i \quad (5)$$

Combining this with Eq. 1 gives the net radiation heat transfer to or from surface “i”

$$Q_i = \frac{E_{b,i} - J_i}{\left(\frac{1 - \epsilon_i}{\epsilon_i A_i} \right)} \equiv \frac{\text{potential difference}}{\text{surface resistance}}$$

this surface resistance represents real surface behavior

Note: for a black surface

$$\epsilon_i = \alpha_i = 1$$

and Eq. 5 becomes

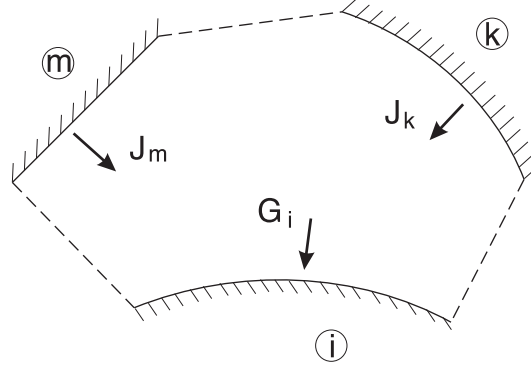
$$J_i = E_{b,i} = \sigma T_i^4 \Leftarrow$$

Radiation Heat Transfer Between Surfaces

- by inspection it is clearly seen that

$$\left\{ \begin{array}{c} \text{irradiation on} \\ \text{surface } i \end{array} \right\} = \left\{ \begin{array}{c} \text{radiation leaving the} \\ \text{remaining surfaces} \end{array} \right\}$$

$$A_i G_i = \sum_{j=1}^N F_{j \rightarrow i} (A_j J_j) = \sum_{j=1}^N A_i F_{i \rightarrow j} J_i \Leftarrow (\text{from reciprocity})$$



Therefore

$$G_i = \sum_{j=1}^N F_{i \rightarrow j} J_j$$

Combining this with Eq. 5 gives

$$J_i = \underbrace{\epsilon_i \sigma T_i^4}_{E_{b,i}} + (1 - \epsilon_i) \sum_{j=1}^N F_{i \rightarrow j} J_j$$

In addition by performing an energy balance at surface “i”, we can write

$$\begin{aligned} \dot{Q}_i &= \text{energy out} - \text{energy in} \\ &= A_i J_i - \sum_{j=1}^N A_i F_{i \rightarrow j} J_j \end{aligned}$$

Since the summation rule states $\sum_{j=1}^N F_{i \rightarrow j} = 1$, the above equation becomes

$$\begin{aligned} \dot{Q}_i &= A_i \left\{ \underbrace{\sum_{j=1}^N F_{i \rightarrow j} J_i}_{\equiv 1} - \sum_{j=1}^N F_{i \rightarrow j} J_j \right\} \\ \dot{Q}_i &= \sum_{j=1}^N A_i F_{i \rightarrow j} (J_i - J_j) \end{aligned}$$

or

$$\dot{Q}_i = \sum_{j=1}^N \frac{J_i - J_j}{\left(\frac{1}{A_i F_{i \rightarrow j}} \right)} \equiv \frac{\text{potential difference}}{\text{space resistance}}$$

- the space resistance can be used for any gray, diffuse and opaque surfaces that form an enclosure

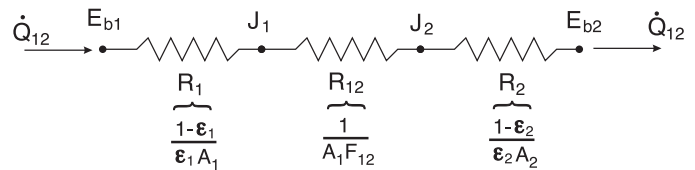
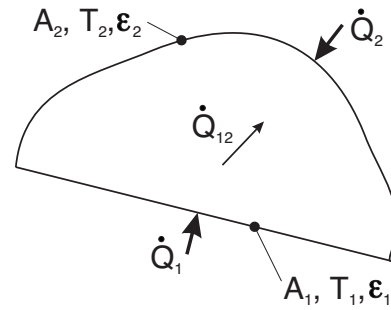
Radiation Exchange in Enclosures

The Two-Surface Enclosure

- radiation from surface 1 must equal radiation to surface 2

$$\dot{Q}_1 = -\dot{Q}_2 = \dot{Q}_{12}$$

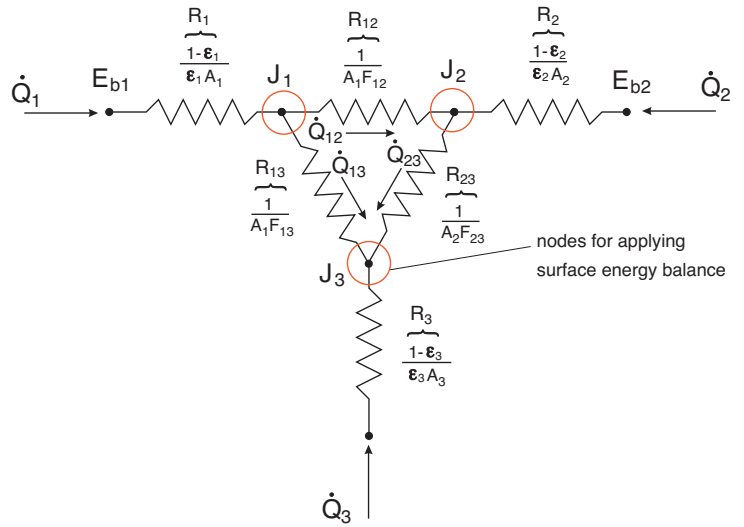
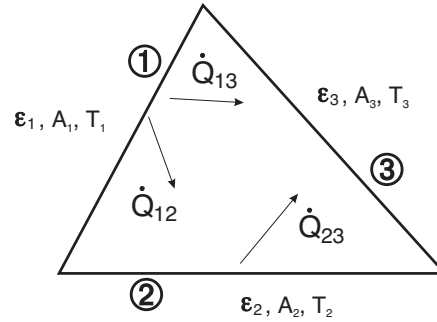
- the resistor network will consist of 2 surface resistances and 1 space resistance
- the net radiation exchange can be determined as follows:



$$\dot{Q}_{12} = \frac{E_{b,1} - E_{b,2}}{R_{total}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1 - \epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1 - \epsilon_2}{\epsilon_2 A_2}}$$

The Three-Surface Enclosure

- radiative heat transfer between all combinations of surfaces must be accounted for
- the resistor network will consist of 3 surface resistances and 3 space resistances
- leads to a system of 3 equations in 3 unknowns
- the algebraic sum of the currents (net radiation transfer) at each node must equal zero. Note: this assumes all heat flow is into the node.
- if the assumed direction of current flow is incorrect, you will get a -ve value of \dot{Q}



Performing an energy balance at each node:

$$\text{at } J_1 \Rightarrow \dot{Q}_1 + \dot{Q}_{12} + \dot{Q}_{13} = 0$$

$$\frac{E_{b1} - J_1}{\frac{1 - \epsilon_1}{\epsilon_1 A_1}} + \frac{J_2 - J_1}{\frac{1}{A_1 F_{12}}} + \frac{J_3 - J_1}{\frac{1}{A_1 F_{13}}} = 0 \quad (5)$$

$$\text{at } J_2 \Rightarrow \dot{Q}_{12} + \dot{Q}_2 + \dot{Q}_{23} = 0$$

$$\frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{E_{b2} - J_2}{\frac{1 - \epsilon_1}{\epsilon_1 A_1}} + \frac{J_3 - J_2}{\frac{1}{A_2 F_{23}}} = 0 \quad (6)$$

$$\text{at } J_3 \Rightarrow \dot{Q}_{13} + \dot{Q}_{23} + \dot{Q}_3 = 0$$

$$\frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}} + \frac{J_2 - J_3}{\frac{1}{A_2 F_{23}}} + \frac{E_{b3} - J_3}{\frac{1 - \epsilon_1}{\epsilon_1 A_1}} = 0 \quad (7)$$

if surface temperature is known

- given T_i , evaluate $E_{bi} = \sigma T_i^4$
- evaluate all space and surface resistances
- solve for J_1 , J_2 and J_3
- determine the heat flow rate as

$$\dot{Q}_i = A_i \sum F_{ij}(J_i - J_j)$$

if surface heat flow rate is known

- replace \dot{Q}_1 , \dot{Q}_2 and/or \dot{Q}_3 in Eqs. 1-3
- solve for J_1 , J_2 and J_3
- determine the surface temperature as

$$\sigma T_i^4 = J_i + \frac{1 - \epsilon_i}{\epsilon_i} \sum F_{ij}(J_i - J_j)$$

Special Cases

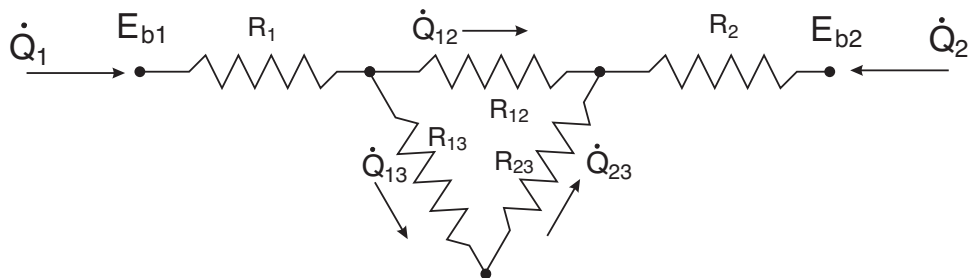
The system of equations for 2 and 3-surface enclosures can simplify further when one or more surfaces are: i) blackbody surfaces or ii) reradiating (fully insulated) surfaces.

blackbody surface: for a blackbody surface $\epsilon = 1$ and the surface resistance goes to zero. As a consequence the radiosity can be calculated directly as a function of surface temperature

$$J_i = E_{bi} = \sigma T_i^4$$

reradiating surface: $\dot{Q}_i = 0$, therefore the heat flow into the radiosity node equals the heat flow out of the node.

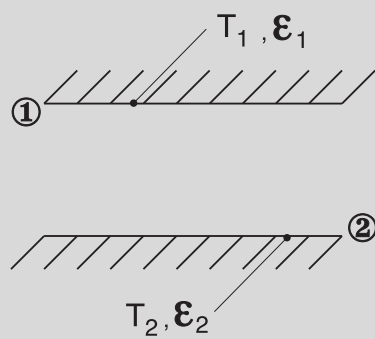
The resistor network simplifies to:



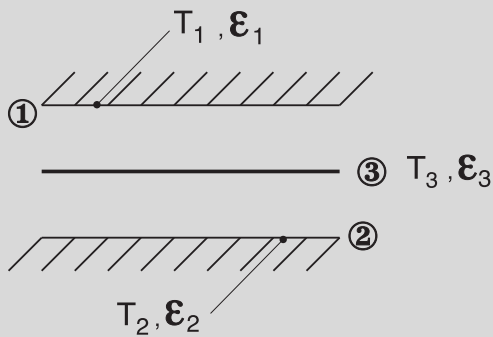
and the system of equations can be easily solved as:

$$\dot{Q}_1 = -\dot{Q}_2 = \frac{E_{b1} - E_{b2}}{R_1 + \underbrace{\left[\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}} \right]^{-1}}_{\text{parallel resistance}} + R_2}$$

Example 7-1: Consider two very large parallel plates with diffuse, gray surfaces. Determine the net rate of radiation heat transfer per unit surface area, \dot{Q}_{12}/A , between the two surfaces. For Case 2, also determine T_3 , the temperature of a radiation shield, positioned midway between surfaces 1 and 2.



CASE 1



CASE 2

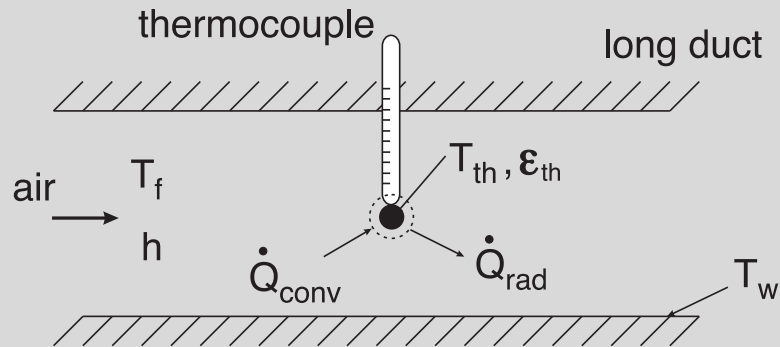
Given:

$$\begin{aligned}\epsilon_1 &= 0.2 \\ \epsilon_2 &= 0.7 \\ \epsilon_3 &= 0.02\end{aligned}$$

$$\begin{aligned}T_1 &= 800 \text{ K} \\ T_2 &= 500 \text{ K} \\ A_1 &= A_2 = A_3 = A\end{aligned}$$

Assume steady state conditions.

Example 7-2: A thermocouple is suspended between two parallel surfaces as shown in the figure below. Find T_f , the temperature of the air stream by performing an energy balance on the thermocouple.



Given:

$$T_w = 400 \text{ K}$$

$$\epsilon_{th} = 0.6$$

$$T_{th} = 650 \text{ K}$$

$$h = 80 \text{ W}/(\text{m}^2 \cdot \text{K})$$

Assume steady state conditions.

Example 7-3: Consider a room that is 4 m long by 3 m wide with a floor-to-ceiling distance of 2.5 m. The four walls of the room are well insulated, while the surface of the floor is maintained at a uniform temperature of 30 °C using an electric resistance heater. Heat loss occurs through the ceiling, which has a surface temperature of 12°C. All surfaces have an emissivity of 0.9.

- determine the rate of heat loss, (W), by radiation from the room.
- determine the temperature, (K), of the walls.

